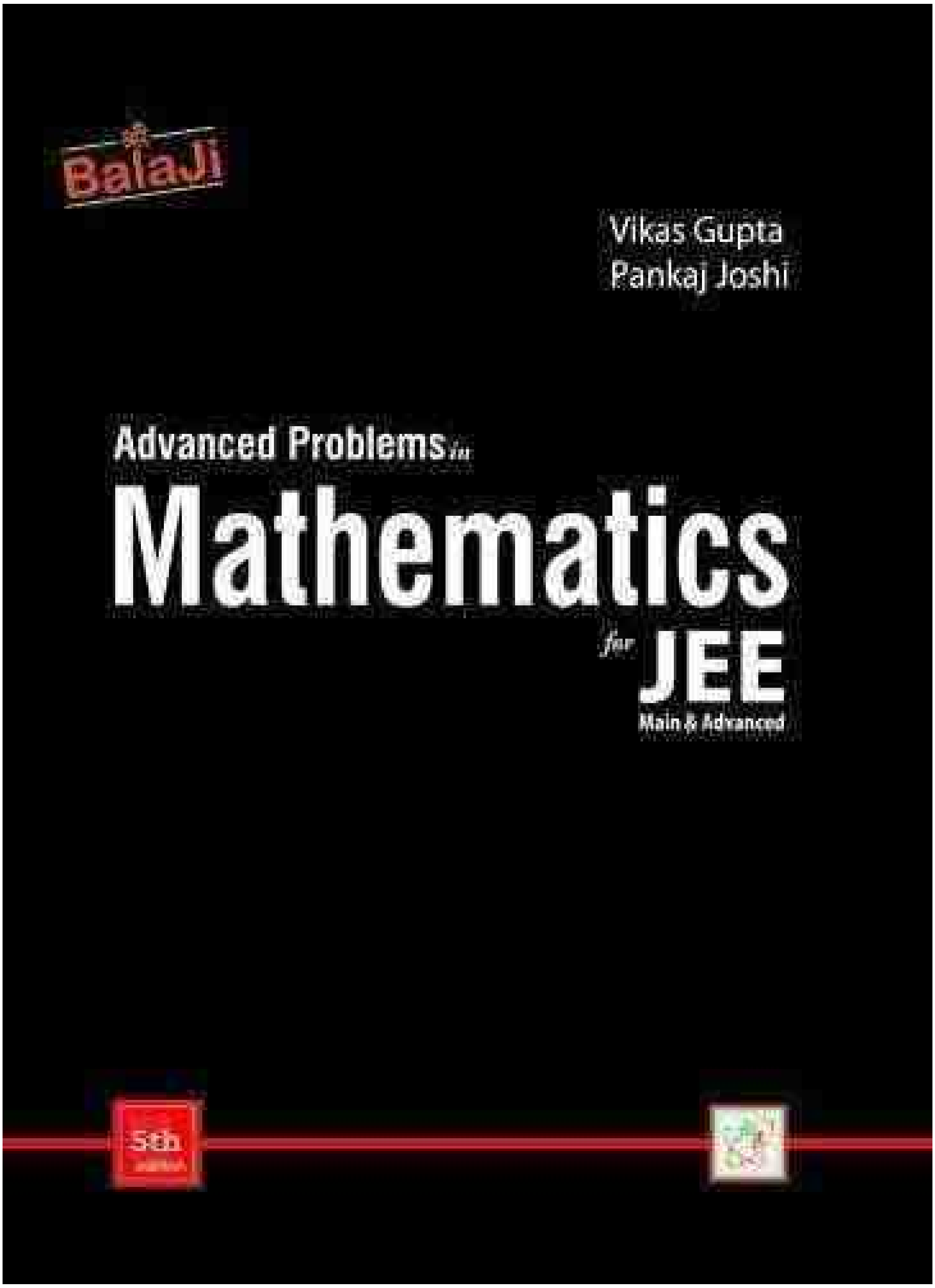
Balaji

Advanced Problems in Mathematics Chapter 1 to 9

for IIT JEE Main and Advanced

by

Vikas Gupta and Pankaj Joshi



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Advanced Problems *in*

MATHEMATICS

for

JEE (MAIN & ADVANCED)

by :

Vikas Gupta
Director
Vibrant Academy India(P) Ltd.
KOTA (Rajasthan)

Pankaj Joshi
Director
Vibrant Academy India(P) Ltd.
KOTA (Rajasthan)

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Chapter 1 – Function

1 FUNCTION

Exercise-1 : Single Choice Problems

1. Range of the function $f(x) = \log_2(2 - \log_{\sqrt{2}}(16\sin^2 x + 1))$ is :

(a) $[0, 1]$ (b) $(-\infty, 1]$ (c) $[-1, 1]$ (d) $(-\infty, \infty)$

2. The value of a and b for which $|e^{|x-b|} - a| = 2$, has four distinct solutions, are :

(a) $a \in (-3, \infty), b = 0$ (b) $a \in (2, \infty), b = 0$ (c) $a \in (3, \infty), b \in R$ (d) $a \in (2, \infty), b = a$

3. The range of the function :

$$f(x) = \tan^{-1} x + \frac{1}{2}\sin^{-1} x$$

(a) $(-\pi/2, \pi/2)$ (b) $[-\pi/2, \pi/2] - \{0\}$ (c) $[-\pi/2, \pi/2]$ (d) $(-3\pi/4, 3\pi/4)$

4. Find the number of real ordered pair(s) (x, y) for which :

$16^{x^2+y} + 16^{x+y^2} = 1$

(a) 0 (b) 1 (c) 2 (d) 3

5. The complete range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|} = x^2 - a$ is satisfied for maximum number of values of x is :

(a) $(-\infty, -1)$ (b) $(-\infty, \infty)$ (c) $(-1, 1)$ (d) $(-1, \infty)$

6. For a real number x, let $[x]$ denotes the greatest integer less than or equal to x. Let $f : R \to R$ be defined by $f(x) = 2x + [x] + \sin x \cos x$. Then f is :

(a) One-one but not onto (b) Onto but not one-one

(c) Both one-one and onto (d) Neither one-one nor onto

7. The maximum value of $\sec^{-1}\left(\frac{7 - 5(x^2 + 3)}{2(x^2 + 2)}\right)$ is :

(a) $\frac{5\pi}{6}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{2\pi}{3}$

8. Number of ordered pair (a, b) from the set $A = \{1, 2, 3, 4, 5\}$ so that the function $f(x) = \frac{x^3}{3} + \frac{a}{2}x^2 + bx + 10$ is an injective mapping $\forall\, x \in R$:

(a) 13 (b) 14 (c) 15 (d) 16

9. Let A be the greatest value of the function $f(x) = \log_x [x]$, (where $[\cdot]$ denotes greatest integer function) and B be the least value of the function $g(x) = |\sin x| + |\cos x|$, then :

(a) $A > B$ (b) $A < B$ (c) $A = B$ (d) $2A + B = 4$

10. Let $A = [a, \infty)$ denotes domain, then $f : [a, \infty) \to B$, $f(x) = 2x^3 - 3x^2 + 6$ will have an inverse for the smallest real value of a, if :

(a) $a = 1, B = [5, \infty)$ (b) $a = 2, B = [10, \infty)$ (c) $a = 0, B = [6, \infty)$ (d) $a = -1, B = [1, \infty)$

11. Solution of the inequation $\{x\}(\{x\} - 1)(\{x\} + 2) \geq 0$

(where $\{\cdot\}$ denotes fractional part function) is :

(a) $x \in (-2, 1)$ (b) $x \in I$ (I denote set of integers)

(c) $x \in [0, 1)$ (d) $x \in [-2, 0)$

12. Let $f(x), g(x)$ be two real valued functions then the function $h(x) = 2\max\{f(x) - g(x), 0\}$ is equal to :

(a) $f(x) - g(x) - |g(x) - f(x)|$ (b) $f(x) + g(x) - |g(x) - f(x)|$

(c) $f(x) - g(x) + |g(x) - f(x)|$ (d) $f(x) + g(x) + |g(x) - f(x)|$

13. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is :

(a) a function (b) reflexive (c) not symmetric (d) transitive

14. The true set of values of 'K' for which $\sin^{-1}\left(\frac{1}{1 + \sin^2 x}\right) = \frac{K\pi}{6}$ may have a solution is :

(a) $\left[\frac{1}{4}, \frac{1}{2}\right]$ (b) $[1, 3]$ (c) $\left[\frac{1}{6}, \frac{1}{2}\right]$ (d) $[2, 4]$

15. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where 'a' is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to :

(a) $-f(x)$ (b) $f(x)$ (c) $f(a) + f(a - x)$ (d) $f(-x)$

16. Let $g : R \to R$ be given by $g(x) = 3 + 4x$ if $g^n(x) = gogogo \ldots\ldots og(x)$ n times. Then inverse of $g^n(x)$ is equal to :

(a) $(x + 1 - 4^n) \cdot 4^{-n}$ (b) $(x - 1 + 4^n)4^{-n}$ (c) $(x + 1 + 4^n)4^{-n}$ (d) None of these

17. Let $f : D \to R$ be defined as : $f(x) = \frac{x^2 + 2x + a}{x^2 + 4x + 3a}$ where D and R denote the domain of f and the set of all real numbers respectively. If f is surjective mapping, then the complete range of a is :

(a) $0 \leq a \leq 1$ (b) $0 < a \leq 1$ (c) $0 \leq a < 1$ (d) $0 < a < 1$

18. If $f:(-\infty, 2] \longrightarrow (-\infty, 4]$, where $f(x) = x(4-x)$, then $f^{-1}(x)$ is given by :

(a) $2-\sqrt{4-x}$ (b) $2+\sqrt{4-x}$ (c) $-2+\sqrt{4-x}$ (d) $-2-\sqrt{4-x}$

19. If $[5\sin x]+[\cos x]+6=0$, then range of $f(x)=\sqrt{3}\cos x+\sin x$ corresponding to solution set of the given equation is : (where $[\cdot]$ denotes greatest integer function)

(a) $[-2,-1)$ (b) $\left(-\frac{3\sqrt{3}+2}{5},-1\right)$ (c) $[-2,-\sqrt{3})$ (d) $\left(-\frac{3\sqrt{3}+4}{5},-1\right)$

20. If $f:R\to R$, $f(x)=ax+\cos x$ is an invertible function, then complete set of values of a is :

(a) $(-2,-1]\cup[1,2)$ (b) $[-1,1]$ (c) $(-\infty,-1]\cup[1,\infty)$ (d) $(-\infty,-2]\cup[2,\infty)$

21. The range of function $f(x)=[1+\sin x]+\left[2+\sin\frac{x}{2}\right]+\left[3+\sin\frac{x}{3}\right]+\ldots+\left[n+\sin\frac{x}{n}\right]\ \forall\ x\in[0,\pi]$, $n\in N$ ($[\cdot]$ denotes greatest integer function) is :

(a) $\left\{\frac{n^2+n-2}{2},\frac{n(n+1)}{2}\right\}$ (b) $\left\{\frac{n(n+1)}{2}\right\}$

(c) $\left\{\frac{n(n+1)}{2},\frac{n^2+n+2}{2},\frac{n^2+n+4}{2}\right\}$ (d) $\left\{\frac{n(n+1)}{2},\frac{n^2+n+2}{2}\right\}$

22. If $f:R\to R$, $f(x)=\frac{x^2+ax+1}{x^2+x+1}$, then the complete set of values of 'a' such that $f(x)$ is onto is :

(a) $(-\infty,\infty)$ (b) $(-\infty,0)$ (c) $(0,\infty)$ (d) not possible

23. If $f(x)$ and $g(x)$ are two functions such that $f(x)=[x]+[-x]$ and $g(x)=\{x\}\ \forall\ x\in R$ and $h(x)=f(g(x))$; then which of the following is incorrect ?

($[\cdot]$ denotes greatest integer function and $\{\cdot\}$ denotes fractional part function)

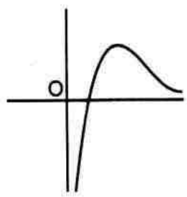
(a) $f(x)$ and $h(x)$ are identical functions (b) $f(x)=g(x)$ has no solution

(c) $f(x)+h(x)>0$ has no solution (d) $f(x)-h(x)$ is a periodic function

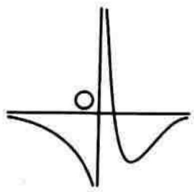
24. Number of elements in the range set of $f(x)=\left[\frac{x}{15}\right]\left[-\frac{15}{x}\right]\ \forall\ x\in(0,90)$; (where $[\cdot]$ denotes greatest integer function) :

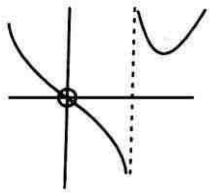
(a) 5 (b) 6 (c) 7 (d) Infinite

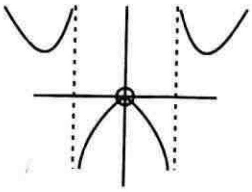
25. The graph of function $f(x)$ is shown below :

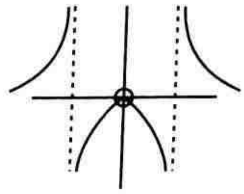


Then the graph of $g(x)=\frac{1}{f(|x|)}$ is :

(a) 

(b) 

(c) 

(d) 

26. Which of the following function is homogeneous ?

(a) $f(x) = x\sin y + y\sin x$

(b) $g(x) = xe^{\frac{y}{x}} + ye^{\frac{x}{y}}$

(c) $h(x) = \dfrac{xy}{x + y^2}$

(d) $\phi(x) = \dfrac{x - y\cos x}{y\sin x + y}$

27. Let $f(x) = \begin{bmatrix} 2x+3 & ; & x \le 1 \\ a^2x+1 & ; & x > 1 \end{bmatrix}$. If the range of $f(x) = R$ (set of real numbers) then number of integral value(s), which a may take :

(a) 2 (b) 3 (c) 4 (d) 5

28. The maximum integral value of x in the domain of $f(x) = \log_{10}(\log_{1/3}(\log_4(x-5)))$ is :

(a) 5 (b) 7 (c) 8 (d) 9

29. Range of the function $f(x) = \log_2\left(\dfrac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$ is :

(a) $(0, \infty)$ (b) $\left[\dfrac{1}{2}, 1\right]$ (c) $[1, 2]$ (d) $\left[\dfrac{1}{4}, 1\right]$

30. Number of integers statisfying the equation $|x^2 + 5x| + |x - x^2| = |6x|$ is :

(a) 3 (b) 5 (c) 7 (d) 9

31. Which of the following is not an odd function ?

(a) $\ln\left(\dfrac{x^4 + x^2 + 1}{(x^2 + x + 1)^2}\right)$

(b) $\text{sgn}(\text{sgn}(x))$

(c) $\sin(\tan x)$

(d) $f(x)$, where $f(x) + f\left(\dfrac{1}{x}\right) = f(x)\cdot f\left(\dfrac{1}{x}\right)\ \forall\, x \in R - \{0\}$ and $f(2) = 33$

32. Which of the following function is periodic with fundamental period π ?

(a) $f(x) = \cos x + \left[\left[\dfrac{\sin x}{2}\right]\right]$; where [·] denotes greatest integer function

(b) $g(x) = \dfrac{\sin x + \sin 7x}{\cos x + \cos 7x} + |\sin x|$

(c) $h(x) = \{x\} + |\cos x|$; where {·} denotes fractional part function

(d) $\phi(x) = |\cos x| + \ln(\sin x)$

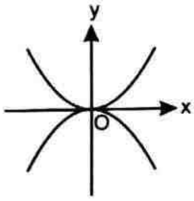
33. Let $f: N \longrightarrow Z$ and $f(x) = \begin{bmatrix} \dfrac{x-1}{2} & ; \text{ when } x \text{ is odd} \\ -\dfrac{x}{2} & ; \text{ when } x \text{ is even} \end{bmatrix}$, then :

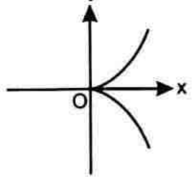
(a) $f(x)$ is bijective
(b) $f(x)$ is injective but not surjective
(c) $f(x)$ is not injective but surjective
(d) $f(x)$ is neither injective nor surjective

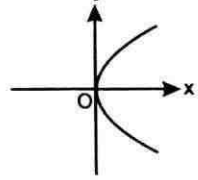
34. Let $g(x)$ be the inverse of $f(x) = \dfrac{2^{x+1} - 2^{1-x}}{2^x + 2^{-x}}$ then $g(x)$ be :

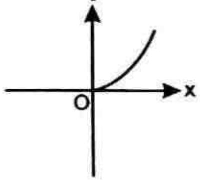
(a) $\dfrac{1}{2}\log_2\left(\dfrac{2+x}{2-x}\right)$ (b) $-\dfrac{1}{2}\log_2\left(\dfrac{2+x}{2-x}\right)$ (c) $\log_2\left(\dfrac{2+x}{2-x}\right)$ (d) $\log_2\left(\dfrac{2-x}{2+x}\right)$

35. Which of the following is the graph of the curve $\sqrt{|y|} = x$ is ?

(a) 

(b) 

(c) 

(d) 

36. Range of $f(x) = \log_{[x]}(9 - x^2)$; where [·] denotes G.I.F. is :

(a) $\{1, 2\}$ (b) $(-\infty, 2)$ (c) $(-\infty, \log_2 5]$ (d) $[\log_2 5, 3]$

37. If $e^x + e^{f(x)} = e$, then for $f(x)$:

(a) Domain is $(-\infty, 1)$ (b) Range is $(-\infty, 1]$ (c) Domain is $(-\infty, 0]$ (d) Range is $(-\infty, 0]$

38. If high voltage current is applied on the field given by the graph $y + |y| - x - |x| = 0$. On which of the following curve a person can move so that he remains safe ?

(a) $y = x^2$ (b) $y = \text{sgn}(-e^2)$ (c) $y = \log_{1/3} x$ (d) $y = m + |x|; m > 3$

39. If $|f(x) + 6 - x^2] = |f(x)| + |4 - x^2| + 2$, then $f(x)$ is necessarily non-negaive for :

(a) $x \in [-2, 2]$
(b) $x \in (-\infty, -2) \cup (2, \infty)$
(c) $x \in [-\sqrt{6}, \sqrt{6}]$
(d) $x \in [-5, -2] \cup [2, 5]$

40. Let $f(x) = \cos(px) + \sin x$ be periodic, then p must be :

(a) Positive real number
(b) Negative real number
(c) Rational
(d) Prime

41. The domain of $f(x)$ is (0, 1), therefore, the domain of $y = f(e^x) + f(\ln|x|)$ is :

(a) $\left(\frac{1}{e}, 1\right)$ (b) $(-e, -1)$ (c) $\left(-1, -\frac{1}{e}\right)$ (d) $(-e, -1) \cup (1, e)$

42. Let $A = \{1, 2, 3, 4\}$ and $f : A \to A$ satisfy $f(1) = 2, f(2) = 3, f(3) = 4, f(4) = 1$.
Suppose $g : A \to A$ satisfies $g(1) = 3$ and $fog = gof$, then $g =$

(a) $\{(1, 3), (2, 1), (3, 2), (4, 4)\}$ (b) $\{(1, 3), (2, 4), (3, 1), (4, 2)\}$

(c) $\{(1, 3), (2, 2), (3, 4), (4, 3)\}$ (d) $\{(1, 3), (2, 4), (3, 2), (4, 1)\}$

43. The number of solutions of the equation $[y + [y]] = 2\cos x$ is :
(where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ and $[\cdot]$ = greatest integer function)

(a) 0 (b) 1 (c) 2 (d) Infinite

44. The function, $f(x) = \begin{cases} \dfrac{(x^{2n})}{(x^{2n}\,\mathrm{sgn}\,x)^{2n+1}}\left(\dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}\right) & x \neq 0 \\ 1 & x = 0 \end{cases}$ $n \in N$ is :

(a) Odd function (b) Even function

(c) Neither odd nor even function (d) Constant function

45. Let $f(1) = 1$, and $f(n) = 2\sum_{r=1}^{n-1} f(r)$. Then $\sum_{r=1}^{m} f(r)$ is equal to :

(a) $\frac{3^m - 1}{2}$ (b) 3^m (c) 3^{m-1} (d) $\frac{3^{m-1} - 1}{2}$

46. Let $f(x) = \frac{x}{\sqrt{1 + x^2}}$, then $\underbrace{fofofo \ldots\ldots of}_{n\text{times}}(x)$ is :

(a) $\frac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} r\right)x^2}}$ (b) $\frac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} 1\right)x^2}}$ (c) $\left(\frac{x}{\sqrt{1 + x^2}}\right)^n$ (d) $\frac{nx}{\sqrt{1 + nx^2}}$

47. Let $f : R \to R$, $f(x) = 2x + |\cos x|$, then f is :

(a) One-one and into (b) One-one and onto

(c) Many-one and into (d) Many-one and onto

48. Let $f : R \to R$, $f(x) = x^3 + x^2 + 3x + \sin x$, then f is :

(a) One-one and into (b) One-one and onto

(c) Many-one and into (d) Many-one and onto

49. $f(x) = \{x\} + \{x + 1\} + \{x + 2\} + \ldots\ldots + \{x + 99\}$, then $[f(\sqrt{2})]$, (where $\{\cdot\}$ denotes fractional part function and $[\cdot]$ denotes the greatest integer function) is equal to :

(a) 5050 (b) 4950 (c) 41 (d) 14

50. If $|\cot x + \text{cosec}\, x| = |\cot x| + |\text{cosec}\, x|$; $x \in [0, 2\pi]$, then complete set of values of x is :

(a) $[0, \pi]$

(b) $\left(0, \frac{\pi}{2}\right]$

(c) $\left(0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right)$

(d) $\left(\pi, \frac{3\pi}{2}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$

51. The function $f(x) = 0$ has eight distinct real solution and f also satisfy $f(4+x) = f(4-x)$. The sum of all the eight solution of $f(x) = 0$ is :

(a) 12 (b) 32 (c) 16 (d) 15

52. Let $f(x)$ be a polynomial of degree 5 with leading coefficient unity such that $f(1) = 5, f(2) = 4, f(3) = 3, f(4) = 2, f(5) = 1$. Then $f(6)$ is equal to :

(a) 0 (b) 24 (c) 120 (d) 720

53. Let $f: A \to B$ be a function such that $f(x) = \sqrt{x-2} + \sqrt{4-x}$, is invertible, then which of the following is not possible ?

(a) $A = [3, 4]$ (b) $A = [2, 3]$ (c) $A = [2, 2\sqrt{3}]$ (d) $[2, 2\sqrt{2}]$

54. The number of positive integral values of x satisfying $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$ is :

(where $[\cdot]$ denotes greatest integer function)

(a) 21 (b) 22 (c) 23 (d) 24

55. The domain of function $f(x) = \log_{\left[x+\frac{1}{2}\right]}(2x^2 + x - 1)$, where $[\cdot]$ denotes the greatest integer function is :

(a) $\left[\frac{3}{2}, \infty\right)$ (b) $(2, \infty)$ (c) $\left(-\frac{1}{2}, \infty\right) - \left\{\frac{1}{2}\right\}$ (d) $\left(\frac{1}{2}, 1\right) \cup (1, \infty)$

56. The solution set of the equation $[x]^2 + [x+1] - 3 = 0$, where $[\cdot]$ represents greatest integer function is :

(a) $[-1, 0) \cup [1, 2)$ (b) $[-2, -1) \cup [1, 2)$ (c) $[1, 2)$ (d) $[-3, -2) \cup [2, 3)$

57. Which among the following relations is a function ?

(a) $x^2 + y^2 = r^2$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2$ (c) $y^2 = 4ax$ (d) $x^2 = 4ay$

(where a, b, r are constants)

58. A funciton $f: R \to R$ is defined as $f(x) = 3x^2 + 1$. Then $f^{-1}(x)$ is :

(a) $\frac{\sqrt{x-1}}{3}$

(b) $\frac{1}{3}\sqrt{x} - 1$

(c) f^{-1} does not exist

(d) $\sqrt{\frac{x-1}{3}}$

59. If $f(x)=\begin{cases}2+x, & x\ge 0\\ 4-x, & x<0\end{cases}$, then $f(f(x))$ is given by :

(a) $f(f(x))=\begin{cases}4+x, & x\ge 0\\ 6-x, & x<0\end{cases}$ (b) $f(f(x))=\begin{cases}4+x, & x\ge 0\\ x, & x<0\end{cases}$

(c) $f(f(x))=\begin{cases}4-x, & x\ge 0\\ x, & x<0\end{cases}$ (d) $f(f(x))=\begin{cases}4-2x, & x\ge 0\\ 4+2x, & x<0\end{cases}$

60. The function $f:R\to R$ defined as $f(x)=\dfrac{3x^2+3x-4}{3+3x-4x^2}$ is :

(a) One to one but not onto (b) Onto but not one to one

(c) Both one to one and onto (d) Neither one to one nor onto

61. The number of solutions of the equation $e^x-\log|x|=0$ is :

(a) 0 (b) 1 (c) 2 (d) 3

62. If complete solution set of $e^{-x}\le 4-x$ is $[\alpha,\beta]$, then $[\alpha]+[\beta]$ is equal to :

(where $[\cdot]$ denotes greatest integer function)

(a) 0 (b) 2 (c) 1 (d) 4

63. Range of $f(x)=\sqrt{\sin(\log_7(\cos(\sin x)))}$ is :

(a) $[0, 1)$ (b) $\{0, 1\}$ (c) $\{0\}$ (d) $[1, 7]$

64. If domain of $y=f(x)$ is $x\in[-3,2]$, then domain of $y=f(|[x]|)$:

(where $[\cdot]$ denotes greatest integer function)

(a) $[-3, 2]$ (b) $[-2, 3)$ (c) $[-3, 3]$ (d) $[-2, 3]$

65. Range of the function $f(x)=\cot^{-1}\{-x\}+\sin^{-1}\{x\}+\cos^{-1}\{x\}$, where $\{\cdot\}$ denotes fractional part function :

(a) $\left(\dfrac{3\pi}{4},\pi\right)$ (b) $\left[\dfrac{3\pi}{4},\pi\right)$ (c) $\left[\dfrac{3\pi}{4},\pi\right]$ (d) $\left(\dfrac{3\pi}{4},\pi\right]$

66. Let $f:R-\left\{\dfrac{3}{2}\right\}\to R$, $f(x)=\dfrac{3x+5}{2x-3}$. Let $f_1(x)=f(x)$, $f_n(x)=f(f_{n-1}(x))$ for $n\ge 2$, $n\in N$, then $f_{2008}(x)+f_{2009}(x)=$

(a) $\dfrac{2x^2+5}{2x-3}$ (b) $\dfrac{x^2+5}{2x-3}$ (c) $\dfrac{2x^2-5}{2x-3}$ (d) $\dfrac{x^2-5}{2x-3}$

67. Range of the function, $f(x)=\dfrac{(1+x+x^2)(1+x^4)}{x^3}$, for $x>0$ is :

(a) $[0,\infty)$ (b) $[2,\infty)$ (c) $[4,\infty)$ (d) $[6,\infty)$

68. The function $f:(-\infty,3]\to(0,e^7]$ defined by $f(x)=e^{x^3-3x^2-9x+2}$ is :

(a) Many-one and onto (b) Many-one and into

(c) One to one and onto (d) One to one and into

69. If $f(x) = \sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}$; $x \in R$, then range of $f(x)$ is given by :

(a) $[-1, 1]$ (b) $[0, 1]$ (c) $(-1, 1)$ (d) None of these

70. Set of values of 'a' for which the function $f: R \to R$, given by $f(x) = x^3 + (a+2)x^2 + 3ax + 10$ is one-one is given by :

(a) $(-\infty, 1] \cup [4, \infty)$ (b) $[1, 4]$ (c) $[1, \infty)$ (d) $[-\infty, 4]$

71. If the range of the function $f(x) = \tan^{-1}(3x^2 + bx + c)$ is $\left[0, \frac{\pi}{2}\right)$; (domain is R), then :

(a) $b^2 = 3c$ (b) $b^2 = 4c$ (c) $b^2 = 12c$ (d) $b^2 = 8c$

72. Let $f(x) = \sin^{-1} x - \cos^{-1} x$, then the set of values of k for which of $|f(x)| = k$ has exactly two distinct solutions is :

(a) $\left(0, \frac{\pi}{2}\right]$ (b) $\left(0, \frac{\pi}{2}\right)$ (c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (d) $\left[\pi, \frac{3\pi}{2}\right]$

73. Let $f: R \to R$ is defined by $f(x) = \begin{cases} (x+1)^3 & ; \ x \le 1 \\ \ln x + (b^2 - 3b + 10) & ; \ x > 1 \end{cases}$. If $f(x)$ is invertible, then the set of all values of 'b' is :

(a) $\{1, 2\}$ (b) ϕ (c) $\{2, 5\}$ (d) None of these

74. Let $f(x)$ is continuous function with range $[-1, 1]$ and $f(x)$ is defined $\forall\ x \in R$. If $g(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}$, then range of $g(x)$ is :

(a) $[0, 1]$ (b) $\left[0, \frac{e^2+1}{e^2-1}\right]$

(c) $\left[0, \frac{e^2-1}{e^2+1}\right]$ (d) $\left[\frac{-e^2+1}{e^2+1}, 0\right]$

75. Consider all functions $f: \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

if $f(k)$ is odd then $f(k+1)$ is even, $k = 1, 2, 3$.

The number of such functions is :

(a) 4 (b) 8 (c) 12 (d) 16

76. Consider the function $f: R - \{1\} \to R - \{2\}$ given by $f(x) = \frac{2x}{x-1}$. Then :

(a) f is one-one but not onto (b) f is onto but not one-one

(c) f is neither one-one nor onto (d) f is both one-one and onto

77. If range of function $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \frac{1}{2} f(2x+1)$ is equal to :

(a) $[-2, 4]$ (b) $[-1, 2]$ (c) $[-3, 9]$ (d) $[-2, 2]$

78. Let $f : R \to R$ and $f(x) = \frac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then $f(x)$ is :

(a) One-one, into (b) Many-one, onto
(c) One-one, onto (d) Many one, into

79. Let $f(x)$ be defined as :

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1|+|x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x))$ is :

(a) $[0, 1]$ (b) $[-1, 0]$ (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-1, 1]$

80. If $[x]^2 - 7[x] + 10 < 0$ and $4[y]^2 - 16[y] + 7 < 0$, then $[x+y]$ cannot be ([·] denotes greatest integer function) :

(a) 7 (b) 8 (c) 9 (d) both (b) and (c)

81. Let $f : R \to R$ be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$, then

(a) $f(x)$ is many one, onto function (b) $f(x)$ is many one, into function
(c) $f(x)$ is decreasing function $\forall x \in R$ (d) $f(x)$ is bijective function

82. The function $f(x)$ satisfy the equation $f(1-x) + 2f(x) = 3x \; \forall x \in R$, then $f(0) =$

(a) -2 (b) -1 (c) 0 (d) 1

83. Let $f : [0, 5] \to [0, 5]$ be an invertible function defined by $f(x) = ax^2 + bx + c$, where $a, b, c \in R$, $abc \neq 0$, then one of the root of the equation $cx^2 + bx + a = 0$ is :

(a) a (b) b (c) c (d) $a+b+c$

84. Let $f(x) = x^2 + \lambda x + \mu \cos x$, λ being an integer and μ is a real number. The number of ordered pairs (λ, μ) for which the equation $f(x) = 0$ and $f(f(x)) = 0$ have the same (non empty) set of real roots is :

(a) 2 (b) 3 (c) 4 (d) 6

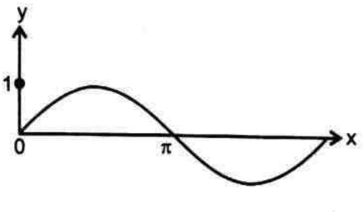
85. Consider all function $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

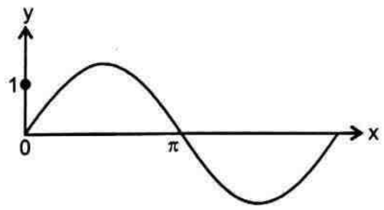
if $f(k)$ is odd then $f(k+1)$ is even, $k = 1, 2, 3$.

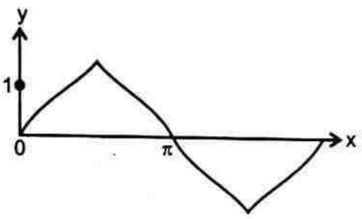
The number of such function is :

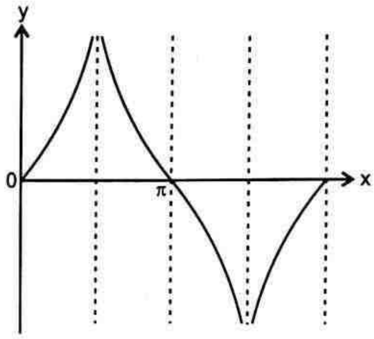
(a) 4 (b) 8 (c) 12 (d) 16

86. Which of the following is closest to the graph of $y = \tan(\sin x), x > 0$?

(a) 

(b) 

(c) 

(d) 

87. Consider the function $f: R-\{1\} \to R-\{2\}$ given by $f(x) = \dfrac{2x}{x-1}$. Then

(a) f is one-one but not onto
(b) f is onto but not one-one
(c) f is neither one-one nor onto
(d) f is both one-one and onto

88. If range of function $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \dfrac{1}{2}f(2x+1)$ is equal to :

(a) $[-2, 4]$ (b) $[-1, 2]$ (c) $[-3, 9]$ (d) $[-2, 2]$

89. Let $f: R \to R$ and $f(x) = \dfrac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then $f(x)$ is :

(a) One-one, into (b) Many one, onto (c) One-one, onto (d) Many one, into

90. Let $f(x)$ be defined as

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1|+|x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x))$ is :

(a) $[0, 1]$ (b) $[-1, 0]$ (c) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (d) $[-1, 1]$

91. The number of integral values of x in the domain of function f defined as $f(x) = \sqrt{\ln|\ln|x||} + \sqrt{7|x|-|x|^2-10}$ is :

(a) 5 (b) 6 (c) 7 (d) 8

92. The complete set of values of x in the domain of function $f(x)=\sqrt{\log_{x+2\{x\}}([x]^2-5[x]+7)}$ (where [·] denote greatest integer function and {·} denote fraction part function) is :

(a) $\left(-\frac{1}{3},0\right)\cup\left(\frac{1}{3},1\right)\cup(2,\infty)$ (b) $(0,1)\cup(1,\infty)$

(c) $\left(-\frac{2}{3},0\right)\cup\left(\frac{1}{3},1\right)\cup(1,\infty)$ (d) $\left(-\frac{1}{3},0\right)\cup\left(\frac{1}{3},1\right)\cup(1,\infty)$

93. The number of integral ordered pair (x, y) that satisfy the system of equation $|x+y-4|=5$ and $|x-3|+|y-1|=5$ is/are :

(a) 2 (b) 4 (c) 6 (d) 12

94. Let $f:R\to R$, where $f(x)=\dfrac{x^2+ax+1}{x^2+x+1}$. Then the complete set of values of 'a' such that $f(x)$ is onto is :

(a) $(-\infty,\infty)$ (b) $(-\infty,0)$ (c) $(0,\infty)$ (d) Empty set

95. If $A=\{1,2,3,4\}$ and $f:A\to A$, then total number of invertible function 'f' such that $f(2)\neq 2$, $f(4)\neq 4$, $f(1)=1$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

96. The domain of definition of $f(x)=\log_{(x^2-x+1)}(2x^2-7x+9)$ is :

(a) R (b) $R-\{0\}$ (c) $R-\{0,1\}$ (d) $R-\{1\}$

97. If $A=\{1,2,3,4\}, B=\{1,2,3,4,5,6\}$ and $f:A\to B$ is an injective mapping satisfying $f(i)\neq i$, then number of such mappings are :

(a) 182 (b) 181 (c) 183 (d) none of these

98. Let $f(x)=x^2-2x-3; x\geq 1$ and $g(x)=1+\sqrt{x+4}; x\geq -4$ then the number of real solutions of equation $f(x)=g(x)$ is/are

(a) 0 (b) 1 (c) 2 (d) 4

Answers

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(b)	**5.**	(d)	**6.**	(a)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(a)
11.	(b)	**12.**	(c)	**13.**	(c)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(d)	**18.**	(a)	**19.**	(d)	**20.**	(c)
21.	(d)	**22.**	(d)	**23.**	(b)	**24.**	(b)	**25.**	(c)	**26.**	(b)	**27.**	(c)	**28.**	(c)	**29.**	(b)	**30.**	(c)
31.	(d)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(b)	**36.**	(c)	**37.**	(a)	**38.**	(d)	**39.**	(a)	**40.**	(c)
41.	(b)	**42.**	(b)	**43.**	(a)	**44.**	(b)	**45.**	(c)	**46.**	(b)	**47.**	(b)	**48.**	(b)	**49.**	(c)	**50.**	(c)
51.	(b)	**52.**	(c)	**53.**	(c)	**54.**	(d)	**55.**	(a)	**56.**	(b)	**57.**	(d)	**58.**	(c)	**59.**	(a)	**60.**	(b)
61.	(b)	**62.**	(c)	**63.**	(c)	**64.**	(b)	**65.**	(d)	**66.**	(a)	**67.**	(d)	**68.**	(a)	**69.**	(a)	**70.**	(b)
71.	(c)	**72.**	(a)	**73.**	(a)	**74.**	(d)	**75.**	(c)	**76.**	(d)	**77.**	(b)	**78.**	(d)	**79.**	(d)	**80.**	(c)
81.	(b)	**82.**	(b)	**83.**	(a)	**84.**	(c)	**85.**	(c)	**86.**	(b)	**87.**	(d)	**88.**	(b)	**89.**	(d)	**90.**	(d)
91.	(b)	**92.**	(d)	**93.**	(d)	**94.**	(d)	**95.**	(c)	**96.**	(c)	**97.**	(b)	**98.**	(b)				

Exercise-2 : One or More than One Answer is/are Correct

1. $f(x)$ is an even periodic function with period 10. In $[0, 5]$, $f(x) = \begin{cases} 2x & 0 \le x < 2 \\ 3x^2 - 8 & 2 \le x < 4 \\ 10x & 4 \le x \le 5 \end{cases}$. Then :

(a) $f(-4) = 40$
(b) $\dfrac{f(-13) - f(11)}{f(13) + f(-11)} = \dfrac{17}{21}$
(c) $f(5)$ is not defined
(d) Range of $f(x)$ is $[0, 50]$

2. Let $f(x) = ||x^2 - 4x + 3| - 2|$. Which of the following is/are correct ?

(a) $f(x) = m$ has exactly two real solutions of different sign $\forall\, m > 2$
(b) $f(x) = m$ has exactly two real solutions $\forall\, m \in (2, \infty) \cup \{0\}$
(c) $f(x) = m$ has no solutions $\forall\, m < 0$
(d) $f(x) = m$ has four distinct real solution $\forall\, m \in (0, 1)$

3. Let $f(x) = \cos^{-1}\left(\dfrac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}\right)$

Which of the following statement(s) is/are correct about $f(x)$?

(a) Domain is R
(b) Range is $[0, \pi]$
(c) $f(x)$ is even
(d) $f(x)$ is derivable in $(\pi, 2\pi)$

4. $|\log_e |x|| = |k - 1| - 3$ has four distinct roots then k satisfies : (where $|x| < e^2, x \ne 0$)

(a) $(-4, -2)$
(b) $(4, 6)$
(c) (e^{-1}, e)
(d) (e^{-2}, e^{-1})

5. Which of the following functions are defined for all $x \in R$?
(Where $[\cdot]$ = denotes greatest integer function)

(a) $f(x) = \sin[x] + \cos[x]$
(b) $f(x) = \sec^{-1}(1 + \sin^2 x)$
(c) $f(x) = \sqrt{\dfrac{9}{8} + \cos x + \cos 2x}$
(d) $f(x) = \tan(\ln(1 + |x|))$

6. Let $f(x) = \begin{cases} x^2 & 0 < x < 2 \\ 2x - 3 & 2 \le x < 3 \\ x + 2 & x \ge 3 \end{cases}$, then the true equations :

(a) $f\left(f\left(f\left(\dfrac{3}{2}\right)\right)\right) = f\left(\dfrac{3}{2}\right)$
(b) $1 + f\left(f\left(f\left(\dfrac{5}{2}\right)\right)\right) = f\left(\dfrac{5}{2}\right)$
(c) $f(f(f(2))) = f(1)$
(d) $\underbrace{f(f(f(\ldots\ldots f(4))\ldots)}_{1004 \text{ times}} = 2012$

7. Let $f : \left[\dfrac{2\pi}{3}, \dfrac{5\pi}{3}\right] \longrightarrow [0, 4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$, then :

(a) $f^{-1}(1) = \dfrac{4\pi}{3}$
(b) $f^{-1}(1) = \pi$
(c) $f^{-1}(2) = \dfrac{5\pi}{6}$
(d) $f^{-1}(2) = \dfrac{7\pi}{6}$

8. Let $f(x)$ be invertible function and let $f^{-1}(x)$ be its inverse. Let equation $f(f^{-1}(x)) = f^{-1}(x)$ has two real roots α and β (with in domain of $f(x)$), then :

(a) $f(x) = x$ also have same two real roots

(b) $f^{-1}(x) = x$ also have same two real roots

(c) $f(x) = f^{-1}(x)$ also have same two real roots

(d) Area of triangle formed by $(0, 0)$, $(\alpha, f(\alpha))$, and $(\beta, f(\beta))$ is 1 unit

9. The function $f(x) = \cos^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right)$, then :

(a) Range of $f(x)$ is $\left[\frac{\pi}{3}, \frac{10\pi}{3}\right]$

(b) Range of $f(x)$ is $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

(c) $f(x)$ is one-one for $x \in \left[-1, \frac{1}{2}\right]$

(d) $f(x)$ is one-one for $x \in \left[\frac{1}{2}, 1\right]$

10. Let $f: R \to R$ defined by $f(x) = \cos^{-1}(-\{-x\})$, where $\{x\}$ is fractional part function. Then which of the following is/are correct ?

(a) f is many-one but not even function

(b) Range of f contains two prime numbers

(c) f is a periodic

(d) Graph of f does not lie below x-axis

11. Which option(s) is/are true ?

(a) $f: R \to R$, $f(x) = e^{|x|} - e^{-x}$ is many-one into function

(b) $f: R \to R$, $f(x) = 2x + |\sin x|$ is one-one onto

(c) $f: R \to R$, $f(x) = \frac{x^2 + 4x + 30}{x^2 - 8x + 18}$ is many-one onto

(d) $f: R \to R$, $f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}$ is many-one into

12. If $h(x) = \left[\ln\frac{x}{e}\right] + \left[\ln\frac{e}{x}\right]$, where $[\cdot]$ denotes greatest integer function, then which of the following are true ?

(a) range of $h(x)$ is $\{-1, 0\}$

(b) If $h(x) = 0$, then x must be irrational

(c) If $h(x) = -1$, then x can be rational as well as irrational

(d) $h(x)$ is periodic function

13. If $f(x) = \begin{cases} x^3 & ; \ x \in Q \\ -x^3 & ; \ x \notin Q \end{cases}$, then :

(a) $f(x)$ is periodic

(b) $f(x)$ is many-one

(c) $f(x)$ is one-one

(d) range of the function is R

14. Let $f(x)$ be a real valued continuous function such that

$f(0) = \frac{1}{2}$ and $f(x+y) = f(x)f(a-y) + f(y)f(a-x)\ \forall\ x, y \in R$,

then for some real a :

(a) $f(x)$ is a periodic function

(b) $f(x)$ is a constant function

(c) $f(x) = \frac{1}{2}$

(d) $f(x) = \frac{\cos x}{2}$

15. $f(x)$ is an even periodic function with period 10. In [0, 5], $f(x) = \begin{cases} 2x & 0 \le x < 2 \\ 3x^2 - 8 & 2 \le x < 4 \\ 10x & 4 \le x \le 5 \end{cases}$. Then :

(a) $f(-4) = 40$

(b) $\frac{f(-13) - f(11)}{f(13) + f(-11)} = \frac{17}{21}$

(c) $f(5)$ is not defined

(d) Range of $f(x)$ is [0, 50]

16. For the equation $\frac{e^{-x}}{1+x} = \lambda$ which of the following statement(s) is/are correct ?

(a) when $\lambda \in (0, \infty)$ equation has 2 real and distinct roots

(b) when $\lambda \in (-\infty, -e^2)$ equation has 2 real and distinct roots

(c) when $\lambda \in (0, \infty)$ equation has 1 real root

(d) when $\lambda \in (-e, 0)$ equation has no real root

17. For $x \in R^+$, if $x, [x], \{x\}$ are in harmonic progression then the value of x can not be equal to :

(where $[\cdot]$ denotes greatest integer function, $\{\cdot\}$ denotes fractional part function)

(a) $\frac{1}{\sqrt{2}}\tan\frac{\pi}{8}$ (b) $\frac{1}{\sqrt{2}}\cot\frac{\pi}{8}$ (c) $\frac{1}{\sqrt{2}}\tan\frac{\pi}{12}$ (d) $\frac{1}{\sqrt{2}}\cot\frac{\pi}{12}$

18. The equation $||x-1|+a| = 4$, $a \in R$, has :

(a) 3 distinct real roots for unique value of a.

(b) 4 distinct real roots for $a \in (-\infty, -4)$

(c) 2 distinct real roots for $|a| < 4$

(d) no real roots for $a > 4$

19. Let $f_n(x) = (\sin x)^{1/n} + (\cos x)^{1/n}$, $x \in R$, then :

(a) $f_2(x) > 1$ for all $x \in \left(2k\pi, (4k+1)\frac{\pi}{2}\right)$, $k \in I$

(b) $f_2(x) = 1$ for $x = 2k\pi$, $k \in I$

(c) $f_2(x) > f_3(x)$ for all $x \in \left(2k\pi, (4k+1)\frac{\pi}{2}\right)$, $k \in I$

(d) $f_3(x) \ge f_5(x)$ for all $x \in \left(2k\pi, (4k+1)\frac{\pi}{2}\right)$, $k \in I$

(Where I denotes set of integers)

20. If the domain of $f(x) = \frac{1}{\pi}\cos^{-1}\left[\log_3\left(\frac{x^2}{3}\right)\right]$ where, $x > 0$ is $[a, b]$ and the range of $f(x)$ is $[c, d]$, then :

(a) a, b are the roots of the equation $x^4 - 3x^3 - x + 3 = 0$

(b) a, b are the roots of the equation $x^4 - x^3 + x^2 - 2x + 1 = 0$

(c) $a^3 + d^3 = 1$

(d) $a^2 + b^2 + c^2 + d^2 = 11$

21. The number of real values of x satisfying the equation ; $\left[\frac{2x+1}{3}\right] + \left[\frac{4x+5}{6}\right] = \frac{3x-1}{2}$ are greater than or equal to {[·] denotes greatest integer function):

(a) 7 (b) 8 (c) 9 (d) 10

22. Let $f(x) = \sin^6\left(\frac{x}{4}\right) + \cos^6\left(\frac{x}{4}\right)$. If $f^n(x)$ denotes n^{th} derivative of f evaluated at x. Then which of the following hold ?

(a) $f^{2014}(0) = -\frac{3}{8}$ (b) $f^{2015}(0) = \frac{3}{8}$ (c) $f^{2010}\left(\frac{\pi}{2}\right) = 0$ (d) $f^{2011}\left(\frac{\pi}{2}\right) = \frac{3}{8}$

23. Which of the following is(are) incorrect ?

(a) If $f(x) = \sin x$ and $g(x) = \ln x$ then range of $g(f(x))$ is $[-1, 1]$

(b) If $x^2 + ax + 9 > x \,\forall\, x \in R$ then $-5 < a < 7$

(c) If $f(x) = (2011 - x^{2012})^{\frac{1}{2012}}$ then $f(f(2)) = \frac{1}{2}$

(d) The function $f: R \to R$ defined as $f(x) = \frac{x^2 + 4x + 30}{x^2 - 8x + 18}$ is not surjective.

24. If $[x]$ denotes the integral part of x for real x, and

$$S = \left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{200}\right] + \left[\frac{1}{4} + \frac{1}{100}\right] + \left[\frac{1}{4} + \frac{3}{200}\right] \ldots\ldots + \left[\frac{1}{4} + \frac{199}{200}\right] \text{ then}$$

(a) S is a composite number (b) Exponent of S in $\underline{|100}$ is 12

(c) Number of factors of S is 10 (d) ${}^{2S}C_r$ is max when $r = 51$

Answers

1.	(a, b, d)	**2.**	(a, b, c)	**3.**	(c, d)	**4.**	(a, b)	**5.**	(a, b, c)	**6.**	(a, b, c, d)
7.	(a, d)	**8.**	(a, b, c)	**9.**	(b, c)	**10.**	(a, b, d)	**11.**	(a, b, d)	**12.**	(a, c)
13.	(c, d)	**14.**	(a, b, c)	**15.**	(a, b, d)	**16.**	(b, c, d)	**17.**	(a, c, d)	**18.**	(a, b, c, d)
19.	(a, b)	**20.**	(a, d)	**21.**	(a, b, c)	**22.**	(a, c, d)	**23.**	a, b)	**24.**	(a, b)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Let $f(x) = \log_{\{x\}}[x]$

$g(x) = \log_{\{x\}}\{x\}$

$h(x)\log_{[x]}\{x\}$

where [ ], { } denotes the greatest integer function and fractional part fucntion respectively.

1. For $x \in (1, 5)$ the $f(x)$ is not defined at how many points :

(a) 5 (b) 4 (c) 3 (d) 2

2. If $A = \{x : x \in \text{ domain of } f(x)\}$ and $B = \{x : x \in \text{domain of } g(x)\}$ then $\forall\ x \in (1, 5)$, $A - B$ will be :

(a) $(2, 3)$ (b) $(1, 3)$ (c) $(1, 2)$ (d) None of these

3. Domain of $h(x)$ is :

(a) $[2, \infty)$ (b) $[1, \infty)$ (c) $[2, \infty) - \{I\}$ (d) $R^+ - \{I\}$

I denotes integers.

Paragraph for Question Nos. 4 to 6

θ is said to be well behaved if it lies in interval $\left[0, \frac{\pi}{2}\right]$. They are intelligent if they make domain of $f + g$ and g equal. The values of θ for which $h(\theta)$ is defined are handsome. Let

$$f(x) = \sqrt{\theta x^2 - 2(\theta^2 - 3)x - 12\theta}, g(x) = \ln(x^2 - 49),$$

$$h(\theta) = \ln\left[\int_0^\theta 4\cos^2 t\, dt - \theta^2\right], \text{ where } \theta \text{ is in radians.}$$

4. Complete set of values of θ which are well behaved as well as intelligent is :

(a) $\left[\frac{3}{4}, \frac{\pi}{2}\right]$ (b) $\left[\frac{3}{5}, \frac{7}{8}\right]$ (c) $\left[\frac{5}{6}, \frac{\pi}{2}\right]$ (d) $\left[\frac{6}{7}, \frac{\pi}{2}\right]$

5. Complete set of values of θ which are intelligent is :

(a) $\left[\frac{6}{7}, \frac{7}{2}\right]$ (b) $\left(0, \frac{\pi}{3}\right]$ (c) $\left[\frac{1}{4}, \frac{6}{7}\right]$ (d) $\left[\frac{1}{2}, \frac{\pi}{2}\right]$

6. Complete set of values of θ which are well behaved, intelligent and handsome is :

(a) $\left(0, \frac{\pi}{2}\right]$ (b) $\left[\frac{6}{7}, \frac{\pi}{2}\right]$ (c) $\left[\frac{3}{4}, \frac{\pi}{2}\right]$ (d) $\left[\frac{3}{5}, \frac{\pi}{2}\right]$

Paragraph for Question Nos. 7 to 8

Let $f(x) = 2-|x-3|, 1 \le x \le 5$ and for rest of the values $f(x)$ can be obtained by using the relation $f(5x) = \alpha f(x) \; \forall \; x \in R$.

7. The maximum value of $f(x)$ in $[5^4, 5^5]$ for $\alpha = 2$ is :

(a) 16 (b) 32 (c) 64 (d) 8

8. The value of $f(2007)$, taking $\alpha = 5$, is :

(a) 1118 (b) 2007 (c) 1250 (d) 132

Paragraph for Question Nos. 9 to 10

An even periodic function $f : R \to R$ with period 4 is such that

$$f(x) = \begin{cases} \max.(|x|, x^2) & ; \; 0 \le x < 1 \\ x & ; \; 1 \le x \le 2 \end{cases}$$

9. The value of $\{f(5.12)\}$ (where $\{\cdot\}$ denotes fractional part function), is :

(a) $\{f(3.26)\}$ (b) $\{f(7.88)\}$ (c) $\{f(2.12)\}$ (d) $\{f(5.88)\}$

10. The number of solutions of $f(x) = |3\sin x|$ for $x \in (-6, 6)$ are :

(a) 5 (b) 3 (c) 7 (d) 9

Paragraph for Question Nos. 11 to 12

Let $f(x) = \dfrac{2|x|-1}{x-3}$

11. Range of $f(x)$:

(a) $R - \{3\}$ (b) $\left(-\infty, \dfrac{1}{3}\right] \cup (2, \infty)$ (c) $\left(-2, \dfrac{1}{3}\right] \cup (2, \infty)$ (d) R

12. Range of the values of 'k' for which $f(x) = k$ has exactly two distinct solutions :

(a) $\left(-2, \dfrac{1}{3}\right)$ (b) $(-2, 1]$ (c) $\left(0, \dfrac{2}{3}\right]$ (d) $(-\infty, -2)$

Paragraph for Question Nos. 13 to 14

Let $f(x)$ be a continuous function (define for all x) which satisfies $f^3(x) - 5f^2(x) + 10f(x) - 12 \ge 0$, $f^2(x) - 4f(x) + 3 \ge 0$ and $f^2(x) - 5f(x) + 6 \le 0$

13. If distinct positive number b_1, b_2 and b_3 are in G.P. then $f(1) + ln\, b_1$, $f(2) + ln\, b_2$, $f(3) + ln\, b_3$ are in :

(a) A.P. (b) G.P. (c) H.P. (d) A.G.P.

14. The equation of tangent that can be drawn from (2, 0) on the curve $y = x^2 f(\sin x)$ is :

(a) $y = 24(x+2)$ (b) $y = 12(x+2)$ (c) $y = 24(x-2)$ (d) $y = 12(x-2)$

Paragraph for Question Nos. 15 to 16

Let $f:[2,\infty)\to[1,\infty)$ defined by $f(x)=2^{x^4-4x^2}$ and $g:\left[\frac{\pi}{2},\pi\right]\to A$ defined by $g(x)=\dfrac{\sin x+4}{\sin x-2}$ be two invertible functions, then

15. $f^{-1}(x)$ is equal to

(a) $\sqrt{2+\sqrt{4-\log_2 x}}$ (b) $\sqrt{2+\sqrt{4+\log_2 x}}$ (c) $\sqrt{4+\sqrt{2+\log_2 x}}$ (d) $\sqrt{4-\sqrt{2+\log_2 x}}$

16. The set 'A' equals to

(a) $[5, 2]$ (b) $[-2, 5]$ (c) $[-5, 2]$ (d) $[-5, -2]$

Answers

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(b)	**7.**	(b)	**8.**	(a)	**9.**	(b)	**10.**	(c)
11.	(b)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(b)	**16.**	(d)								

Exercise-4 : Matching Type Problems

1. If $x, y, z \in R$ satisfies the system of equations $x + [y] + \{z\} = 12.7$, $[x] + \{y\} + z = 4.1$ and $\{x\} + y + [z] = 2$
(where $\{\cdot\}$ and $[\cdot]$ denotes the fractional and integral parts respectively), then match the following :

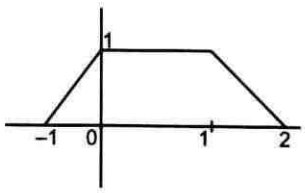
	Column-I		Column-II
(A)	$\{x\} + \{y\} =$	**(P)**	7.7
(B)	$[z] + [x] =$	**(Q)**	1.1
(C)	$x + \{z\} =$	**(R)**	1
(D)	$z + [y] - \{x\} =$	**(S)**	3
		(T)	4

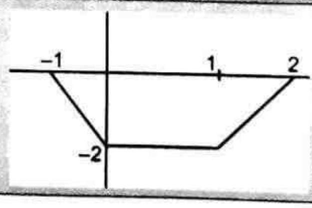
2. Consider $ax^4 + (7a - 2b)x^3 + (12a - 14b - c)x^2 - (24b + 7c)x + 1 - 12c = 0$, has no real roots and $f_1(x) = \dfrac{\sqrt{\log_{(\pi+e)}(ax^4 + (7a-2b)x^3 + (12a - 14b - c)x^2 - (24b + 7c)x + 1 - 12c)}}{\sqrt{a}\sqrt{-\operatorname{sgn}(1 + ac + b^2)}}$

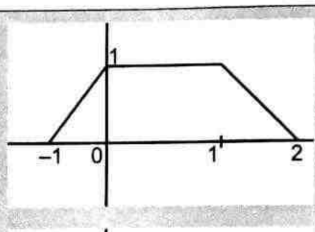
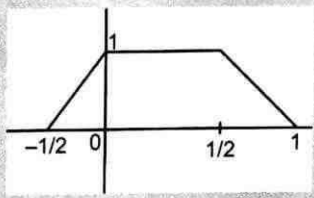
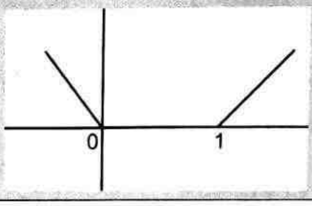
$f_2(x) = -2 + 2\log_{\sqrt{2}} \cos\left(\tan^{-1}(\sin(\pi(\cos(\pi(x + \frac{7}{2})))))\right)$. Then match the following :

	Column-I		Column-II
(A)	Domain of $f_1(x)$ is	**(P)**	$[-3, -2]$
(B)	Range of $f_2(x)$ in the domain of $f_1(x)$ is	**(Q)**	$[-4, -2]$
(C)	Range of $f_2(x)$ is	**(R)**	$(-\infty, \infty)$
(D)	Domain of $f_2(x)$ is	**(S)**	$(-\infty, -4] \cup [-3, \infty)$
		(T)	$[0, 1]$

3. Given the graph of $y = f(x)$



	Column-I		Column-II
(A)	$y = f(1 - x)$	**(P)**	

(B)	$y = f(2x)$	**(Q)**	
(C)	$y = -2f(x)$	**(R)**	
(D)	$y = 1 - f(x)$	**(S)**	

4.

	Column-I		Column-II
(A)	$f(x) = \sin^2 2x - 2\sin^2 x$	**(P)**	Range contains no natural number
(B)	$f(x) = \frac{4}{\pi}(\sin^{-1}(\sin \pi x))$	**(Q)**	Range contains atleast one integer
(C)	$f(x) = \sqrt{\ln(\cos(\sin x))}$	**(R)**	Many one but not even function
(D)	$f(x) = \tan^{-1}\left(\frac{x^2+1}{x^2+\sqrt{3}}\right)$	**(S)**	Both many one and even function
		(T)	Periodic but not odd function

5.

	Column-I		Column-II
(A)	If $\lvert x^2 - x \rvert \ge x^2 + x$, then complete set of values of x is	**(P)**	$(0, \infty)$
(B)	If $\lvert x + y \rvert > x - y$, where $x > 0$, then complete set of values of y is	**(Q)**	$(-\infty, 0]$
(C)	If $\log_2 x \ge \log_2(x^2)$, then complete set of values of x is	**(R)**	$[-1, \infty)$

(D)	$[x]+2 \geq \|x\|$, (where $[\cdot]$ denotes the greatest integer function) then complete set of values of x is	**(S)**	$(0,1]$
		(T)	$[1,\infty)$

6.

	Column-I		Column-II
(A)	Domain of $f(x) = \ln \tan^{-1}\{(x^3 - 6x^2 + 11x - 6)\,x(e^x - 1)\}$ is	**(P)**	$\left[-1, \frac{5}{4}\right]$
(B)	Range of $f(x) = \sin^2\frac{x}{4} + \cos\frac{x}{4}$ is	**(Q)**	$[2,\infty)$
(C)	The domain of function $f(x) = \sqrt{\log_{(\|x\|-1)}(x^2+4x+4)}$ is	**(R)**	$(1,2) \cup (3,\infty)$
(D)	Let $f(x) = \begin{cases} x^2 & x<1 \\ x+1 & x \geq 1 \end{cases}; g(x) = \begin{cases} x+2 & x<1 \\ x^2 & x \geq 1 \end{cases}$	**(S)**	$[0,\infty)$
	Then range of function $f(g(x))$ is	**(T)**	$(-\infty,-3) \cup (-2,-1) \cup (2,\infty)$

7. Let $f(x) \begin{bmatrix} 1+x; \; 0 \leq x \leq 2 \\ 3-x; \; 2 < x \leq 3 \end{bmatrix}$;

$g(x) = f(f(x))$:

	Column-I		Column-II
(A)	If domain of $g(x)$ is $[a, b]$ then $b-a$ is	**(P)**	1
(B)	If range of $g(x)$ is $[c, d]$ then $c+d$ is	**(Q)**	2
(C)	$f(f(f(2))) + f(f(f(3)))$, is	**(R)**	3
(D)	m = maximum value of $g(x)$ then $2m-2$ is :	**(S)**	4

Answers

1. A → R ; B → S ; C → P ; D → Q

2. A → S ; B → P ; C → Q ; D → R

3. A → Q ; B → R ; C → P ; D → S

4. A → P, Q, S, T ; B → Q, R ; C → P, Q, S ; D → P, S

5. A → Q; B → P; C → S; D → R

6. A → R; B → P; C → T; D → S

7. A → R ; B → R ; C → R ; D → S

Exercise-5 : Subjective Type Problems

1. Let $f(x)$ be a polynomial of degree 6 with leading coefficient 2009. Suppose further, that $f(1)=1, f(2)=3, f(3)=5, f(4)=7, f(5)=9, f'(2)=2$, then the sum of all the digits of $f(6)$ is
2. Let $f(x)=x^3-3x+1$. Find the number of different real solution of the equation $f(f(x))=0$.
3. If $f(x+y+1)=(\sqrt{f(x)}+\sqrt{f(y)})^2\ \forall\, x, y \in R$ and $f(0)=1$, then $f(2)=$......
4. If the domain of $f(x)=\sqrt{12-3^x-3^{3-x}}+\sin^{-1}\left(\frac{2x}{3}\right)$ is $[a, b]$, then $a=$......
5. The number of elements in the range of the function :
$y=\sin^{-1}\left[x^2+\frac{5}{9}\right]+\cos^{-1}\left[x^2-\frac{4}{9}\right]$ where $[\cdot]$ denotes the greatest integer function is
6. The number of solutions of the equation $f(x-1)+f(x+1)=\sin\alpha,\ 0<\alpha<\frac{\pi}{2}$, where $f(x)=\begin{cases}1-|x| & , & |x|\le 1\\ 0 & , & |x|>1\end{cases}$ is
7. The number of integers in the range of function $f(x)=[\sin x]+[\cos x]+[\sin x+\cos x]$ is (where $[\cdot]$ = denotes greatest integer function)
8. If $P(x)$ is a polynomial of degree 4 such that $P(-1)=P(1)=5$ and $P(-2)=P(0)=P(2)=2$, then find the maximum value of $P(x)$.
9. The number of integral value(s) of k for which the curve $y=\sqrt{-x^2-2x}$ and $x+y-k=0$ intersect at 2 distinct points is/are
10. Let the solution set of the equation :
$$\sqrt{\left[x+\left[\frac{x}{2}\right]\right]}+\left[\sqrt{\{x\}}+\left[\frac{x}{3}\right]\right]=3$$
is $[a, b)$. Find the product ab.
(where $[\cdot]$ and $\{\cdot\}$ denote greatest integer and fractional part function respectively).
11. For all real number x, let $f(x)=\frac{1}{\sqrt[2011]{1-x^{2011}}}$. Find the number of real roots of the equation
$$f(f(\ldots\ldots(f(x))\ldots\ldots))=\{-x\}$$
where f is applied 2013 times and $\{\cdot\}$ denotes fractional part function.
12. Find the number of elements contained in the range of the function $f(x)=\left[\frac{x}{6}\right]\left[\frac{-6}{x}\right]\ \forall\, x\in (0, 30]$ (where $[\cdot]$ denotes greatest integer function)
13. Let $f(x, y)=x^2-y^2$ and $g(x, y)=2xy$.
such that $(f(x, y))^2-(g(x, y))^2=\frac{1}{2}$ and $f(x, y)\cdot g(x, y)=\frac{\sqrt{3}}{4}$
Find the number of ordered pairs (x, y) ?

14. Let $f(x) = \dfrac{x+5}{\sqrt{x^2+1}}$ $\forall\, x \in R$, then the smallest integral value of k for which $f(x) \le k$ $\forall\, x \in R$ is

15. In the above problem, $f(x)$ is injective in the interval $x \in (-\infty, a]$, and λ is the largest possible value of a, then $[\lambda] =$

(where $[x]$ denote greatest integer $\le x$)

16. The number of integral values of m for which $f: R \to R;\, f(x) = \dfrac{x^3}{3} + (m-1)x^2 + (m+5)x + n$ is bijective is :

17. The number of roots of equation :

$$\left(\frac{(x-1)(x-3)}{(x-2)(x-4)} - e^x\right)\left(\frac{(x+1)(x+3)e^x}{(x+2)(x+4)} - 1\right)(x^3 - \cos x) = 0$$

18. The number of solutions of the equation $\cos^{-1}\left(\dfrac{1-x^2-2x}{(x+1)^2}\right) = \pi(1-\{x\})$, for $x \in [0, 76]$ is equal to. (where $\{\cdot\}$ denote fraction part function)

19. Let $f(x) = x^2 - bx + c$, b is an odd positive integer. Given that $f(x) = 0$ has two prime numbers as roots and $b + c = 35$. If the least value of $f(x)\ \forall\, x \in R$ is λ, then $\left[\left|\dfrac{\lambda}{3}\right|\right]$ is equal to

(where $[\cdot]$ denotes greatest integer function)

20. Let $f(x)$ be continuous function such that $f(0) = 1$ and $f(x) - f\left(\dfrac{x}{7}\right) = \dfrac{x}{7}$ $\forall\, x \in R$, then $f(42) =$

21. If $f(x) = 4x^3 - x^2 - 2x + 1$ and $g(x) = \begin{cases} \min\{f(t): 0 \le t \le x\} & ;\ 0 \le x \le 1 \\ 3 - x & ;\ 1 < x \le 2 \end{cases}$ and if $\lambda = g\left(\dfrac{1}{4}\right) + g\left(\dfrac{3}{4}\right) + g\left(\dfrac{5}{4}\right)$, then $2\lambda =$

22. If $x = 10\displaystyle\sum_{r=3}^{100} \frac{1}{(r^2-4)}$, then $[x] =$

(where $[\cdot]$ denotes greatest integer function)

23. Let $f(x) = \dfrac{ax+b}{cx+d}$, where a, b, c, d are non zero. If $f(7) = 7$, $f(11) = 11$ and $f(f(x)) = x$ for all x except $-\dfrac{d}{c}$. The unique number which is not in the range of f is

24. Let $A = \{x \mid x^2 - 4x + 3 < 0, x \in R\}$

$B = \{x \mid 2^{1-x} + p \le 0;\, x^2 - 2(p+7)x + 5 \le 0\}$

If $A \subseteq B$, then the range of real number $p \in [a, b]$ where a, b are integers.

Find the value of $(b - a)$.

25. Let the maximum value of expression $y = \dfrac{x^4 - x^2}{x^6 + 2x^3 - 1}$ for $x > 1$ is $\dfrac{p}{q}$, where p and q are relatively prime natural numbers, then $p + q =$

26. If $f(x)$ is an even function, then the number of distinct real numbers x such that $f(x) = f\left(\dfrac{x+1}{x+2}\right)$ is :

27. The least integral value of $m, m \in R$ for which the range of function $f(x) = \dfrac{x+m}{x^2+1}$ contains the interval $[0, 1]$ is :

28. Let x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in G.P. where x_1, x_2, x_3 are positive numbers. Then the maximum value of $[\beta] + [\gamma] + 4$ is where $[\cdot]$ denotes greatest integer function is :

29. Let $A = \{1, 2, 3, 4\}$ and $B = \{0, 1, 2, 3, 4, 5\}$. If 'm' is the number of strictly increasing function f, $f : A \to B$ and n is the number of onto functions $g, g : B \to A$. Then the last digit of $n - m$ is.

30. If $\sum_{r=1}^{n} [\log_2 r] = 2010$, where $[\cdot]$ denotes greatest integer function, then the sum of the digits of n is :

31. Let $f(x) = \dfrac{ax+b}{cx+d}$, where a, b, c, d are non-zero. If $f(7) = 7$, $f(11) = 11$ and $f(f(x)) = x$ for all x except $-\dfrac{d}{c}$. The unique number which is not in the range of f is

32. It is pouring down rain, and the amount of rain hitting point (x, y) is given by $f(x, y) = |x^3 + 2x^2y - 5xy^2 - 6y^3|$. If Mr. '$A$' starts at $(0, 0)$; find number of possible value(s) for 'm' such that $y = mx$ is a line along which Mr. 'A' could walk without any rain falling on him.

33. Let $P(x)$ be a cubic polynomical with leading co-efficient unity. Let the remainder when $P(x)$ is divided by $x^2 - 5x + 6$ equals 2 times the remainder when $P(x)$ is divided by $x^2 - 5x + 4$. If $P(0) = 100$, find the sum of the digits of $P(5)$:

34. Let $f(x) = x^2 + 10x + 20$. Find the number of real solution of the equation $f(f(f(f(x)))) = 0$

35. If range of $f(x) = \dfrac{(\ln x)(\ln x^2) + \ln x^3 + 3}{\ln^2 x + \ln x^2 + 2}$ can be expressed as $\left[\dfrac{a}{b}, \dfrac{c}{d}\right]$ where a, b, c and d are prime numbers (not necessarily distinct) then find the value of $\dfrac{(a+b+c+d)}{2}$.

36. Polynomial $P(x)$ contains only terms of odd degree. When $P(x)$ is divided by $(x-3)$, then remainder is 6. If $P(x)$ is divided by $(x^2 - 9)$ then remainder is $g(x)$. Find the value of $g(2)$.

37. The equation $2x^3 - 3x^2 + p = 0$ has three real roots. Then find the minimum value of p.

38. Find the number of integers in the domain of $f(x) = \dfrac{1}{\sqrt{\ln \cos^{-1} x}}$.

Answers

1.	26	**2.**	7	**3.**	9	**4.**	1	**5.**	1	**6.**	4	**7.**	5
8.	6	**9.**	1	**10.**	12	**11.**	1	**12.**	6	**13.**	4	**14.**	6
15.	0	**16.**	6	**17.**	7	**18.**	76	**19.**	6	**20.**	8	**21.**	5
22.	5	**23.**	9	**24.**	3	**25.**	7	**26.**	4	**27.**	1	**28.**	3
29.	5	**30.**	8	**31.**	9	**32.**	3	**33.**	2	**34.**	2	**35.**	6
36.	4	**37.**	0	**38.**	2								

❑❑❑

Chapter 2 – Limit

2

LIMIT

Exercise-1 : Single Choice Problems

1. $\lim\limits_{x\to 0} \dfrac{\cos(\tan x) - \cos x}{x^4} =$

(a) $\dfrac{1}{6}$ (b) $-\dfrac{1}{3}$ (c) $-\dfrac{1}{6}$ (d) $\dfrac{1}{3}$

2. The value of $\lim\limits_{x\to 0} \dfrac{(\sin x - \tan x)^2 - (1-\cos 2x)^4 + x^5}{7(\tan^{-1} x)^7 + (\sin^{-1} x)^6 + 3\sin^5 x}$ equal to :

(a) 0 (b) 1 (c) 2 (d) $\dfrac{1}{3}$

3. Let $a = \lim\limits_{x\to 0} \dfrac{\ln(\cos 2x)}{3x^2}$, $b = \lim\limits_{x\to 0} \dfrac{\sin^2 2x}{x(1-e^x)}$, $c = \lim\limits_{x\to 1} \dfrac{\sqrt{x} - x}{\ln x}$.

Then a, b, c satisfy :

(a) $a < b < c$ (b) $b < c < a$ (c) $a < c < b$ (d) $b < a < c$

4. If $f(x) = \cot^{-1}\left(\dfrac{3x - x^3}{1-3x^2}\right)$ and $g(x) = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, then $\lim\limits_{x\to a} \dfrac{f(x) - f(a)}{g(x) - g(a)}, 0 < a < \dfrac{1}{2}$ is :

(a) $\dfrac{3}{2(1+a^2)}$ (b) $\dfrac{3}{2}$ (c) $\dfrac{-3}{2(1+a^2)}$ (d) $-\dfrac{3}{2}$

5. $\lim\limits_{x\to 0} \left(\dfrac{(1+x)^{\frac{2}{x}}}{e^2}\right)^{\frac{4}{\sin x}}$ is :

(a) e^4 (b) e^{-4} (c) e^8 (d) e^{-8}

6. $\lim\limits_{x\to\infty} \dfrac{3}{x}\left[\dfrac{x}{4}\right] = \dfrac{p}{q}$ (where [·] denotes greatest integer function), then $p + q$ (where p, q are relative prime) is :

(a) 2 (b) 7 (c) 5 (d) 6

7. $f(x) = \lim\limits_{n\to\infty} \dfrac{x^n + \left(\dfrac{\pi}{3}\right)^n}{x^{n-1} + \left(\dfrac{\pi}{3}\right)^{n-1}}$, ($n$ is an even integer), then which of the following is incorrect ?

(a) If $f: \left[\dfrac{\pi}{3}, \infty\right) \to \left[\dfrac{\pi}{3}, \infty\right)$, then function is invertible

(b) $f(x) = f(-x)$ has infinite number of solutions

(c) $f(x) = |f(x)|$ has infinite number of solutions

(d) $f(x)$ is one-one function for all $x \in R$

8. $\lim\limits_{x\to 0} \dfrac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2} =$

(a) π (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{2}$ (d) none of these

9. If $f(x) = \begin{cases} \dfrac{(e^{(x+3)\ln 27})^{\frac{x}{27}} - 9}{3^x - 27} & ; \ x < 3 \\ \lambda \dfrac{1 - \cos(x-3)}{(x-3)\tan(x-3)} & ; \ x > 3 \end{cases}$

If $\lim\limits_{x\to 3} f(x)$ exist, then $\lambda =$

(a) $\dfrac{9}{2}$ (b) $\dfrac{2}{9}$ (c) $\dfrac{2}{3}$ (d) none of these

10. $\lim\limits_{x\to \frac{\pi}{3}} \dfrac{\sin\left(\dfrac{\pi}{3} - x\right)}{2\cos x - 1}$ is equal to :

(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\dfrac{1}{2}$

11. $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\sin x}{\cos^{-1}\left[\dfrac{1}{4}(3\sin x - \sin 3x)\right]}$, (where [·] denotes greatest integer function) is :

(a) $\dfrac{2}{\pi}$ (b) 1 (c) $\dfrac{4}{\pi}$ (d) does not exist

12. Let f be a continuous function on R such that $f\left(\dfrac{1}{4^n}\right) = (\sin e^n)\, e^{-n^2} + \dfrac{n^2}{n^2+1}$, then $f(0) =$

(a) 1 (b) 0 (c) -1 (d) $\dfrac{1}{4}$

13. $\lim\limits_{x \to I^-} \dfrac{e^{\{x\}} - \{x\} - 1}{\{x\}^2}$ equals, where $\{\cdot\}$ is fractional part function and I is an integer, to :

(a) $\dfrac{I}{2}$ (b) $e-2$ (c) I (d) does not exist

14. $\lim\limits_{x \to \infty} (e^{11x} - 7x)^{\frac{1}{3x}}$ is equal to :

(a) $\dfrac{11}{3}$ (b) $\dfrac{3}{11}$ (c) $e^{\frac{3}{11}}$ (d) $e^{\frac{11}{3}}$

15. The value of $\lim\limits_{x \to 0} \left[(1-2x)^n \sum\limits_{r=0}^{n} {}^nC_r \left(\dfrac{x+x^2}{1-2x} \right)^r \right]^{1/x}$ is :

(a) e^n (b) e^{-n} (c) e^{3n} (d) e^{-3n}

16. For a certain value of 'c', $\lim\limits_{x \to \infty} [(x^5 + 7x^4 + 2)^c - x]$ is finite and non-zero. Then the value of limit is :

(a) $\dfrac{7}{5}$ (b) 1 (c) $\dfrac{2}{5}$ (d) None of these

17. The number of non-negative integral values of n for which $\lim\limits_{x \to 0} \dfrac{(\cos x - 1)(\cos x - e^x)}{x^n} = 0$ is :

(a) 1 (b) 2 (c) 3 (d) 4

18. The value of $\lim\limits_{x \to 0} \left(\dfrac{\sin x}{x} \right)^{\frac{1}{1-\cos x}}$:

(a) $e^{-1/3}$ (b) $e^{1/3}$ (c) $e^{-1/6}$ (d) $e^{1/6}$

19. If $\lim\limits_{x \to \infty} (\sqrt{x^2 - x + 1} - ax - b) = 0$, then for $k \geq 2, (k \in N)$ $\lim\limits_{n \to \infty} \sec^{2n}(k!\pi b) =$

(a) a (b) $-a$ (c) $2a$ (d) b

20. If f is a positive function such that $f(x+T) = f(x)\,(T > 0)$, $\forall\, x \in R$, then

$$\lim_{n \to \infty} n \left(\frac{f(x+T) + 2f(x+2T) + \ldots\ldots + nf(x+nT)}{f(x+T) + 4f(x+4T) + \ldots\ldots + n^2 f(x+n^2T)} \right) =$$

(a) 2 (b) $\dfrac{2}{3}$ (c) $\dfrac{3}{2}$ (d) None of these

21. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$$265 \left(\lim_{h \to 0} \frac{h^4 + 3h^2}{(f(1-h) - f(1)) \sin 5h} \right) =$$

(a) 1 (b) 2 (c) 3 (d) -3

22. $\lim_{x\to 0}\left(\frac{\cos x-\sec x}{x^2(x+1)}\right)=$

(a) 0 (b) $-\frac{1}{2}$ (c) -1 (d) -2

23. Let $f(x)$ be a continuous and differentiable function satisfying $f(x+y)=f(x)f(y)\ \forall\ x, y\in R$ if $f(x)$ can be expressed as $f(x)=1+xP(x)+x^2Q(x)$ where $\lim_{x\to 0}P(x)=a$ and $\lim_{x\to 0}Q(x)=b$, then $f'(x)$ is equal to :

(a) $a\,f(x)$ (b) $b\,f(x)$

(c) $(a+b)\,f(x)$ (d) $(a+2b)\,f(x)$

24. $\lim_{x\to\frac{\pi}{2}}\frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}=$

(a) not exist (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$

25. $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to :

(a) e (b) e^{-1} (c) e^{-5} (d) e^5

26. $\lim_{x\to\frac{\pi}{2}}(\cos x)^{\cos x}$ is :

(a) 1 (b) 0 (c) $\frac{1}{e}$ (d) $\frac{2}{e}$

27. If $\lim_{x\to c^-}\{\ln x\}$ and $\lim_{x\to c^+}\{\ln x\}$ exists finitely but they are not equal (where $\{\cdot\}$ denotes fractional part function), then :

(a) 'c' can take only rational values

(b) 'c' can take only irrational values

(c) 'c' can take infinite values in which only one is irrational

(d) 'c' can take infinite values in which only one is rational

28. $\lim_{x\to 0}\left(1+\frac{a\sin bx}{\cos x}\right)^{\frac{1}{x}}$, where a, b are non-zero constants is equal to :

(a) $e^{a/b}$ (b) ab

(c) e^{ab} (d) $e^{b/a}$

29. The value of $\lim_{x\to 0}\left((\cos x)^{\frac{1}{\sin^2 x}} + \dfrac{\sin 2x + 2\tan^{-1} 3x + 3x^2}{\ln(1+3x+\sin^2 x) + xe^x}\right)$ is :

(a) $\sqrt{e}+\dfrac{3}{2}$ (b) $\dfrac{1}{\sqrt{e}}+\dfrac{3}{2}$ (c) $\sqrt{e}+2$ (d) $\dfrac{1}{\sqrt{e}}+2$

30. Let $a = \lim_{x\to 1}\left(\dfrac{x}{\ln x} - \dfrac{1}{x\ln x}\right)$; $b = \lim_{x\to 0}\dfrac{x^3-16x}{4x+x^2}$; $c = \lim_{x\to 0}\dfrac{\ln(1+\sin x)}{x}$ and

$d = \lim_{x\to -1}\dfrac{(x+1)^3}{3[\sin(x+1)-(x+1)]}$, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is :

(a) Idempotent (b) Involutary

(c) Non-singular (d) Nilpotent

31. The integral value of n so that $\lim_{x\to 0} f(x)$ where $f(x) = \dfrac{(\sin x - x)\left(2\sin x - \ln\left(\dfrac{1+x}{1-x}\right)\right)}{x^n}$ is a finite non-zero number, is :

(a) 2 (b) 4 (c) 6 (d) 8

32. Consider the function $f(x) = \begin{cases} \dfrac{\max\left(x, \dfrac{1}{x}\right)}{\min\left(x, \dfrac{1}{x}\right)} & , \text{ if } x \neq 0 \\ 1 & , \text{ if } x = 0 \end{cases}$, then $\lim_{x\to 0^-}\{f(x)\} + \lim_{x\to 1^-}\{f(x)\} + \lim_{x\to -1^-}[f(x)] =$

(where $\{\cdot\}$ denotes fraction part function and $[\cdot]$ denotes greatest integer function)

(a) 0 (b) 1 (c) 2 (d) 3

33. $\lim_{x\to\left(\frac{1}{\sqrt{2}}\right)^+}\dfrac{\cos^{-1}(2x\sqrt{1-x^2})}{\left(x-\dfrac{1}{\sqrt{2}}\right)} - \lim_{x\to\left(\frac{1}{\sqrt{2}}\right)^-}\dfrac{\cos^{-1}(2x\sqrt{1-x^2})}{\left(x-\dfrac{1}{\sqrt{2}}\right)} =$

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 0

34. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sin\dfrac{\pi}{2k} - \cos\dfrac{\pi}{2k} - \sin\left(\dfrac{\pi}{2(k+2)}\right) + \cos\dfrac{\pi}{2(k+2)}\right) =$

(a) 0 (b) 1

(c) 2 (d) 3

35. $\lim_{x\to 0^+}[1+[x]]^{2/x}$, where $[\cdot]$ is greatest integer function, is equal to :

(a) 0 (b) 1

(c) e^2 (d) Does not exist

36. If m and n are positive integers, then $\lim_{x\to 0}\frac{(\cos x)^{1/m}-(\cos x)^{1/n}}{x^2}$ equals to :

(a) $m-n$ (b) $\frac{1}{n}-\frac{1}{m}$

(c) $\frac{m-n}{2mn}$ (d) None of these

37. The value of ordered pair (a, b) such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$, is :

(a) $\left(-\frac{5}{2},-\frac{3}{2}\right)$ (b) $\left(\frac{5}{2},\frac{3}{2}\right)$ (c) $\left(-\frac{5}{2},\frac{3}{2}\right)$ (d) $\left(\frac{5}{2},-\frac{3}{2}\right)$

38. What is the value of $a+b$, if $\lim_{x\to 0}\frac{\sin(ax)-\ln(e^x\cos x)}{x\sin(bx)}=\frac{1}{2}$?

(a) 1 (b) 2 (c) 3 (d) $-\frac{1}{2}$

39. Let $\alpha=\lim_{n\to\infty}\frac{(1^3-1^2)+(2^3-2^2)+\ldots+(n^3-n^2)}{n^4}$, then α is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) non existent

40. The value of $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to :

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

41. The value of ordered pair (a, b) such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$, is :

(a) $\left(-\frac{5}{2},-\frac{3}{2}\right)$ (b) $\left(\frac{5}{2},\frac{3}{2}\right)$ (c) $\left(-\frac{5}{2},\frac{3}{2}\right)$ (d) $\left(\frac{5}{2},-\frac{3}{2}\right)$

42. Consider the sequence :

$$u_n=\sum_{r=1}^{n}\frac{r}{2^r},\quad n\geq 1$$

Then the limit of u_n as $n\to\infty$ is :

(a) 1 (b) e (c) $\frac{1}{2}$ (d) 2

43. The value of $\lim_{x\to 0}\left((\cos x)^{\frac{1}{\sin^2 x}}+\frac{\sin 2x+2\tan^{-1}3x+3x^2}{\ln(1+3x+\sin^2 x)+xe^x}\right)$ is :

(a) $\sqrt{e}+\frac{3}{2}$ (b) $\frac{1}{\sqrt{e}}+\frac{3}{2}$ (c) $\sqrt{e}+2$ (d) $\frac{1}{\sqrt{e}}+2$

44. For $n \in N$, let $f_n(x) = \tan\frac{x}{2}(1+\sec x)(1+\sec 2x)(1+\sec 4x)\ldots\ldots(1+\sec 2^n x)$, the $\lim\limits_{x\to 0}\frac{f_n(x)}{2x}$ is equal to :

(a) 0 (b) 2^n (c) 2^{n-1} (d) 2^{n+1}

45. The value of $\lim\limits_{x\to\frac{\pi}{4}}(1+[x])^{\frac{1}{ln(\tan x)}}$ is :

(where [.] denotes greatest integer function).

(a) 0 (b) 1 (c) e (d) $\frac{1}{e}$

46. If $\lim\limits_{x\to 0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2}=0$, $n \neq 0$ then a is equal to :

(a) 0 (b) $1+\frac{1}{n}$ (c) n (d) $n+\frac{1}{n}$

47. The value of $\lim\limits_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{3n^3+4}{4n^4-1}}$, $n \in N$ is equal to :

(a) $\left(\frac{1}{e}\right)^{3/4}$ (b) $e^{3/4}$ (c) e^{-1} (d) 0

48. The value of $\lim\limits_{x\to\infty}\frac{ax^2+bx+c}{dx+e}$ $(a, b, c, d, e \in R-\{0\})$ depends on the sign of :

(a) a only (b) d only

(c) a and d only (d) a, b and d only

49. Let $f(x)=\lim\limits_{n\to\infty}\tan^{-1}\left(4n^2\left(1-\cos\frac{x}{n}\right)\right)$ and $g(x)=\lim\limits_{n\to\infty}\frac{n^2}{2}\ln\cos\left(\frac{2x}{n}\right)$ then $\lim\limits_{x\to 0}\frac{e^{-2g(x)}-e^{f(x)}}{x^6}$ equals.

(a) $\frac{8}{3}$ (b) $\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{2}{3}$

50. If $f(x)$ be a cubic polynomial and $\lim\limits_{x\to 0}\frac{\sin^2 x}{f(x)}=\frac{1}{3}$ then $f(1)$ can not be equal to :

(a) 0 (b) -5 (c) 3 (d) -2

51. $\lim\limits_{x\to 0}\frac{2e^{\sin x}-e^{-\sin x}-1}{x^2+2x}$ equals to :

(a) $\frac{3}{2}$ (b) $e^{3/2}$ (c) 2 (d) e^2

52. If $x_1, x_2, x_3, \ldots\ldots, x_n$ are the roots of $x^n + ax + b = 0$, then the value of $(x_1 - x_2)(x_1 - x_3)$ $(x_1 - x_4)\ldots\ldots(x_1 - x_n)$ is equal to :

(a) $nx_1 + b$ (b) $nx_1^{n-1} + a$ (c) nx_1^{n-1} (d) nx_1^{n-1}

53. $\lim\limits_{x \to 0} \dfrac{\sqrt[3]{1+\sin^2 x} - \sqrt[4]{1-2\tan x}}{\sin x + \tan^2 x}$ is equal to :

(a) -1 (b) 1 (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

54. If $f(x) = \begin{vmatrix} x\cos x & 2x\sin x & x\tan x \\ 1 & x & 1 \\ 1 & 2x & 1 \end{vmatrix}$, find $\lim\limits_{x \to 0} \dfrac{f(x)}{x^2}$.

(a) 0 (b) 1 (c) -1 (d) Does not exist

Answers

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(d)	**5.**	(b)	**6.**	(b)	**7.**	(d)	**8.**	(a)	**9.**	(c)	**10.**	(b)
11.	(a)	**12.**	(a)	**13.**	(b)	**14.**	(d)	**15.**	(b)	**16.**	(a)	**17.**	(c)	**18.**	(a)	**19.**	(a)	**20.**	(c)
21.	(c)	**22.**	(c)	**23.**	(a)	**24.**	(d)	**25.**	(c)	**26.**	(a)	**27.**	(d)	**28.**	(c)	**29.**	(d)	**30.**	(d)
31.	(c)	**32.**	(a)	**33.**	(c)	**34.**	(d)	**35.**	(b)	**36.**	(c)	**37.**	(a)	**38.**	(b)	**39.**	(b)	**40.**	(b)
41.	(a)	**42.**	(d)	**43.**	(d)	**44.**	(c)	**45.**	(b)	**46.**	(d)	**47.**	(a)	**48.**	(c)	**49.**	(a)	**50.**	(c)
51.	(a)	**52.**	(b)	**53.**	(c)	**54.**	(c)												

Exercise-2 : One or More than One Answer is/are Correct

1. If $\lim_{x\to 0}(p\tan qx^2 - 3\cos^2 x + 4)^{1/(3x^2)} = e^{5/3}$; $p, q \in R$ then :

(a) $p=\sqrt{2}, q=\frac{1}{2\sqrt{2}}$ (b) $p=\frac{1}{\sqrt{2}}, q=2\sqrt{2}$ (c) $p=1, q=2$ (d) $p=2, q=4$

2. $\lim_{x\to\infty} 2\left(\sqrt{25x^2+x}-5x\right)$ is equal to :

(a) $\lim_{x\to 0}\frac{2x-\log_e(1+x)^2}{5x^2}$

(b) $\lim_{x\to 0}\frac{e^{-x}-1+x}{x^2}$

(c) $\lim_{x\to 0}\frac{2(1-\cos x^2)}{5x^4}$

(d) $\lim_{x\to 0}\frac{\sin\frac{x}{5}}{x}$

3. Let $\lim_{x\to\infty}(2^x+a^x+e^x)^{1/x}=L$

which of the following statement(s) is(are) correct ?

(a) if $L=a\,(a>0)$, then the range of a is $[e,\infty)$

(b) if $L=2e(a>0)$, then the range of a is $\{2e\}$

(c) if $L=e\,(a>0)$, then the range of a is $(0,e]$

(d) if $L=2a\,(a>1)$, then the range of a is $\left(\frac{e}{2},\infty\right)$

4. Let $\tan\alpha\cdot x+\sin\alpha\cdot y=\alpha$ and $\alpha\operatorname{cosec}\alpha\cdot x+\cos\alpha\cdot y=1$ be two variable straight lines, α being the parameter. Let P be the point of intersection of the lines. In the limiting position when $\alpha\to 0$, the point P lies on the line :

(a) $x=2$ (b) $x=-1$ (c) $y+1=0$ (d) $y=2$

5. Let $f:R\to[-1,1]$ be defined as $f(x)=\cos(\sin x)$, then which of the following is(are) correct ?

(a) f is periodic with fundamental period 2π (b) Range of $f=[\cos 1, 1]$

(c) $\lim_{x\to\frac{\pi}{2}}\left(f\left(\frac{\pi}{2}-x\right)+f\left(\frac{\pi}{2}+x\right)\right)=2$ (d) f is neither even nor odd function

6. Let $f(x)=x+\sqrt{x^2+2x}$ and $g(x)=\sqrt{x^2+2x}-x$, then :

(a) $\lim_{x\to\infty}g(x)=1$ (b) $\lim_{x\to\infty}f(x)=1$ (c) $\lim_{x\to-\infty}f(x)=-1$ (d) $\lim_{x\to\infty}g(x)=-1$

7. Which of the following limits does not exist ?

(a) $\lim_{x\to\infty}\operatorname{cosec}^{-1}\left(\frac{x}{x+7}\right)$

(b) $\lim_{x\to 1}\sec^{-1}(\sin^{-1}x)$

(c) $\lim_{x\to 0^+}x^{\frac{1}{x}}$

(d) $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{8}+x\right)\right)^{\cot x}$

8. If $f(x) = \lim_{n\to\infty} x\left(\frac{3}{2}+[\cos x]\left(\sqrt{n^2+1}-\sqrt{n^2-3n+1}\right)\right)$ where $[y]$ denotes largest integer $\le y$, then identify the correct statement(s).

(a) $\lim_{x\to 0} f(x) = 0$

(b) $\lim_{x\to \frac{\pi}{2}} f(x) = \frac{3\pi}{4}$

(c) $f(x) = \frac{3x}{2}\ \forall\, x \in \left[0, \frac{\pi}{2}\right]$

(d) $f(x) = 0\ \forall\, x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

9. Let $f: R \to R$; $f(x) = \begin{cases} (-1)^n & \text{if } \ x = \frac{1}{2^{2^n}},\quad n = 1, 2, 3 \ldots\ldots \\ 0 & \text{otherwise} \end{cases}$

then identify the correct statement(s).

(a) $\lim_{x\to 0} f(x) = 0$

(b) $\lim_{x\to 0} f(x)$ does not exist

(c) $\lim_{x\to 0} f(x) f(2x) = 0$

(d) $\lim_{x\to 0} f(x) f(2x)$ does not exist

10. If $\lim_{x\to a} f(x) = \lim_{x\to a} [f(x)]$ ([·] denotes the greatest integer function) and $f(x)$ is non-constant continuous function, then :

(a) $\lim_{x\to a} f(x)$ is an integer

(b) $\lim_{x\to a} f(x)$ is non-integer

(c) $f(x)$ has local maximum at $x = a$

(d) $f(x)$ has local minimum at $x = a$

11. Let $f(x) = \frac{\cos^{-1}(1-\{x\})\sin^{-1}(1-\{x\})}{\sqrt{2\{x\}}(1-\{x\})}$ where $\{x\}$ denotes the fractional part of x, then :

(a) $\lim_{x\to 0^+} f(x) = \frac{\pi}{4}$

(b) $\lim_{x\to 0^+} f(x) = \sqrt{2} \lim_{x\to 0^-} f(x)$

(c) $\lim_{x\to 0^-} f(x) = \frac{\pi}{4\sqrt{2}}$

(d) $\lim_{x\to 0^-} f(x) = \frac{\pi}{2\sqrt{2}}$

12. If $\lim_{x\to 0} \frac{(\sin(\sin x) - \sin x)}{ax^3 + bx^5 + c} = -\frac{1}{12}$, then:

(a) $a = 2$

(b) $a = -2$

(c) $c = 0$

(d) $b \in R$

13. If $f(x) = \lim_{n\to\infty} (n(x^{1/n} - 1))$ for $x > 0$, then which of the following is/are true ?

(a) $f\left(\frac{1}{x}\right) = 0$

(b) $f\left(\frac{1}{x}\right) = \frac{1}{f(x)}$

(c) $f\left(\frac{1}{x}\right) = -f(x)$

(d) $f(xy) = f(x) + f(y)$

14. The value of $\lim\limits_{n\to\infty} \cos^2 (\pi (\sqrt[3]{n^3 + n^2 + 2n}))$ (where $n \in N$):

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{9}$

15. If $\alpha, \beta \in \left(-\frac{\pi}{2}, 0\right)$ such that $(\sin\alpha + \sin\beta) + \frac{\sin\alpha}{\sin\beta} = 0$ and $(\sin\alpha + \sin\beta) \frac{\sin\alpha}{\sin\beta} = -1$ and

$\lambda = \lim\limits_{n\to\infty} \frac{1 + (2\sin\alpha)^{2n}}{(2\sin\beta)^{2n}}$ then :

(a) $a = -\frac{\pi}{6}$ (b) $\lambda = 2$ (c) $\alpha = -\frac{\pi}{3}$ (d) $\lambda = 1$

16. Let $f(x) = \begin{cases} |x-2| + a^2 - 6a + 9 & , \; x < 2 \\ 5 - 2x & , \; x \geq 2 \end{cases}$

If $\lim\limits_{x\to 2} [f(x)]$ exists, the possible values a can take is/are (where $[\cdot]$ represents the greatest integer function)

(a) 2 (b) $\frac{5}{2}$ (c) 3 (d) $\frac{7}{2}$

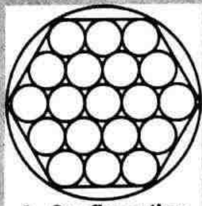
Answers

1.	(b, c)	2.	(a, c, d)	3.	(a, b, c)	4.	(a, c)	5.	(b, c)	6.	(a, c)
7.	(a, d)	8.	(a, c, d)	9.	(b, c)	10.	(a, d)	11.	(b, d)	12.	(a, c)
13.	(c, d)	14.	(c)	15.	(a, b)	16.	(b)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

A circular disk of unit radius is filled with a number of smaller circular disks arranged in the form of hexagon. Let A_n denotes a stack of disks arranged in the shape of a hexagon having 'n' disks on a side. The figure shows the configuration A_3. If 'A' be the area of large disk, S_n be the number of disks in A_n configuration and r_n be the radius of each disk in A_n configuration, then



A_3 Configuration

1. $\lim\limits_{n\to\infty} \dfrac{S_n}{n^2}$:

(a) 3 (b) 4 (c) 1 (d) 11

2. $\lim\limits_{n\to\infty} nr_n$:

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{11}$

Paragraph for Question Nos. 3 to 4

Let $f(x) = \begin{bmatrix} x+3 & ; & -2 < x < 0 \\ 4 & ; & x = 0 \\ 2x+5 & ; & 0 < x < 1 \end{bmatrix}$, then

3. $\lim\limits_{x\to 0^-} f([x - \tan x])$ is : ([·] denotes greatest integer fucntion)

(a) 2 (b) 4 (c) 5 (d) None of these

4. $\lim\limits_{x\to 0^+} f\left(\left\{\dfrac{x}{\tan x}\right\}\right)$ is : ({·} denotes fractional part of function)

(a) 4 (b) 5 (c) 7 (d) None of these

Paragraph for Question Nos. 5 to 6

A certain function $f(x)$ has the property that $f(3x) = \alpha f(x)$ for all positive real values of x and $f(x) = 1 - |x-2|$ for $1 \le x \le 3$.

5. $\lim\limits_{x\to 2} (f(x))^{\operatorname{cosec}\left(\frac{\pi x}{2}\right)}$ is :

(a) $\dfrac{2}{\pi}$ (b) $-\dfrac{2}{\pi}$

(c) $e^{2/\pi}$ (d) Non-existent

6. If the total area bounded by $y = f(x)$ and x-axis in $[1, \infty)$ converges to a finite quantity, then the range of α is :

(a) $(-1, 1)$ (b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (c) $\left(-\frac{1}{3}, \frac{1}{3}\right)$ (d) $\left(-\frac{1}{4}, \frac{1}{4}\right)$

Paragraph for Question Nos. 7 to 9

Consider the limit $\lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{(1+ax)}{(1+bx)}\right)$ exists, finite and has the value equal to l (where a, b are real constants), then :

7. $a =$

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

8. $a + b =$

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

9. $\left|\frac{b}{l}\right| =$

(a) 38 (b) 16 (c) 72 (d) 24

Paragraph for Question Nos. 10 to 11

For the curve $\sin x + \sin y = 1$ lying in the first quadrant there exists a constant α for which

$$\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2} = L \text{ (not zero)}$$

10. The value of α :

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{3}{2}$ (d) 2

11. The value of L :

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{2\sqrt{3}}$

Answers

1.	(a)	2.	(b)	3.	(b)	4.	(c)	5.	(d)	6.	(c)	7.	(d)	8.	(c)	9.	(d)	10.	(c)
11.	(c)																		

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	$\lim\limits_{n\to\infty}\left(\dfrac{1+\sqrt[n]{4}}{2}\right)^n =$	**(P)**	2
(B)	Let $f(x)=\lim\limits_{n\to\infty}\dfrac{2x}{\pi}\tan^{-1}(nx)$, then $\lim\limits_{x\to 0^+} f(x) =$	**(Q)**	0
(C)	$\lim\limits_{x\to\frac{\pi}{2}^+}\dfrac{\cos(\tan^{-1}(\tan x))}{x-\frac{\pi}{2}} =$	**(R)**	1
(D)	If $\lim\limits_{x\to 0^+}(x)^{\frac{1}{\ln\sin x}} = e^L$, then $L+2 =$	**(S)**	3
		(T)	Non-existent

2. [·] represents greatest integer function :

	Column-I		Column-II
(A)	If $f(x)=\sin^{-1}x$ and $\lim\limits_{x\to\frac{1}{2}^+} f(3x-4x^3) = a-3\lim\limits_{x\to\frac{1}{2}^-} f(x)$, then $[a] =$	**(P)**	2
(B)	If $f(x)=\tan^{-1}g(x)$ where $g(x)=\dfrac{3x-x^3}{1-3x^2}$ and then find $\left[\lim\limits_{h\to 0}\dfrac{f\left(\frac{1}{2}+6h\right)-f\left(\frac{1}{2}\right)}{6h}\right] =$	**(Q)**	3
(C)	If $\cos^{-1}(4x^3-3x) = a+b\cos^{-1}x$ for $-1<x<\dfrac{-1}{2}$, then $[a+b+2] =$	**(R)**	4
(D)	If $f(x)=\cos^{-1}(4x^3-3x)$ and $\lim\limits_{x\to\frac{1}{2}^+} f'(x)=a$ and $\lim\limits_{x\to\frac{1}{2}^-} f'(x)=b$, then $a+b+3 =$	**(S)**	−2
		(T)	Non existent

Answers

1. A → P ; B → Q ; C → R ; D → S

2. A → Q ; B → P ; C → S ; D → Q

Exercise-5 : Subjective Type Problems

1. If $\lim_{x\to 0} \dfrac{\ln \cot\left(\dfrac{\pi}{4}-\beta x\right)}{\tan \alpha x} = 1$, then $\dfrac{\alpha}{\beta} =$......

2. If $\lim_{x\to 0} \dfrac{f(x)}{\sin^2 x} = 8$, $\lim_{x\to 0} \dfrac{g(x)}{2\cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x\to 0} (1+2f(x))^{\frac{1}{g(x)}} = \dfrac{1}{e}$, then $\lambda =$

3. If α, β are two distinct real roots of the equation $ax^3 + x - 1 - a = 0$, $(a \neq -1, 0)$, none of which is equal to unity, then the value of $\lim_{x\to(1/\alpha)} \dfrac{(1+a)x^3 - x^2 - a}{(e^{1-\alpha x}-1)(x-1)}$ is $\dfrac{al(k\alpha - \beta)}{\alpha}$. Find the value of kl.

4. The value of $\lim_{x\to 0} \dfrac{(140)^x - (35)^x - (28)^x - (20)^x + 7^x + 5^x + 4^x - 1}{x \sin^2 x} = 2 \ln 2 \ln k \ln 7$, then $k =$

5. If $\lim_{x\to 0} \dfrac{a \cot x}{x} + \dfrac{b}{x^2} = \dfrac{1}{3}$, then $b - a =$

6. Find the value of $\lim_{x\to\infty} \left(x + \dfrac{1}{x}\right) e^{1/x} - x$.

7. Find $\lim_{x\to \alpha^+} \left[\dfrac{\min(\sin x, \{x\})}{x-1}\right]$ where α is root of equation $\sin x + 1 = x$ (here $[\cdot]$ represent greatest integer and $\{\cdot\}$ represent fractional part function)

Answers

1.	2	**2.**	8	**3.**	1	**4.**	5	**5.**	2	**6.**	1	**7.**	0

□□□

Chapter 3 – Continuity, Differentiability and Differentiation

3 CONTINUITY, DIFFERENTIABILITY AND DIFFERENTIATION

Exercise-1 : Single Choice Problems

1. Let 'f' be a differentiable real valued function satisfying $f(x+2y)=f(x)+f(2y)+6xy(x+2y)\ \forall\ x,y\in R$. Then $f''(0), f''(1), f''(2)\ldots\ldots$ are in :

(a) AP (b) GP (c) HP (d) None of these

2. The number of points of non-differentiability for $f(x)=\max\left\{||x|-1|, \frac{1}{2}\right\}$ is :

(a) 4 (b) 3 (c) 2 (d) 5

3. Number of points of discontinuity of $f(x)=\left\{\frac{x}{5}\right\}+\left[\frac{x}{2}\right]$ in $x\in[0,100]$ is/are (where $[\cdot]$ denotes greatest integer function and $\{\cdot\}$ denotes fractional part function)

(a) 50 (b) 51 (c) 52 (d) 61

4. If $f(x)$ has isolated point of discontinuity at $x=a$ such that $|f(x)|$ is continuous at $x=a$ then :

(a) $\lim_{x\to a} f(x)$ does not exist (b) $\lim_{x\to a} f(x)+f(a)=0$

(c) $f(a)=0$ (d) None of these

5. If $f(x)$ is a thrice differentiable function such that, $\lim_{x\to 0}\frac{f(4x)-3f(3x)+3f(2x)-f(x)}{x^3}=12$ then the value of $f'''(0)$ equals to :

(a) 0 (b) 1 (c) 12 (d) None of these

6. $y=\frac{1}{1+(\tan\theta)^{\sin\theta-\cos\theta}+(\cot\theta)^{\cos\theta-\cot\theta}}+\frac{1}{1+(\tan\theta)^{\cos\theta-\sin\theta}+(\cot\theta)^{\sin\theta-\cot\theta}}$

$+\frac{1}{1+(\tan\theta)^{\cos\theta-\cot\theta}+(\cot\theta)^{\cot\theta-\sin\theta}}$ then $\frac{dy}{dx}$ at $\theta=\pi/3$ is :

(a) 0 (b) 1

(c) $\sqrt{3}$ (d) None of these

7. Let $f'(x)=\sin(x^2)$ and $y=f(x^2+1)$ then $\frac{dy}{dx}$ at $x=1$ is :

(a) $2\sin 2$ (b) $2\cos 2$ (c) $2\sin 4$ (d) $\cos 2$

8. If $f(x) = |\sin x - |\cos x||$, then $f'\left(\frac{7\pi}{6}\right) =$

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{1-\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}-1}{2}$ (d) $\frac{-1-\sqrt{3}}{2}$

9. If $2\sin x \cdot \cos y = 1$, then $\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is ....

(a) -4 (b) -2 (c) -6 (d) 0

10. f is a differentiable function such that $x = f(t^2), y = f(t^3)$ and $f'(1) \neq 0$ if $\left(\frac{d^2y}{dx^2}\right)_{t=1} =$

(a) $\frac{3}{4}\left(\frac{f''(1)+f'(1)}{(f'(1))^2}\right)$ (b) $\frac{3}{4}\left(\frac{f'(1)\cdot f''(1)-f''(1)}{(f'(1))^2}\right)$

(c) $\frac{4}{3}\frac{f''(1)}{(f'(1))^2}$ (d) $\frac{4}{3}\left(\frac{f'(1)f''(1)-f''(1)}{(f'(1))^2}\right)$

11. Let $f(x) = \begin{cases} ax+1 & \text{if } x<1 \\ 3 & \text{if } x=1 \\ bx^2+1 & \text{if } x>1 \end{cases}$. If $f(x)$ is continuous at $x = 1$ then $(a-b)$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 4

12. If $y = 1 + \frac{\alpha}{\left(\frac{1}{x}-\alpha\right)} + \frac{\beta/x}{\left(\frac{1}{x}-\alpha\right)\left(\frac{1}{x}-\beta\right)} + \frac{\gamma/x^2}{\left(\frac{1}{x}-\alpha\right)\left(\frac{1}{x}-\beta\right)\left(\frac{1}{x}-\gamma\right)}$, then $\frac{dy}{dx}$ is :

(a) $y\left(\frac{\alpha}{\alpha-x}+\frac{\beta}{\beta-x}+\frac{\gamma}{\gamma-x}\right)$ (b) $\frac{y}{x}\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$

(c) $y\left(\frac{\alpha}{1/x-\alpha}+\frac{\beta}{1/x-\beta}+\frac{\gamma}{1/x-\gamma}\right)$ (d) $\frac{y}{x}\left(\frac{\alpha/x}{1/x-\alpha}+\frac{\beta/x}{1/x-\beta}+\frac{\gamma/x}{1/x-\gamma}\right)$

13. If $f(x) = \sqrt{\frac{1+\sin^{-1}x}{1-\tan^{-1}x}}$; then $f'(0)$ is equal to :

(a) 4 (b) 3 (c) 2 (d) 1

14. Let $f(x) = \begin{cases} \sin^2 x & , x \text{ is rational} \\ -\sin^2 x & , x \text{ is irrational} \end{cases}$, then set of points, where $f(x)$ is continuous, is :

(a) $\left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$ (b) a null set

(c) $\{n\pi, n \in I\}$ (d) set of all rational numbers

15. The number of values of x in $(0, 2\pi)$ where the function $f(x) = \frac{\tan x + \cot x}{2} - \left|\frac{\tan x - \cot x}{2}\right|$ is continuous but non-derivable :

(a) 3 (b) 4 (c) 0 (d) 1

16. If $f(x) = |x-1|$ and $g(x) = f(f(f(x)))$, then $g'(x)$ is equal to :

(a) 1 for $x > 2$ (b) 1 for $2 < x < 3$ (c) -1 for $2 < x < 3$ (d) -1 for $x > 3$

17. If $f(x)$ is a continuous function $\forall\, x \in R$ and the range of $f(x)$ is $(2, \sqrt{26})$ and $g(x) = \left[\frac{f(x)}{C}\right]$ is continuous $\forall\, x \in R$, then the least positive integral value of C is : (where $[\cdot]$ denotes the greatest integer function.)

(a) 3 (b) 5 (c) 6 (d) 7

18. If $y = x + e^x$, then $\left(\frac{d^2x}{dy^2}\right)_{x=\ln 2}$ is :

(a) $-\frac{1}{9}$ (b) $-\frac{2}{27}$ (c) $\frac{2}{27}$ (d) $\frac{1}{9}$

19. Let $f(x) = x^3 + 4x^2 + 6x$ and $g(x)$ be its inverse then the value of $g'(-4)$:

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) None of these

20. If $f(x) = 2 + |x| - |x-1| - |x+1|$, then $f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) + f'\left(\frac{5}{2}\right)$ is equal to :

(a) 1 (b) -1 (c) 2 (d) -2

21. If $f(x) = \cos(x^2 - 4[x])$; $0 < x < 1$, (where $[\cdot]$ denotes greatest integer function) then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is equal to :

(a) $-\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) 0 (d) $\sqrt{\frac{\pi}{4}}$

22. Let $g(x)$ be the inverse of $f(x)$ such that $f'(x) = \frac{1}{1+x^5}$, then $\frac{d^2(g(x))}{dx^2}$ is equal to :

(a) $\frac{1}{1+(g(x))^5}$ (b) $\frac{g'(x)}{1+(g(x))^5}$

(c) $5(g(x))^4(1+(g(x))^5)$ (d) $1+(g(x))^5$

23. Let $f(x) = \begin{cases} \min(x, x^2) & x \geq 0 \\ \max(2x, x-1) & x < 0 \end{cases}$, then which of the following is not true ?

(a) $f(x)$ is not differentiable at $x = 0$

(b) $f(x)$ is not differentiable at exactly two points

(c) $f(x)$ is continuous everywhere

(d) $f(x)$ is strictly increasing $\forall\, x \in R$

24. If $f(x) = \lim\limits_{n\to\infty}\left(\prod_{i=1}^{n}\cos\left(\frac{x}{2^i}\right)\right)$ then $f'(x)$ is equal to :

(a) $\frac{\sin x}{x}$ (b) $\frac{x}{\sin x}$ (c) $\frac{x\cos x - \sin x}{x^2}$ (d) $\frac{\sin x - x\cos x}{\sin^2 x}$

25. Let $f(x) = \begin{cases} \frac{1-\tan x}{4x-\pi} & x \neq \frac{\pi}{4} \\ \lambda & x = \frac{\pi}{4} \end{cases}; x \in \left[0, \frac{\pi}{2}\right)$.

If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right)$ then λ is equal to :

(a) 1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -1

26. Let $f(x) = \begin{cases} e^{-\frac{1}{x^2}}\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$, then $f'(0) =$

(a) 1 (b) -1 (c) 0 (d) Does not exist

27. Let f be a differentiable function satisfying $f'(x) = 2f(x) + 10\ \forall\, x \in R$ and $f(0) = 0$, then the number of real roots of the equation $f(x) + 5\sec^2 x = 0$ in $(0, 2\pi)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

28. If $f(x) = \begin{cases} \frac{\sin\{\cos x\}}{x - \frac{\pi}{2}} & x \neq \frac{\pi}{2} \\ 1 & x = \frac{\pi}{2} \end{cases}$, where $\{k\}$ represents the fractional part of k, then :

(a) $f(x)$ is continuous at $x = \frac{\pi}{2}$

(b) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ does not exist

(c) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x = \frac{\pi}{2}$

(d) $\lim\limits_{x\to\frac{\pi}{2}^-} f(x) = 1$

29. Let $f(x)$ be a polynomial in x. The second derivative of $f(e^x)$ w.r.t. x is :

(a) $f''(e^x)e^x + f'(e^x)$ (b) $f''(e^x)e^{2x} + f'(e^x)e^{2x}$

(c) $f''(e^x)e^x + f'(e^x)e^{2x}$ (d) $f''(e^x)e^{2x} + e^x f'(e^x)$

30. If $e^{f(x)} = \log_e x$ and $g(x)$ is the inverse function of $f(x)$, then $g'(x)$ is equal to :

(a) $e^x + x$ (b) $e^{e^{e^x}} e^{e^x} e^x$ (c) $e^{e^x + x}$ (d) e^{e^x}

31. If $y = f(x)$ is differentiable $\forall\, x \in R$, then

(a) $y = |f(x)|$ is differentiable $\forall\, x \in R$

(b) $y = f^2(x)$ is non-differentiable for atleast one x

(c) $y = f(x)|f(x)|$ is non-differentiable for atleast one x

(d) $y = |f(x)|^3$ is differentiable $\forall\, x \in R$

32. If $f(x) = (x-1)^4 (x-2)^3 (x-3)^2$ then the value of $f'''(1) + f''(2) + f'(3)$ is :

(a) 0 (b) 1 (c) 2 (d) 6

33. If $f(x) = \left(\frac{x}{2}\right) - 1$, then on the interval $[0, \pi]$:

(a) $\tan(f(x))$ and $\frac{1}{f(x)}$ are both continuous

(b) $\tan(f(x))$ and $\frac{1}{f(x)}$ are both discontinuous

(c) $\tan(f(x))$ and $f^{-1}(x)$ are both continuous

(d) $\tan f(x)$ is continuous but $f^{-1}(x)$ is not

34. Let $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x-2}} - 3}{3^{\frac{1}{x-2}} + 1} & x > 2 \\ \dfrac{b \sin\{-x\}}{\{-x\}} & x < 2 \\ c & x = 2 \end{cases}$, where $\{\cdot\}$ denotes fraction part function, is continuous at $x = 2$,

then $b + c =$

(a) 0 (b) 1 (c) 2 (d) 4

35. Let $f(x) = \dfrac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continuous function at $x = 0$. The value of $f(0)$ equals :

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 2

36. Let $f(x) = \begin{cases} (1+ax)^{1/x} & x < 0 \\ b & x = 0 \\ \dfrac{(x+c)^{1/3} - 1}{(x+1)^{1/2} - 1} & x > 0 \end{cases}$, is continuous at $x = 0$, then $3(e^a + b + c)$ is equal to :

(a) 3 (b) 6 (c) 7 (d) 8

37. If $\sqrt{x+y} + \sqrt{y-x} = 5$, then $\dfrac{d^2y}{dx^2} =$

(a) $\dfrac{2}{5}$ (b) $\dfrac{4}{25}$ (c) $\dfrac{2}{25}$ (d) $\dfrac{1}{25}$

38. If $f(x) = x^3 + x^4 + \log x$ and g is the inverse of f, then $g'(2)$ is :

(a) 8 (b) $\dfrac{1}{8}$ (c) 2 (d) $\dfrac{1}{4}$

39. The number of points at which the function,

$$f(x) = \begin{cases} \min\{|x|, x^2\} & \text{if } x \in (-\infty, 1) \\ \min\{2x-1, x^2\} & \text{otherwise} \end{cases}$$

is not differentiable is :

(a) 0 (b) 1 (c) 2 (d) 3

40. If $f(x)$ is a function such that $f(x) + f''(x) = 0$ and $g(x) = (f(x))^2 + (f'(x))^2$ and $g(3) = 8$, then $g(8) =$

(a) 0 (b) 3 (c) 5 (d) 8

41. Let f is twice differentiable on R such that $f(0) = 1, f'(0) = 0$ and $f''(0) = -1$, then for $a \in R$,

$$\lim_{x \to \infty} \left(f\left(\frac{a}{\sqrt{x}} \right) \right)^x =$$

(a) e^{-a^2} (b) $e^{-\frac{a^2}{4}}$ (c) $e^{-\frac{a^2}{2}}$ (d) e^{-2a^2}

42. Let $f_1(x) = e^x$ and $f_{n+1}(x) = e^{f_n(x)}$ for any $n \geq 1, n \in N$. Then for any fixed n, the value of $\dfrac{d}{dx} f_n(x)$ equals :

(a) $f_n(x)$ (b) $f_n(x) f_{n-1}(x) \ldots\ldots f_2(x) f_1(x)$

(c) $f_n(x) f_{n-1}(x)$ (d) $f_n(x) f_{n-1}(x) \ldots\ldots f_2(x) f_1(x) e^x$

43. If $y = \tan^{-1}\left(\dfrac{x^{1/3} - a^{1/3}}{1 + x^{1/3} a^{1/3}} \right)$, $x > 0, a > 0$, then $\dfrac{dy}{dx}$ is :

(a) $\dfrac{1}{x^{2/3}(1 + x^{2/3})}$ (b) $\dfrac{3}{x^{2/3}(1 + x^{2/3})}$ (c) $\dfrac{1}{3x^{2/3}(1 + x^{2/3})}$ (d) $\dfrac{1}{3x^{1/3}(1 + x^{2/3})}$

44. The value of $k + f(0)$ so that $f(x) = \begin{cases} \dfrac{\sin(4k-1)x}{3x} & , \quad x < 0 \\ \dfrac{\tan(4k+1)x}{5x} & , \quad 0 < x < \dfrac{\pi}{2} \\ 1 & , \quad x = 0 \end{cases}$ can be made continuous at $x = 0$ is :

(a) 1 (b) 2 (c) $\frac{5}{4}$ (d) 0

45. If $y = \tan^{-1}\left(\dfrac{x}{1+\sqrt{1-x^2}}\right)$, $|x| \le 1$, then $\dfrac{dy}{dx}$ at $\left(\dfrac{1}{2}\right)$ is :

(a) $\frac{1}{\sqrt{3}}$ (b) 3 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

46. Let $f(x) = \dfrac{e^x x\cos x - x\log_e(1+x) - x}{x^2}$, $x \ne 0$. If $f(x)$ is continuous at $x = 0$, then $f(0)$ is equal to :

(a) 0 (b) 1 (c) -1 (d) 2

47. A function $f(x) = \max(\sin x, \cos x, 1 - \cos x)$ is non-derivable for n values of $x \in [0, 2\pi]$. Then the value of n is :

(a) 2 (b) 1 (c) 3 (d) 4

48. Let g be the inverse function of a differentiable function f and $G(x) = \dfrac{1}{g(x)}$. If $f(4) = 2$ and $f'(4) = \dfrac{1}{16}$, then the value of $(G'(2))^2$ equals to :

(a) 1 (b) 4 (c) 16 (d) 64

49. If $f(x) = \max.\left(x^4, x^2, \dfrac{1}{81}\right)$ $\forall\, x \in [0, \infty)$, then the sum of the square of reciprocal of all the values of x where $f(x)$ is non-differentiable, is equal to :

(a) 1 (b) 81 (c) 82 (d) $\frac{82}{81}$

50. If $f(x)$ is derivable at $x = 2$ such that $f(2) = 2$ and $f'(2) = 4$, then the value of

$$\lim_{h\to 0} \frac{1}{h^2}(\ln(f(2+h^2)) - \ln(f(2-h^2)))$$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

51. Let $f(x) = (x^2 - 3x + 2)\,|(x^3 - 6x^2 + 11x - 6)| + \left|\sin\left(x + \dfrac{\pi}{4}\right)\right|$.

Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$, is :

(a) 5 (b) 4 (c) 3 (d) 2

52. Let f and g be differentiable functions on R (the set of all real numbers) such that $g(1)=2=g'(1)$ and $f'(0)=4$. If $h(x)=f(2xg(x)+\cos\pi x-3)$ then $h'(1)$ is equal to :

(a) 28 (b) 24 (c) 32 (d) 18

53. If $f(x)=\dfrac{(x+1)^7\sqrt{1+x^2}}{(x^2-x+1)^6}$, then the value of $f'(0)$ is equal to :

(a) 10 (b) 11 (c) 13 (d) 15

54. Statement-1 : The function $f(x)=\lim\limits_{n\to\infty}\dfrac{\log_e(1+x)-x^{2n}\sin(2x)}{1+x^{2n}}$ is discontinuous at $x=1$.

Statement-2 : L. H. L = R. H. L $\neq f(1)$.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

55. If $f(x)=\begin{bmatrix} x & ; & \text{if } x \text{ is rational} \\ 1-x & ; & \text{if } x \text{ is irrational}\end{bmatrix}$, then number of points for $x\in R$, where $y=f(f(x))$ is discontinuous is :

(a) 0 (b) 1 (c) 2 (d) Infinitely many

56. Number of points where $f(x)=\begin{cases}\max(|x^2-x-2|, x^2-3x) & ; \ x\geq 0 \\ \max(\ln(-x), e^x) & ; \ x<0\end{cases}$

is non-differentiable will be :

(a) 1 (b) 2 (c) 3 (d) None of these

57. If the function $f(x)=-4e^{\frac{1-x}{2}}+1+x+\dfrac{x^2}{2}+\dfrac{x^3}{3}$ and $g(x)=f^{-1}(x)$, then the value of $g'\left(\dfrac{-7}{6}\right)$ equals to :

(a) $\dfrac{1}{5}$ (b) $-\dfrac{1}{5}$ (c) $\dfrac{6}{7}$ (d) $-\dfrac{6}{7}$

58. Find k; if possible ; so that

$$f(x)=\begin{bmatrix}\dfrac{\ln(2-\cos 2x)}{\ln^2(1+\sin 3x)} & ; \ x<0 \\ k & ; \ x=0 \\ \dfrac{e^{\sin 2x}-1}{\ln(1+\tan 9x)} & ; \ x>0\end{bmatrix}$$

is continuous at $x=0$.

(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{9}$ (c) $\dfrac{2}{9}$ (d) Not possible

59. Let $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}$ then the value of $\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3$ is :

(a) 2 (b) 0 (c) –1 (d) –2

60. If $y^{-2} = 1 + 2\sqrt{2}\cos 2x$, then :

$$\frac{d^2y}{dx^2} = y\,(py^2 + 1)(qy^2 - 1) \text{ then the value of } (p+q) \text{ equals to :}$$

(a) 7 (b) 8 (c) 9 (d) 10

61. Let $f: R \to R$ is not identically zero, differentiable function and satisfy the equations $f(xy) = f(x)f(y)$ and $f(x+z) = f(x) + f(z)$, then $f(5) =$

(a) 3 (b) 5 (c) 10 (d) 15

62. Number of points at which the function $f(x) = \begin{cases} \min.(x, x^2) & \text{if } -\infty < x < 1 \\ \min.(2x-1, x^2) & \text{if } x \geq 1 \end{cases}$ is not derivable is :

(a) 0 (b) 1 (c) 2 (d) 3

63. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is :

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

64. If $g(x) = f\left(x - \sqrt{1-x^2}\right)$ and $f'(x) = 1 - x^2$ then $g'(x)$ equals to :

(a) $1-x^2$ (b) $\sqrt{1-x^2}$ (c) $2x\left(x+\sqrt{1-x^2}\right)$ (d) $2x\left(x-\sqrt{1-x^2}\right)$

65. Let $f(x) = \lim\limits_{n\to\infty} \frac{\log_e(2+x) - x^{2n}\sin x}{1+x^{2n}}$ then :

(a) $f(x)$ is continuous at $x = 1$ (b) $\lim\limits_{x\to 1^-} f(x) = \log_e 3$

(c) $\lim\limits_{x\to 1^+} f(x) = -\sin 1$ (d) $\lim\limits_{x\to 1^+} f(x)$ does not exist

66. Let $f(x+y) = f(x)f(y)$ for all x and y, and $f(5) = -2$, $f'(0) = 3$, then $f'(5)$ is equal to :

(a) 3 (b) 1 (c) –6 (d) 6

67. Let $f(x) = \lim\limits_{n\to\infty} \frac{\log_e(2+x) - x^{2n}\sin x}{1+x^{2n}}$ then :

(a) $f(x)$ is continuous at $x = 1$ (b) $\lim\limits_{x\to 1^+} f(x) = \log_e 3$

(c) $\lim\limits_{x\to 1^+} f(x) = -\sin 1$ (d) $\lim\limits_{x\to 1^-} f(x)$ does not exist

68. If $f(x)=\begin{cases}\dfrac{x-e^x+1-\{1-\cos 2x\}}{x^2} & x\neq 0\\ k & x=0\end{cases}$ is continuous at $x=0$ then, which of the following statement is false ?

(a) $k=\dfrac{-5}{2}$ (b) $\{k\}=\dfrac{1}{2}$ (c) $[k]=-2$ (d) $[k]\{k\}=\dfrac{-3}{2}$

(where [·] denotes greatest integer function and {·} denotes fraction part function.)

69. Let $f(x)=||x^2-10x+21|-p|$; then the exhaustive set of values of p for which $f(x)$ has exactly 6 points of non-derivability; is :

(a) $(4,\infty)$ (b) $(0, 4)$ (c) $[0, 4]$ (d) $(-4, 4)$

70. If $f(x)=\sqrt{\dfrac{1+\sin^{-1}x}{1-\tan^{-1}x}}$; then $f'(0)$ is equal to :

(a) 4 (b) 3 (c) 2 (d) 1

71. For $t\in(0, 1)$; let $x=\sqrt{2^{\sin^{-1}t}}$ and $y=\sqrt{2^{\cos^{-1}t}}$,

then $1+\left(\dfrac{dy}{dx}\right)^2$ equals :

(a) $\dfrac{x^2}{y^2}$ (b) $\dfrac{y^2}{x^2}$ (c) $\dfrac{x^2+y^2}{y^2}$ (d) $\dfrac{x^2+y^2}{x^2}$

72. Let $f(x)=-1+|x-2|$ and $g(x)=1-|x|$ then set of all possible value(s) of x for which (fog) (x) is discontinuous is :

(a) $\{0, 1, 2\}$ (b) $\{0, 2\}$ (c) $\{0\}$ (d) an empty set

73. If $f(x)=[x]\tan(\pi x)$ then $f'(K^+)$ is equal to ($k\in I$ and [.] denotes greatest integer function):

(a) $(k-1)\pi(-1)^k$ (b) $k\pi$ (c) $k\pi(-1)^{k+1}$ (d) $(k-1)\pi(-1)^{k+1}$

74. If $f(x)=\begin{cases}\dfrac{ae^{\sin x}+be^{-\sin x}-c}{x^2}; & x\neq 0\\ 2 & ;\ x=0\end{cases}$ is continuous at $x=0$; then :

(a) $a=b=c$ (b) $a=2b=3c$ (c) $a=b=2c$ (d) $2a=2b=c$

75. If $\tan x\cdot\cot y=\sec\alpha$ where α is constant and $\alpha\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ then $\dfrac{d^2y}{dx^2}$ at $\left(\dfrac{\pi}{4},\dfrac{\pi}{4}\right)$ equals to :

(a) 0 (b) 1 (c) 2 (d) 3

76. If $y=(x-3)(x-2)(x-1)\times(x+1)(x+2)(x+3)$, then $\dfrac{d^2y}{dx^2}$ at $x=1$ is :

(a) -101 (b) 48 (c) 56 (d) 190

77. Let $f(x+y)=f(x)f(y)\ \forall\, x,y\in R, f(0)\neq 0$. If $f(x)$ is continuous at $x=0$, then $f(x)$ is continuous at :

(a) all natural numbers only
(b) all integers only
(c) all rational numbers only
(d) all real numbers

78. If $f(x)=3x^9-2x^4+2x^3-3x^2+x+\cos x+5$ and $g(x)=f^{-1}(x)$; then the value of $g'(6)$ equals :

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

79. If $y=f(x)$ and $z=g(x)$ then $\frac{d^2y}{dz^2}$ equals

(a) $\frac{g'f''-f'g''}{(g')^2}$ (b) $\frac{g'f''-f'g''}{(g')^3}$ (c) $\frac{f'g''-g'f''}{(g')^3}$ (d) None of these

80. Let $f(x)=\begin{cases} x+1 & ; & x<0 \\ |x-1| & ; & x\geq 0\end{cases}$ and $g(x)=\begin{cases} x+1 & ; & x<0 \\ (x-1)^2 & ; & x\geq 0\end{cases}$ then

the number of points where $g(f(x))$ is not differentiable.

(a) 0 (b) 1 (c) 2 (d) None of these

81. Let $f(x)=[\sin x]+[\cos x]$, $x\in[0,2\pi]$, where $[\cdot]$ denotes the greatest integer function, total number of points where $f(x)$ is non differentiable is equal to :

(a) 2 (b) 3 (c) 4 (d) 5

82. Let $f(x)=\cos x, g(x)=\begin{cases}\min\{f(t):0\leq t\leq x\} & , & x\in[0,\pi] \\ (\sin x)-1 & , & x>\pi\end{cases}$

Then

(a) $g(x)$ is discontinuous at $x=\pi$
(b) $g(x)$ is continuous for $x\in[0,\infty)$
(c) $g(x)$ is differentiable at $x=\pi$
(d) $g(x)$ is differentiable for $x\in[0,\infty)$

83. If $f(x)=(4+x)^n, n\in N$ and $f^r(0)$ represents the r^{th} derivative of $f(x)$ at $x=0$, then the value of $\sum_{r=0}^{\infty}\frac{f^r(0)}{r!}$ is equal to :

(a) 2^n (b) 3^n (c) 5^n (d) 4^n

84. Let $f(x)=\begin{cases}\frac{x}{1+|x|} & , & |x|\geq 1 \\ \frac{x}{1-|x|} & , & |x|<1\end{cases}$, then domain of $f'(x)$ is :

(a) $(-\infty,\infty)$ (b) $(-\infty,\infty)-\{-1,0,1\}$ (c) $(-\infty,\infty)-\{-1,1\}$ (d) $(-\infty,\infty)-\{0\}$

85. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the value of $g'\left(\frac{-7}{6}\right)$ equals :

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) $\frac{6}{7}$ (d) $-\frac{6}{7}$

86. The number of points at which the function $f(x) = (x-|x|)^2(1-x+|x|)^2$ is not differentiable in the interval $(-3, 4)$ is :

(a) Zero (b) One (c) Two (d) Three

87. If $f(x) = \sqrt{\frac{1+\sin^{-1} x}{1-\tan^{-1} x}}$; then $f'(0)$ is equal to :

(a) 4 (b) 3 (c) 2 (d) 1

88. If $f(x) = \begin{cases} e^{x-1} & ; \ 0 \le x \le 1 \\ x+1-\{x\} & ; \ 1 < x < 3 \end{cases}$ and $g(x) = x^2 - ax + b$ such that $f(x)g(x)$ is continuous in $[0, 3)$ then the ordered pair (a, b) is (where $\{\cdot\}$ denotes fractional part function) :

(a) $(2, 3)$ (b) $(1, 2)$ (c) $(3, 2)$ (d) $(2, 2)$

89. Use the following table and the fact that $f(x)$ is invertible and differentiable everywhere to find $f^{-1}(3)$:

x	$f(x)$	$f'(x)$
3	1	7
6	2	10
9	3	5

(a) 0 (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{7}$

90. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x} & , \ x \ne 0 \\ 0 & , \ x = 0 \end{cases}$

Such that $f(x)$ is continuous at $x = 0$; $f'(0)$ is real and finite; and $\lim_{x\to 0} f'(x)$ does not exist. This holds true for which of the following values of n ?

(a) 0 (b) 1 (c) 2 (d) 3

Answers

1.	(a)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(a)	**7.**	(c)	**8.**	(c)	**9.**	(a)	**10.**	(a)
11.	(a)	**12.**	(b)	**13.**	(d)	**14.**	(c)	**15.**	(b)	**16.**	(c)	**17.**	(c)	**18.**	(b)	**19.**	(c)	**20.**	(d)
21.	(a)	**22.**	(c)	**23.**	(b)	**24.**	(c)	**25.**	(c)	**26.**	(c)	**27.**	(a)	**28.**	(b)	**29.**	(d)	**30.**	(c)
31.	(d)	**32.**	(a)	**33.**	(c)	**34.**	(a)	**35.**	(c)	**36.**	(c)	**37.**	(c)	**38.**	(b)	**39.**	(b)	**40.**	(d)
41.	(c)	**42.**	(b)	**43.**	(c)	**44.**	(b)	**45.**	(a)	**46.**	(a)	**47.**	(c)	**48.**	(a)	**49.**	(c)	**50.**	(d)
51.	(c)	**52.**	(c)	**53.**	(c)	**54.**	(c)	**55.**	(a)	**56.**	(c)	**57.**	(a)	**58.**	(c)	**59.**	(c)	**60.**	(d)
61.	(b)	**62.**	(b)	**63.**	(a)	**64.**	(c)	**65.**	(c)	**66.**	(c)	**67.**	(c)	**68.**	(c)	**69.**	(b)	**70.**	(d)
71.	(d)	**72.**	(d)	**73.**	(b)	**74.**	(d)	**75.**	(a)	**76.**	(c)	**77.**	(d)	**78.**	(a)	**79.**	(b)	**80.**	(c)
81.	(d)	**82.**	(b)	**83.**	(c)	**84.**	(c)	**85.**	(a)	**86.**	(a)	**87.**	(d)	**88.**	(c)	**89.**	(b)	**90.**	(c)

Exercise-2 : One or More than One Answer is/are Correct

1. If $f(x) = \tan^{-1}(\text{sgn}(x^2 - \lambda x + 1))$ has exactly one point of discontinuity, then the value of λ can be :

(a) 1 (b) −1 (c) 2 (d) −2

2. $f(x) = \begin{cases} 2(x+1) & ; \quad x \le -1 \\ \sqrt{1-x^2} & ; \quad -1 < x < 1 \\ ||x|-1|-1| & ; \quad x \ge 1 \end{cases}$, then :

(a) $f(x)$ is non-differentiable at exactly three points

(b) $f(x)$ is continuous in $(-\infty, 1]$

(c) $f(x)$ is differentiable in $(-\infty, -1)$

(d) $f(x)$ is finite type of discontinuity at $x = 1$, but continuous at $x = -1$

3. Let $f(x) = \begin{cases} \dfrac{x(3e^{1/x} + 4)}{2 - e^{1/x}} & ; \quad x \ne 0 \quad x \ne \dfrac{1}{\ln 2} \\ 0 & ; \quad x = 0 \end{cases}$

which of the following statement(s) is/are correct ?

(a) $f(x)$ is continuous at $x = 0$ (b) $f(x)$ is non-derivable at $x = 0$

(c) $f'(0^+) = -3$ (d) $f'(0^-)$ does not exist

4. Let $|f(x)| \le \sin^2 x, \forall x \in R$, then

(a) $f(x)$ is continuous at $x = 0$

(b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ is continuous but not differentiable at $x = 0$

(d) $f(0) = 0$

5. Let $f(x) = \begin{cases} \dfrac{a(1 - x\sin x) + b\cos x + 5}{x^2} & ; \quad x < 0 \\ 3 & ; \quad x = 0 \\ \left(1 + \left(\dfrac{cx + dx^3}{x^2}\right)\right)^{\frac{1}{x}} & ; \quad x > 0 \end{cases}$

If f is continuous at $x = 0$ then correct statement(s) is/are :

(a) $a + c = -1$ (b) $b + c = -4$

(c) $a + b = -5$ (d) $c + d =$ an irrational number

6. If $f(x) = |||x| - 2| + p|$ have more than 3 points of non-derivability then the value of p can be :

(a) 0 (b) −1

(c) −2 (d) 2

7. Identify the options having correct statement :

(a) $f(x)=\sqrt[3]{x^2|x|}-1-|x|$ is no where non-differentiable

(b) $\lim_{x\to\infty}((x+5)\tan^{-1}(x+1))-((x+1)\tan^{-1}(x+1))=2\pi$

(c) $f(x)=\sin(\ln(x+\sqrt{x^2+1}))$ is an odd function

(d) $f(x)=\dfrac{4-x^2}{4x-x^3}$ is discontinuous at exactly one point

8. A twice differentiable function $f(x)$ is defined for all real numbers and satisfies the following conditions :

$f(0)=2;\quad f'(0)=-5\quad$ and $\quad f''(0)=3.$

The function $g(x)$ is defined by $g(x)=e^{ax}+f(x)\ \forall\, x\in R$, where '$a$' is any constant. If $g'(0)+g''(0)=0$ then 'a' can be equal to :

(a) 1 (b) -1 (c) 2 (d) -2

9. If $f(x)=|x|\sin x$, then f is :

(a) differentiable everywhere (b) not differentiable at $x=n\pi, n\in I$

(c) not differentiable at $x=0$ (d) continuous at $x=0$

10. Let [ ] denotes the greatest integer function and $f(x)=[\tan^2 x]$, then

(a) $\lim_{x\to 0} f(x)$ does not exist (b) $f(x)$ is continuous at $x=0$

(c) $f(x)$ is not differentiable at $x=0$ (d) $f'(0)=0$

11. Let f be a differentiable function satisfying $f'(x)=f'(-x)\ \forall\, x\in R$. Then

(a) If $f(1)=f(2)$, then $f(-1)=f(-2)$

(b) $\frac{1}{2}f(x)+\frac{1}{2}f(y)=f\left(\frac{1}{2}(x+y)\right)$ for all real values of x, y

(c) Let $f(x)$ be an even function, then $f(x)=0\ \forall\, x\in R$

(d) $f(x)+f(-x)=2f(0)\ \forall\, x\in R$

12. Let $f:R\to R$ be a function, such that $|f(x)|\le x^{4n}$, $n\in N\ \forall\, x\in R$ then $f(x)$ is :

(a) discontinuous at $x=0$ (b) continuous at $x=0$

(c) non-differentiable at $x=0$ (d) differentiable at $x=0$

13. Let $f(x)=[x]$ and $g(x)=0$ when x is an integer and $g(x)=x^2$ when x is not an integer ([ ] is the greatest integer function) then :

(a) $\lim_{x\to 1} g(x)$ exists, but $g(x)$ is not continuous at $x=1$

(b) $\lim_{x\to 1} f(x)$ does not exist

(c) gof is continuous for all x

(d) fog is continuous for all x

14. Let the function f be defined by $f(x) = \begin{cases} p + qx + x^2 & , \quad x < 2 \\ 2px + 3qx^2 & , \quad x \geq 2 \end{cases}$. Then :

(a) $f(x)$ is continuous in R if $3p + 10q = 4$

(b) $f(x)$ is differentiable in R if $p = q = \dfrac{4}{13}$

(c) If $p = -2, q = 1$, then $f(x)$ is continuous in R

(d) $f(x)$ is differentiable in R if $2p + 11q = 4$

15. Let $f(x) = |2x - 9| + |2x| + |2x + 9|$. Which of the following are true ?

(a) $f(x)$ is not differentiable at $x = \dfrac{9}{2}$

(b) $f(x)$ is not differentiable at $x = \dfrac{-9}{2}$

(c) $f(x)$ is not differentiable at $x = 0$

(d) $f(x)$ is differentiable at $x = \dfrac{-9}{2}, 0, \dfrac{9}{2}$

16. Let $f(x) = \max(x, x^2, x^3)$ in $-2 \leq x \leq 2$. Then :

(a) $f(x)$ is continuous in $-2 \leq x \leq 2$

(b) $f(x)$ is not differentiable at $x = 1$

(c) $f(-1) + f\left(\dfrac{3}{2}\right) = \dfrac{35}{8}$

(d) $f'(-1) f'\left(\dfrac{3}{2}\right) = \dfrac{-35}{4}$

17. If $f(x)$ be a differentiable function satisfying $f(y) f\left(\dfrac{x}{y}\right) = f(x) \; \forall \; x, y \in R, \; y \neq 0$ and $f(1) \neq 0$, $f'(1) = 3$, then :

(a) $\operatorname{sgn}(f(x))$ is non-differentiable at exactly one point

(b) $\lim\limits_{x \to 0} \dfrac{x^2(\cos x - 1)}{f(x)} = 0$

(c) $f(x) = x$ has 3 solutions

(d) $f(f(x)) - f^3(x) = 0$ has infinitely many solutions

18. Let $f(x) = (x^2 - 3x + 2)(x^2 + 3x + 2)$ and α, β, γ satisfy $\alpha < \beta < \gamma$ are the roots of $f'(x) = 0$ then which of the following is/are correct ([·] denotes greatest integer function) ?

(a) $[\alpha] = -2$

(b) $[\beta] = -1$

(c) $[\beta] = 0$

(d) $[\alpha] = 1$

19. Let the function f be defined by $f(x) = \begin{cases} p + qx + x^2, & x < 2 \\ 2px + 3qx^2, & x \geq 2 \end{cases}$. Then :

(a) $f(x)$ is continuous in R if $3p + 10q = 4$

(b) $f(x)$ is differentiable in R if $p = q = \dfrac{4}{13}$

(c) If $p = -2$, $q = 1$, then $f(x)$ is continuous in R

(d) $f(x)$ is differentiable in R if $2p + 11q = 4$

20. If $y = e^{x\sin(x^3)} + (\tan x)^x$ then $\frac{dy}{dx}$ may be equal to :

(a) $e^{x\sin(x^3)}[3x^3\cos(x^3) + \sin(x^3)] + (\tan x)^x[\ln\tan x + 2x\operatorname{cosec}2x]$

(b) $e^{x\sin(x^3)}[x^3\cos(x^3) + \sin(x^3)] + (\tan x)^x[\ln\tan x + 2x\operatorname{cosec}2x]$

(c) $e^{x\sin(x^3)}[x^3\sin(x^3) + \cos(x^3)] + (\tan x)^x[\ln\tan x + 2\operatorname{cosec}2x]$

(d) $e^{x\sin(x^3)}[3x^3\cos(x^3) + \sin(x^3)] + (\tan x)^x\left[\ln\tan x + \frac{x\sec^2 x}{\tan x}\right]$

21. Let $f(x) = x + (1-x)x^2 + (1-x)(1-x^2)x^3 + \ldots\ldots + (1-x)(1-x^2)\ldots\ldots(1-x^{n-1})x^n; (n \ge 4)$ then :

(a) $f(x) = -\prod_{r=1}^{n}(1-x^r)$

(b) $f(x) = 1-\prod_{r=1}^{n}(1-x^r)$

(c) $f'(x) = (1-f(x))\left(\sum_{r=1}^{n}\frac{rx^{r-1}}{(1-x^r)}\right)$

(d) $f'(x) = f(x)\left(\sum_{r=1}^{n}\frac{rx^{r-1}}{(1-x^r)}\right)$

22. Let $f(x) = \begin{cases} x^2 + a\ ;\ 0 \le x < 1 \\ 2x + b\ ;\ 1 \le x \le 2 \end{cases}$ and $g(x) = \begin{cases} 3x + b\ ;\ 0 \le x < 1 \\ x^3\ ;\ 1 \le x \le 2 \end{cases}$

If derivative of $f(x)$ w.r.t. $g(x)$ at $x = 1$ exists and is equal to λ, then which of the following is/are correct ?

(a) $a + b = -3$ (b) $a - b = 1$ (c) $\frac{ab}{\lambda} = 3$ (d) $\frac{-b}{\lambda} = 3$

23. If $f(x) = \begin{cases} \frac{\sin[x^2]\pi}{x^2 - 3x + 8} + ax^3 + b\ ;\ 0 \le x \le 1 \\ 2\cos\pi x + \tan^{-1}x\ ;\ 1 < x \le 2 \end{cases}$ is differentiable in [0, 2] then :

([·] denotes greatest integer function)

(a) $a = \frac{1}{3}$ (b) $a = \frac{1}{6}$ (c) $b = \frac{\pi}{4} - \frac{13}{6}$ (d) $b = \frac{\pi}{4} - \frac{7}{3}$

24. If $f(x) = \begin{cases} 1 + x & 0 \le x \le 2 \\ 3 - x & 2 < x \le 3 \end{cases}$, then $f(f(x))$ is not differentiable at :

(a) $x = 1$ (b) $x = 2$ (c) $x = \frac{5}{2}$ (d) $x = 3$

25. Let $f(x) = (x+1)(x+2)(x+3)\ldots\ldots(x+100)$ and $g(x) = f(x)f''(x) - (f'(x))^2$. Let n be the number of real roots of $g(x) = 0$, then :

(a) $n < 2$ (b) $n > 2$ (c) $n < 100$ (d) $n > 100$

26. If $f(x) = \begin{cases} |x|-3 & , \quad x<1 \\ |x-2|+a & , \quad x \geq 1 \end{cases}$, $g(x) = \begin{cases} 2-|x| & , \quad x<2 \\ \text{sgn}(x)-b & , \quad x \geq 2 \end{cases}$

If $h(x) = f(x) + g(x)$ is discontinuous at exactly one point, then which of the following are correct ?

(a) $a=-3, b=0$ (b) $a=-3, b=-1$ (c) $a=2, b=1$ (d) $a=0, b=1$

27. Let $f(x)$ be a continuous function in $[-1, 1]$ such that

$$f(x) = \begin{cases} \dfrac{\ln(ax^2+bx+c)}{x^2} & ; \ -1 \leq x < 0 \\ 1 & ; \quad x = 0 \\ \dfrac{\sin(e^{x^2}-1)}{x^2} & ; \ 0 < x \leq 1 \end{cases}$$

Then which of the following is/are correct ?

(a) $a+b+c=0$ (b) $b=a+c$ (c) $c=1+b$ (d) $b^2+c^2=1$

28. $f(x)$ is differentiable function satisfying the relationship $f^2(x)+f^2(y)+2(xy-1)=f^2(x+y)$ $\forall\, x, y \in R$

Also $f(x)>0 \ \forall\, x \in R$ and $f(\sqrt{2})=2$. Then which of the following statement(s) is/are correct about $f(x)$?

(a) $[f(3)]=3$ ([·] denotes greatest integer function)

(b) $f(\sqrt{7})=3$

(c) $f(x)$ is even

(d) $f'(0)=0$

29. The function $f(x) = \left[\sqrt{1-\sqrt{1-x^2}}\right]$, (where [·] denotes greatest integer function) :

(a) has domain $[-1, 1]$

(b) is discontinuous at two points in its domain

(c) is discontinuous at $x=0$

(d) is discontinuous at $x=1$

30. A function $f(x)$ satisfies the relation :

$f(x+y)=f(x)+f(y)+xy(x+y) \ \forall\, x, y \in R$. If $f'(0)=-1$, then :

(a) $f(x)$ is a polynomial function

(b) $f(x)$ is an exponential function

(c) $f(x)$ is twice differentiable for all $x \in R$

(d) $f'(3)=8$

31. The points of discontinuities of $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$ in $\left[\frac{\pi}{6}, \pi\right]$ is/are :

(where [·] denotes greatest integer function)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

32. Let $f(x) = \begin{cases} \frac{x^2}{2} & 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2} & 1 \le x \le 2 \end{cases}$, then in $[0, 2]$:

(a) $f(x), f'(x)$ are continuous

(b) $f'(x)$ is continuous, $f''(x)$ is not continuous

(c) $f''(x)$ is continuous

(d) $f''(x)$ is non differentiable

33. If $x = \phi(t), y = \psi(t)$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$ (b) $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$ (c) $\frac{\psi''}{\phi'} - \frac{\psi'\phi''}{(\phi')^2}$ (d) $\frac{\psi''}{(\phi')^2} - \frac{\psi'\phi''}{(\phi')^3}$

34. $f(x) = [x]$ and $g(x) = \begin{cases} 0 & , \ x \in I \\ x^2 & , \ x \notin I \end{cases}$ where [·] denotes the greatest integer function. Then

(a) gof is continuous for all x

(b) gof is not continuous for all x

(c) fog is continuous everywhere

(d) fog is not continuous everywhere

35. Let $f : R^+ \to R$ defined as $f(x) = e^x + \ln x$ and $g = f^{-1}$ then correct statement(s) is/are :

(a) $g''(e) = \frac{1-e}{(1+e)^3}$ (b) $g''(e) = \frac{e-1}{(1+e)^3}$ (c) $g'(e) = e + 1$ (d) $g'(e) = \frac{1}{e+1}$

36. Let $f(x) = \begin{cases} \frac{3x - x^2}{2} & ; \quad x < 2 \\ [x-1] & ; \quad 2 \le x < 3 \\ x^2 - 8x + 17 & ; \quad x \ge 3 \end{cases}$; then which of the following hold(s) good ?

([·] denotes greatest integer function)

(a) $\lim_{x \to 2} f(x) = 1$ (b) $f(x)$ is differentiable at $x = 2$

(c) $f(x)$ is continuous at $x = 2$ (d) $f(x)$ is discontinuus at $x = 3$

Answers

1.	(c, d)	**2.**	(a, c, d)	**3.**	(a, b, c)	**4.**	(a, b, d)	**5.**	(a, b, c, d)	**6.**	(b, c)
7.	(a, b, c)	**8.**	(a, d)	**9.**	(a, d)	**10.**	(b, d)	**11.**	(a, d)	**12.**	(b, d)
13.	(a, b, c)	**14.**	(a, b, c)	**15.**	(a, b, c)	**16.**	(a, b, c)	**17.**	(a, b, c, d)	**18.**	(a, c)
19.	(a, b, c)	**20.**	(a, d)	**21.**	(b, c)	**22.**	(a, b, c, d)	**23.**	(b, c)	**24.**	(a, b)
25.	(a, c)	**26.**	(a, b, c, d)	**27.**	(c, d)	**28.**	(a, b, c, d)	**29.**	(a, b, d)	**30.**	(a, c, d)
31.	(b, c)	**32.**	(a, b, d)	**33.**	(b, d)	**34.**	(a)	**35.**	(a, d)	**36.**	(a, c, d)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $f(x) = \lim_{n \to \infty} n^2 \tan\left(\ln\left(\sec\frac{x}{n}\right)\right)$ and $g(x) = \min(f(x), \{x\})$

(where $\{\cdot\}$ denotes fractional part function)

1. Left hand derivative of $\phi(x) = e^{\sqrt{2f(x)}}$ at $x = 0$ is :

(a) 0 (b) 1 (c) -1 (d) Does not exist

2. Number of points in $x \in [-1, 2]$ at which $g(x)$ is discontinuous :

(a) 2 (b) 1 (c) 0 (d) 3

Paragraph for Question Nos. 3 to 4

Let $f(x)$ and $g(x)$ be two differentiable functions, defined as :

$$f(x) = x^2 + xg'(1) + g''(2) \quad \text{and} \quad g(x) = f(1)x^2 + xf'(x) + f''(x).$$

3. The value of $f(1) + g(-1)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

4. The number of integers in the domain of the function $F(x) = \sqrt{-\frac{f(x)}{g(x)}} + \sqrt{3-x}$ is :

(a) 0 (b) 1 (c) 2 (d) Infinite

Paragraph for Question Nos. 5 to 6

Define : $f(x) = |x^2 - 4x + 3| \ln x + 2(x-2)^{1/3}, x > 0$

$$h(x) = \begin{cases} x-1 & , x \in Q \\ x^2 - x - 2 & , x \notin Q \end{cases}$$

5. $f(x)$ is non-differentiable at .... points and the sum of corresponding x value(s) is ....

(a) 3, 6 (b) 2, 3 (c) 2, 4 (d) 2, 5

6. $h(x)$ is discontinuous at $x =$......

(a) $1 + \sqrt{2}$ (b) $\tan\frac{3\pi}{8}$ (c) $\tan\frac{7\pi}{8}$ (d) $\sqrt{2} - 1$

Paragraph for Question Nos. 7 to 8

Consider a function defined in $[-2, 2]$

$$f(x) = \begin{cases} \{x\} & -2 \le x < -1 \\ |\operatorname{sgn} x| & -1 \le x \le 1 \\ \{-x\} & 1 < x \le 2 \end{cases}$$

where $\{\cdot\}$ denotes the fractional part function.

7. The total number of points of discontinuity of $f(x)$ for $x \in [-2, 2]$ is :

(a) 0 (b) 1 (c) 2 (d) 4

8. The number of points for $x \in [-2, 2]$ where $f(x)$ is non-differentiable is :

(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Question Nos. 9 to 10

Consider a function $f(x)$ in $[0, 2\pi]$ defined as :

$$f(x) = \begin{cases} [\sin x] + [\cos x] & ; \quad 0 \le x \le \pi \\ [\sin x] - [\cos x] & ; \quad \pi < x \le 2\pi \end{cases}$$

where $[\cdot]$ denotes greatest integer function then

9. Number of points where $f(x)$ is non-derivable :

(a) 2 (b) 3 (c) 4 (d) 5

10. $\lim\limits_{x \to \left(\frac{3\pi}{2}\right)^+} f(x)$ equals

(a) 0 (b) 1 (c) -1 (d) 2

Paragraph for Question Nos. 11 to 13

Let $f(x) = \begin{cases} x[x] & 0 \le x < 2 \\ (x-1)[x] & 2 \le x \le 3 \end{cases}$ where $[x]$ = greatest integer less than or equal to x, then :

11. The number of values of x for $x \in [0, 3]$ where $f(x)$ is discontinuous is :

(a) 0 (b) 1 (c) 2 (d) 3

12. The number of values of x for $x \in [0, 3]$ where $f(x)$ is non-differentiable is :

(a) 0 (b) 1 (c) 2 (d) 3

13. The number of integers in the range of $y = f(x)$ is :

(a) 3 (b) 4 (c) 5 (d) 6

Paragraph for Question Nos. 14 to 16

Let $f: R \to R$ be a continuous and differentiable function such that $f(x+y) = f(x) \cdot f(y)$ $\forall\, x, y, f(x) \ne 0$ and $f(0) = 1$ and $f'(0) = 2$.

Let $g(xy) = g(x) \cdot g(y)\ \forall\, x, y$ and $g'(1) = 2; g(1) \ne 0$

14. Identify the correct option :

(a) $f(2) = e^4$ (b) $f(2) = 2e^2$ (c) $f(1) < 4$ (d) $f(3) > 729$

15. Identify the correct option :

(a) $g(2) = 2$ (b) $g(3) = 3$ (c) $g(3) = 9$ (d) $g(3) = 6$

16. The number of values of x, where $f(x) = g(x)$:

(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Question Nos. 17 to 18

Let $f(x) = \dfrac{\cos^2 x}{1 + \cos x + \cos^2 x}$ and $g(x) = \lambda \tan x + (1 - \lambda) \sin x - x$, where $\lambda \in R$ and $x \in [0, \pi/2)$.

17. $g'(x)$ equals

(a) $\dfrac{(1 - \cos x)(f(x) - \lambda)}{\cos x}$ (b) $\dfrac{(1 - \cos x)(\lambda - f(x))}{\cos x}$

(c) $\dfrac{(1 - \cos x)(\lambda - f(x))}{f(x)}$ (d) $\dfrac{(1 - \cos x)(\lambda - f(x))}{(f(x))^2}$

18. The exhaustive set of values of 'λ' such that $g'(x) \geq 0$ for any $x \in [0, \pi/2)$:

(a) $[1, \infty)$ (b) $[0, \infty)$ (c) $\left[\dfrac{1}{2}, \infty\right)$ (d) $\left[\dfrac{1}{3}, \infty\right)$

Paragraph for Question Nos. 19 to 21

Let $f(x) = \lim\limits_{n \to \infty} \dfrac{x^2 + 2(x+1)^{2n}}{(x+1)^{2n+1} + x^2 + 1}, n \in N$ and

$g(x) = \tan\left(\dfrac{1}{2} \sin^{-1}\left(\dfrac{2f(x)}{1 + f^2(x)}\right)\right)$, then

19. The number of points where $g(x)$ is non-differentiable $\forall\, x \in R$ is :

(a) 1 (b) 2 (c) 3 (d) 4

20. $\lim\limits_{x \to -3} \dfrac{(x^2 + 4x + 3)}{\sin(x+3)\, g(x)}$ is equal to :

(a) 1 (b) 2 (c) 4 (d) Non-existent

21. $\lim\limits_{x \to 0^-} \left\{\dfrac{f(x)}{\tan^2 x}\right\} + \left|\lim\limits_{x \to -2^-} f(x)\right| + \lim\limits_{x \to -2^+} (5f(x))$ is equal to

(where $\{\cdot\}$ denotes fraction part function)

(a) 7 (b) 8 (c) 12 (d) Non-existent

Paragraph for Question Nos. 22 to 24

Let f and g be two differentiable functions such that :

$$f(x) = g'(1)\sin x + (g''(2) - 1)x$$

$$g(x) = x^2 - f'\left(\frac{\pi}{2}\right)x + f''\left(-\frac{\pi}{2}\right)$$

22. The number of solution(s) of the equation $f(x) = g(x)$ is/are :

(a) 1 (b) 2 (c) 3 (d) infinite

23. If $\int \frac{g(\cos x)}{f(x) - x} dx = \cos x + ln\ (h(x)) + C$ where C is constant and $h\left(\frac{\pi}{2}\right) = 1$ then $\left|h\left(\frac{2\pi}{3}\right)\right|$ is :

(a) $3\sqrt{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

24. If $\phi(x) = f^{-1}(x)$ then $\phi'\left(\frac{\pi}{2} + 1\right)$ equals to :

(a) $\frac{\pi}{2} + 1$ (b) $\frac{\pi}{2}$ (c) 1 (d) 0

Paragraph for Question Nos. 25 to 26

Suppose a function $f(x)$ satisfies the following conditions

$$f(x+y) = \frac{f(x) + f(y)}{1 + f(x)\,f(y)},\ \forall\ x, y \in R \text{ and } f'(0) = 1$$

Also $-1 < f(x) < 1,\ \forall\ x \in R$

25. $f(x)$ increases in the complete interval :

(a) $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$ (b) $(-\infty, \infty)$

(c) $(-\infty, -1) \cup (-1, 0)$ (d) $(0, 1) \cup (1, \infty)$

26. The value of the limit $\underset{x \to \infty}{lt}\ (f(x))^x$ is :

(a) 0 (b) 1 (c) e (d) e^2

Paragraph for Question Nos. 27 to 28

Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^4 f(x)}{x^8 + 1} = 3$

$f(2) = 5, f(3) = 10, f(-1) = 2, f(-6) = 37$

27. The value of $\lim\limits_{x\to -6}\dfrac{f(x)-x^2-1}{3(x+6)}$ equals to :

(a) $-\lfloor 6$ (b) $\lfloor 6$ (c) $\dfrac{\lfloor 6}{2}$ (d) $\dfrac{-\lfloor 6}{2}$

28. The number of points of discontinuity of $g(x)=\dfrac{1}{x^2+1-f(x)}$ in $\left[\dfrac{-15}{2},\dfrac{5}{2}\right]$ equals :

(a) 4 (b) 3 (c) 1 (d) 0

Paragraph for Question Nos. 29 to 30

Consider $f(x)=x^{\ln x}$ and $g(x)=e^2x$. Let α and β be two values of x satisfying $f(x)=g(x)$ $(\alpha<\beta)$

29. If $\lim\limits_{x\to\beta}\dfrac{f(x)-c\beta}{g(x)-\beta^2}=l$ then the value of $c-l$ equals to :

(a) $4-e^2$ (b) e^2-4 (c) $4-e$ (d) $e-4$

30. If $h(x)=\dfrac{f(x)}{g(x)}$ then $h'(\alpha)$ equals to :

(a) e (b) $-e$ (c) $3e$ (d) $-3e$

Paragraph for Question Nos. 31 to 32

Let $f_n(x)+f_n(y)=\dfrac{x^n+y^n}{x^ny^n}\ \forall\ x,y\in R-\{0\}$ where $n\in N$ and

$g(x)=\max\left\{f_2(x), f_3(x), \dfrac{1}{2}\right\}\ \forall\ x\in R-\{0\}$

31. The minimum value of $\sum\limits_{k=1}^{\infty}f_{2k}(\operatorname{cosec}\theta)+\sum\limits_{k=1}^{\infty}f_{2k}(\sec\theta)$, where $\theta\neq\dfrac{k\pi}{2}$; $k\in I$ is :

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 4

32. The number of values of x for which $g(x)$ is non-differentiable $(x\in R-\{0\})$:

(a) 3 (b) 4 (c) 5 (d) 1

Answers

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(d)	**6.**	(d)	**7.**	(b)	**8.**	(d)	**9.**	(c)	**10.**	(c)
11.	(c)	**12.**	(d)	**13.**	(c)	**14.**	(a)	**15.**	(c)	**16.**	(b)	**17.**	(c)	**18.**	(d)	**19.**	(d)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(b)	**24.**	(c)	**25.**	(b)	**26.**	(b)	**27.**	(d)	**28.**	(b)	**29.**	(b)	**30.**	(d)
31.	(b)	**32.**	(a)																

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If $\int_0^{\pi} \frac{\log \sin x}{\cos^2 x} dx = -K$ then the value of $\frac{3k}{\pi}$ is greater than	**(P)**	0
(B)	If $e^{x+y} + e^{y-x} = 1$ and $y'' - (y')^2 + K = 0$, then K is equal to	**(Q)**	1
(C)	If $f(x) = x \ln x$ then $2(f^{-1})'(\ln 4)$ is more than	**(R)**	2
(D)	$\lim_{x\to\infty} (x \ln x)^{\frac{1}{x^2+1}}$ is less than	**(S)**	4
		(T)	5

2. Let $f(x) = \begin{cases} [x] & , \quad -2 \le x < 0 \\ |x| & , \quad 0 \le x \le 2 \end{cases}$

(where [·] denotes the greatest integer function) $g(x) = \sec x$, $x \in R - (2n+1)\frac{\pi}{2}, n \in I$

Match the following statements in column I with their values in column II in the interval $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$.

	Column-I		Column-II
(A)	Abscissa of points where limit of $fog(x)$ exist is/are	**(P)**	-1
(B)	Abscissa of points in domain of $gof(x)$, where limit of $gof(x)$ does not exist is/are	**(Q)**	π
(C)	Abscissa of points of discontinuity of $fog(x)$ is/are	**(R)**	$\frac{5\pi}{6}$
(D)	Abscissa of points of differentiability of $fog(x)$ is/are	**(S)**	$-\pi$
		(T)	0

3. Let a function $f(x) = [x]\{x\} - |x|$ where [·], {·} are greatest integer and fractional part respectively then match the following List-I with List-II.

	Column-I		Column-II
(A)	$f(x)$ is continuous at x equal to	**(P)**	3
(B)	$\frac{4}{3}\left\lvert \int_2^3 f(x)\,dx \right\rvert$ is equal to	**(Q)**	1

(C)	If $g(x)=x-1$ and if $f(x)=g(x)$ where $x\in(-3,\infty)$, then number of solutions	**(R)**	4
(D)	If $l=\lim\limits_{x\to4^+} f(x)$, then $-l$ is equal to	**(S)**	2

4.

	Column-I		Column-II
(A)	$\lim\limits_{x\to\infty}\left(\dfrac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{2x-1}} =$	**(P)**	$\dfrac{1}{2}$
(B)	$\lim\limits_{x\to0}\dfrac{\log_{\sec x/2}\cos x}{\log_{\sec x}\cos\dfrac{x}{2}} =$	**(Q)**	2
(C)	Let $f(x) = \max.(\cos x, x, 2x-1)$ where $x \ge 0$ then number of points of non-differentiability of $f(x)$ is	**(R)**	5
(D)	If $f(x)=[2+3\sin x]$, $0<x<\pi$ then number of points at which the function is discontinuous, is	**(S)**	16

5. The function
$$
\begin{aligned}
f(x) &= ax(x-1)+b & x<1\\
&= x-1 & 1\le x\le 3\\
&= px^2+qx+2 & x>3
\end{aligned}
$$

if (i) $f(x)$ is continuous for all x

(ii) $f'(1)$ does not exist

(iii) $f'(x)$ is continuous at $x=3$, then

	Column-I		Column-II
(A)	a cannot has value	**(P)**	1/3
(B)	b has value	**(Q)**	0
(C)	p has value	**(R)**	−1
(D)	q has value	**(S)**	1

Answers

1.	A → P, Q, R ; B → Q ; C → P, Q ; D → R, S, T
2.	A → P, Q, R, S, T ; B → P, T ; C → Q, S ; D → P, R, T
3.	A → Q ; B → S ; C → P ; D → R
4.	A → P ; B → S ; C → Q ; D → R
5.	A → S ; B → Q ; C → P ; D → R

Exercise-5 : Subjective Type Problems

1. Let $f(x)=\begin{cases} ax(x-1)+b & ; \quad x<1 \\ x+2 & ; \quad 1\le x\le 3 \\ px^2+qx+2 & ; \quad x>3 \end{cases}$ is continuous $\forall x\in R$ except $x=1$ but $|f(x)|$ is differentiable everywhere and $f'(x)$ is continuous at $x=3$ and $|a+p+b+q|=k$, then $k=$

2. If $y=\sin(8\sin^{-1}x)$ then $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=-ky$, where $k=$

3. If $y^2=4ax$, then $\dfrac{d^2y}{dx^2}=\dfrac{ka^2}{y^3}$, where $k^2=$

4. The number of values of $x, x\in[-2,3]$ where $f(x)=[x^2]\sin(\pi x)$ is discontinuous is (where $[\cdot]$ denotes greatest integer function)

5. If $f(x)$ is continuous and differentiable in $[-3,9]$ and $f'(x)\in[-2,8]\ \forall x\in(-3,9)$. Let N be the number of divisors of the greatest possible value of $f(9)-f(-3)$, then find the sum of digits of N.

6. If $f(x)=\begin{cases} \cos x^3 & ; \quad x<0 \\ \sin x^3-|x^3-1| & ; \quad x\ge 0 \end{cases}$

then find the number of points where $g(x)=f(|x|)$ is non-differentiable.

7. Let $f(x)=x^2+ax+3$ and $g(x)=x+b$, where $F(x)=\lim\limits_{n\to\infty}\dfrac{f(x)+(x^2)^n g(x)}{1+(x^2)^n}$. If $F(x)$ is continuous at $x=1$ and $x=-1$ then find the value of (a^2+b^2).

8. Let $f(x)=\begin{cases} 2-x & , \quad -3\le x\le 0 \\ x-2 & , \quad 0<x<4 \end{cases}$

Then $f^{-1}(x)$ is discontinuous at $x=$

9. If $f(x)+2f(1-x)=x^2+2\ \forall x\in R$ and $f(x)$ is a differentiable function, then the value of $f'(8)$ is

10. Let $f(x)=signum\,(x)$ and $g(x)=x(x^2-10x+21)$, then the number of points of discontinuity of $f[g(x)]$ is

11. If $\dfrac{d^2}{dx^2}\left(\dfrac{\sin^4 x+\sin^2 x+1}{\sin^2 x+\sin x+1}\right)=a\sin^2 x+b\sin x+c$ then the value of $b+c-a$ is

12. If $f(x)=a\cos(\pi x)+b$, $f'\left(\dfrac{1}{2}\right)=\pi$ and $\displaystyle\int_{1/2}^{3/2} f(x)\,dx=\dfrac{2}{\pi}+1$, then find the value of $-\dfrac{12}{\pi}\left(\dfrac{\sin^{-1}a}{3}+\cos^{-1}b\right)$.

13. Let $\alpha(x) = f(x) - f(2x)$ and $\beta(x) = f(x) - f(4x)$

and $\alpha'(1) = 5\ \alpha'(2) = 7$

then find the value of $\beta'(1) - 10$

14. Let $f(x) = -4 \cdot e^{\frac{1-x}{2}} + \frac{x^3}{3} + \frac{x^2}{2} + x + 1$ and g be inverse function of f and $h(x) = \frac{a + bx^{3/2}}{x^{5/4}}$, $h'(5) = 0$, then $\frac{a^2}{5b^2 g'\left(\frac{-7}{6}\right)} =$

15. If $y = e^{2\sin^{-1} x}$ then $\left|\frac{(x^2 - 1)y'' + xy'}{y}\right|$ is equal to

16. Let f be a continuous function on $[0, \infty)$ such that $\lim_{x \to \infty}\left(f(x) + \int_0^x f(t)\,dt\right)$ exists. Find $\lim_{x \to \infty} f(x)$.

17. Let $f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$ and let $g(x) = f^{-1}(x)$. Find $g'''(0)$.

18. If $f(x) = \begin{cases} \cos x^3 & ; \ x < 0 \\ \sin x^3 - |x^3 - 1| & ; \ x \geq 0 \end{cases}$

then find the number of points where $g(x) = f(|x|)$ is non-differentiable.

19. Let $f: R^+ \longrightarrow R$ be a differentiable function satisfying :

$f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}\ \forall x, y \in R^+$ also $f(1) = 0; f'(1) = 1$

find $\lim_{x \to e}\left[\frac{1}{f(x)}\right]$ (where $[\cdot]$ denotes greatest integer function).

20. For the curve $\sin x + \sin y = 1$ lying in the first quadrant there exists a constant α for which $\lim_{x \to 0} x^\alpha \frac{d^2y}{dx^2} = L$ (not zero), then $2\alpha =$

21. Let $f(x) = x\tan^{-1}(x^2) + x^4$. Let $f^k(x)$ denotes k^{th} derivative of $f(x)$ w.r.t. x, $k \in N$. If $f^{2m}(0) \neq 0, m \in N$, then $m =$

22. If $x = \cos\theta$ and $y = \sin^3\theta$, then $\left|\frac{yd^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right|$ at $\theta = \frac{\pi}{2}$ is :

23. The value of x, $x \in (2, \infty)$ where $f(x) = \sqrt{x + \sqrt{8x - 16}} + \sqrt{x - \sqrt{8x - 16}}$ is not differentiable is :

24. The number of non differentiability points of function $f(x) = \min\left([x], \{x\}, \left|x - \frac{3}{2}\right|\right)$ for $x \in (0, 2)$, where $[\cdot]$ and $\{\cdot\}$ denote greatest integer function and fractional part function respectively.

Answers

1.	3	**2.**	64	**3.**	16	**4.**	8	**5.**	3	**6.**	2	**7.**	17
8.	2	**9.**	4	**10.**	3	**11.**	7	**12.**	2	**13.**	9	**14.**	5
15.	4	**16.**	0	**17.**	1	**18.**	2	**19.**	2	**20.**	3	**21.**	2
22.	3	**23.**	4	**24.**	3								

❑❑❑

4 APPLICATION OF DERIVATIVES

Exercise-1 : Single Choice Problems

1. The difference between the maximum and minimum value of the function $f(x) = 3\sin^4 x - \cos^6 x$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 4

2. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is :

(a) $(x-1)^2$ (b) $(x-1)^3$ (c) $(x+1)^3$ (d) $(x+1)^2$

3. If the subnormal at any point on the curve $y = 3^{1-k} \cdot x^k$ is of constant length then k equals to :

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 0

4. If $x^5 - 5qx + 4r$ is divisible by $(x-c)^2$ then which of the following must hold true $\forall\, q, r, c \in R$?

(a) $q = r$ (b) $q + r = 0$ (c) $q^5 = r^4$ (d) $q^4 = r^5$

5. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\,\text{cm}^3/\text{min}$. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is :

(a) $\frac{1}{36\pi}$ cm/min (b) $\frac{1}{18\pi}$ cm/min (c) $\frac{1}{54\pi}$ cm/min (d) $\frac{5}{6\pi}$ cm/min

6. If $f(x) = \frac{(x-1)(x-2)}{(x-3)(x-4)}$, then number of local extremas for $g(x)$, where $g(x) = f(|x|)$:

(a) 3 (b) 4 (c) 5 (d) None of these

7. Two straight roads OA and OB intersect at an angle 60°. A car approaches O from A, where $OA = 700$ m at a uniform speed of 20 m/s, Simultaneously, a runner starts running from O towards B at a uniform speed of 5 m/s. The time after start when the car and the runner are closest is :

(a) 10 sec (b) 15 sec

(c) 20 sec (d) 30 sec

8. Let $f(x) = \begin{cases} a-3x & ; & -2 \le x < 0 \\ 4x+3 & ; & 0 \le x < 1 \end{cases}$; if $f(x)$ has smallest value at $x = 0$, then range of a, is :

(a) $(-\infty, 3)$ (b) $(-\infty, 3]$ (c) $(3, \infty)$ (d) $[3, \infty)$

9. $f(x) = \begin{cases} 3+|x-k| & , & x \le k \\ a^2 - 2 + \dfrac{\sin(x-k)}{(x-k)} & , & x > k \end{cases}$ has minimum at $x = k$, then :

(a) $a \in R$ (b) $|a| < 2$ (c) $|a| > 2$ (d) $1 < |a| < 2$

10. For a certain curve $\dfrac{d^2y}{dx^2} = 6x - 4$ and curve has local minimum value 5 at $x = 1$. Let the global maximum and global minimum values, where $0 \le x \le 2$; are M and m. Then the value of $(M - m)$ equals to :

(a) -2 (b) 2 (c) 12 (d) -12

11. The tangent to $y = ax^2 + bx + \dfrac{7}{2}$ at (1, 2) is parallel to the normal at the point (–2, 2) on the curve $y = x^2 + 6x + 10$. Then the value of $\dfrac{a}{2} - b$ is :

(a) 2 (b) 0 (c) 3 (d) 1

12. If (a, b) be the point on the curve $9y^2 = x^3$ where normal to the curve make equal intercepts with the axis, then the value of $(a + b)$ is :

(a) 0 (b) $\dfrac{10}{3}$ (c) $\dfrac{20}{3}$ (d) None of these

13. The curve $y = f(x)$ satisfies $\dfrac{d^2y}{dx^2} = 6x - 4$ and $f(x)$ has a local minimum value 5 when $x = 1$. Then $f(0)$ is equal to :

(a) 1 (b) 0 (c) 5 (d) None of these

14. Let A be the point where the curve $5\alpha^2x^3 + 10\alpha x^2 + x + 2y - 4 = 0$ $(\alpha \in R, \alpha \ne 0)$ meets the y-axis, then the equation of tangent to the curve at the point where normal at A meets the curve again, is :

(a) $x - \alpha y + 2\alpha = 0$ (b) $\alpha x + y - 2 = 0$ (c) $2x - y + 2 = 0$ (d) $x + 2y - 4 = 0$

15. The difference between the greatest and the least value of the function $f(x) = \cos x + \dfrac{1}{2}\cos 2x - \dfrac{1}{3}\cos 3x$

(a) $\dfrac{11}{5}$ (b) $\dfrac{13}{6}$ (c) $\dfrac{9}{4}$ (d) $\dfrac{7}{3}$

16. The x co-ordinate of the point on the curve $y = \sqrt{x}$ which is closest to the point (2, 1) is :

(a) $\dfrac{2+\sqrt{3}}{2}$ (b) $\dfrac{1+\sqrt{3}}{2}$ (c) $\dfrac{-1+\sqrt{3}}{2}$ (d) 1

17. The tangent at a point P on the curve $y = \ln\left(\frac{2+\sqrt{4-x^2}}{2-\sqrt{4-x^2}}\right) - \sqrt{4-x^2}$ meets the y-axis at T ; then PT^2 equals to :

(a) 2 (b) 4 (c) 8 (d) 16

18. Let $f(x) = \int_{x^2}^{x^3} \frac{dt}{\ln t}$ for $x > 1$

and $g(x) = \int_1^x (2t^2 - \ln t) f(t)\, dt$ $(x > 1)$, then :

(a) g is increasing on $(1, \infty)$
(b) g is decreasing on $(1, \infty)$
(c) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$
(d) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

19. Let $f(x) = x^3 + 6x^2 + ax + 2$, if $(-3, -1)$ is the largest possible interval for which $f(x)$ is decreasing function, then $a =$

(a) 3 (b) 9 (c) -2 (d) 1

20. Let $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right)$. Then difference of the greatest and least value of $f(x)$ on $[0, 1]$ is :

(a) $\pi/2$ (b) $\pi/4$ (c) π (d) $\pi/3$

21. The number of integral values of a for which $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is monotonic in $\forall x \in R$.

(a) 2 (b) 4 (c) 6 (d) 7

22. The number of critical points of $f(x) = \left(\int_0^x (\cos^2 t - \sqrt[3]{t})\, dt\right) + \frac{3}{4}x^{4/3} - \frac{x+1}{2}$ in $[0, 6\pi]$ is :

(a) 10 (b) 8 (c) 6 (d) 12

23. Let $f(x) = \min\left(\frac{1}{2} - \frac{3x^2}{4}, \frac{5x^2}{4}\right)$ for $0 \le x \le 1$, then maximum value of $f(x)$ is :

(a) 0 (b) $\frac{5}{64}$

(c) $\frac{5}{4}$ (d) $\frac{5}{16}$

24. Let $f(x) = \begin{cases} 2 - |x^2 + 5x + 6| & x \ne -2 \\ b^2 + 1 & x = -2 \end{cases}$

Has relative maximum at $x = -2$, then complete set of values b can take is :

(a) $|b| \ge 1$ (b) $|b| < 1$ (c) $b > 1$ (d) $b < 1$

25. Let for the function $f(x) = \begin{cases} \cos^{-1} x & ; & -1 \le x \le 0 \\ mx + c & ; & 0 < x \le 1 \end{cases}$;

Lagrange's mean value theorem is applicable in $[-1, 1]$ then ordered pair (m, c) is :

(a) $\left(1, -\frac{\pi}{2}\right)$ (b) $\left(1, \frac{\pi}{2}\right)$ (c) $\left(-1, -\frac{\pi}{2}\right)$ (d) $\left(-1, \frac{\pi}{2}\right)$

26. Tangents are drawn to $y = \cos x$ from origin then points of contact of these tangents will always lie on :

(a) $\frac{1}{x^2} = \frac{1}{y^2} + 1$ (b) $\frac{1}{x^2} = \frac{1}{y^2} - 2$ (c) $\frac{1}{y^2} = \frac{1}{x^2} + 1$ (d) $\frac{1}{y^2} = \frac{1}{x^2} - 2$

27. Least natural number a for which $x + ax^{-2} > 2 \,\forall\, x \in (0, \infty)$ is :

(a) 1 (b) 2 (c) 5 (d) None of these

28. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at points (2, 0) and (3, 0) is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

29. Difference between the greatest and least values of the function $f(x) = \int_0^x (\cos^2 t + \cos t + 2)\, dt$ in the interval $[0, 2\pi]$ is $K\pi$, then K is equal to :

(a) 1 (b) 3 (c) 5 (d) None of these

30. The range of the function $f(\theta) = \frac{\sin\theta}{\theta} + \frac{\theta}{\tan\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is equal to :

(a) $(0, \infty)$ (b) $\left(\frac{1}{\pi}, 2\right)$ (c) $(2, \infty)$ (d) $\left(\frac{2}{\pi}, 2\right)$

31. Number of integers in the range of c so that the equation $x^3 - 3x + c = 0$ has all its roots real and distinct is :

(a) 2 (b) 3 (c) 4 (d) 5

32. Let $f(x) = \int e^x (x-1)(x-2)\, dx$. Then $f(x)$ decreases in the interval :

(a) $(2, \infty)$ (b) $(-2, -1)$

(c) $(1, 2)$ (d) $(-\infty, 1) \cup (2, \infty)$

33. If the cubic polynomial $y = ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R)$ has only one critical point in its entire domain and $ac = 2$, then the value of $|b|$ is :

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{6}$

34. On the curve $y = \frac{1}{1 + x^2}$, the point at which $\left|\frac{dy}{dx}\right|$ is greatest in the first quadrant is :

(a) $\left(\frac{1}{2}, \frac{4}{5}\right)$ (b) $\left(1, \frac{1}{2}\right)$ (c) $\left(\frac{1}{\sqrt{2}}, \frac{2}{3}\right)$ (d) $\left(\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$

35. If $f(x) = 2x, g(x) = 3\sin x - x\cos x$, then for $x \in \left(0, \frac{\pi}{2}\right)$:

(a) $f(x) > g(x)$ (b) $f(x) < g(x)$

(c) $f(x) = g(x)$ has exactly one real root. (d) $f(x) = g(x)$ has exactly two real roots

36. Let $f(x) = \sin^{-1}\left(\frac{2g(x)}{1+g(x)^2}\right)$, then which are correct ?

(i) $f(x)$ is decreasing if $g(x)$ is increasing and $|g(x)| > 1$

(ii) $f(x)$ is an increasing function if $g(x)$ is increasing and $|g(x)| \le 1$

(iii) $f(x)$ is decreasing function if $g(x)$ is decreasing and $|g(x)| > 1$

(a) (i) and (iii) (b) (i) and (ii) (c) (i), (ii) and (iii) (d) (iii)

37. The graph of the function $y = f(x)$ has a unique tangent at $(e^a, 0)$ through which the graph passes then $\lim\limits_{x \to e^a} \frac{\ln(1+7f(x)) - \sin(f(x))}{3f(x)}$ is equal to :

(a) 1 (b) 3 (c) 2 (d) 7

38. Let $f(x)$ be a function such that $f'(x) = \log_{1/3}(\log_3(\sin x + a))$. The complete set of values of 'a' for which $f(x)$ is strictly decreasing for all real values of x is :

(a) $[4, \infty)$ (b) $[3, 4]$ (c) $(-\infty, 4)$ (d) $[2, \infty)$

39. If $f(x) = a\ln|x| + bx^2 + x$ has extremas at $x = 1$ and $x = 3$, then :

(a) $a = \frac{3}{4}, b = -\frac{1}{8}$ (b) $a = \frac{3}{4}, b = \frac{1}{8}$ (c) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (d) $a = -\frac{3}{4}, b = \frac{1}{8}$

40. Let $f(x) = \begin{cases} 1 + \sin x, & x < 0 \\ x^2 - x + 1, & x \ge 0 \end{cases}$, then :

(a) f has a local maximum at $x = 0$ (b) f has a local minimum at $x = 0$

(c) f is increasing everywhere (d) f is decreasing everywhere

41. If m and n are positive integers and $f(x) = \int_1^x (t-a)^{2n}(t-b)^{2m+1}\,dt,\ a \ne b$, then :

(a) $x = b$ is a point of local minimum (b) $x = b$ is a point of local maximum

(c) $x = a$ is a point of local minimum (d) $x = a$ is a point of local maximum

42. For any real θ, the maximum value of $\cos^2(\cos\theta) + \sin^2(\sin\theta)$ is :

(a) 1 (b) $1 + \sin^2 1$

(c) $1 + \cos^2 1$ (d) Does not exist

43. If the tangent at P of the curve $y^2 = x^3$ intersects the curve again at Q and the straight line OP, OQ have inclinations a, b where O is origin, then $\left(\frac{\tan\alpha}{\tan\beta}\right)$ has the value, equals to :

(a) -1 (b) -2 (c) 2 (d) $\sqrt{2}$

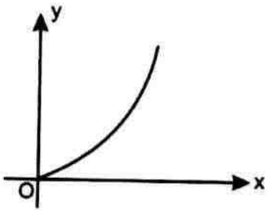
44. If $x + 4y = 14$ is a normal to the curve $y^2 = \alpha x^3 - \beta$ at (2, 3), then value of $\alpha + \beta$ is :

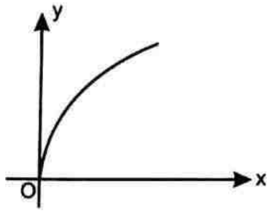
(a) 9 (b) −5 (c) 7 (d) −7

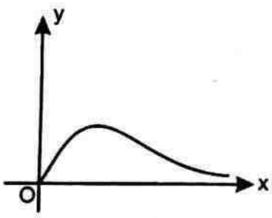
45. The tangent to the curve $y = e^{kx}$ at a point (0, 1) meets the x-axis at $(a, 0)$ where $a \in [-2, -1]$, then $k \in$:

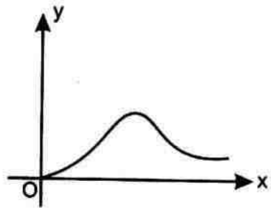
(a) $\left[-\frac{1}{2}, 0\right]$ (b) $\left[-1, -\frac{1}{2}\right]$ (c) $[0, 1]$ (d) $\left[\frac{1}{2}, 1\right]$

46. Which of the following graph represent the function $f(x) = \int_0^{\sqrt{x}} e^{-\frac{u^2}{x}} du$, for $x > 0$ and $f(0) = 0$?

(a)


(b)


(c)


(d)


47. Let $f(x) = (x-a)(x-b)(x-c)$ be a real valued function where $a < b < c$ $(a, b, c \in R)$ such that $f''(\alpha) = 0$. Then if $\alpha \in (c_1, c_2)$, which one of the following is correct ?

(a) $a < c_1 < b$ and $b < c_2 < c$ (b) $a < c_1, c_2 < b$

(c) $b < c_1, c_2 < c$ (d) None of these

48. $f(x) = x^6 - x - 1, x \in [1, 2]$. Consider the following statements :

(1) f is increasing on [1, 2] (2) f has a root in [1, 2]

(3) f is decreasing on [1, 2] (4) f has no root in [1, 2]

Which of the above are correct?

(a) 1 and 2 (b) 1 and 4 (c) 2 and 3 (d) 3 and 4

49. Which one of the following curves is the orthogonal trajectory of straight lines passing through a fixed point (a, b)?

(a) $x - a = k(y - b)$ (b) $(x - a)(y - b) = k$

(c) $(x - a)^2 = k(y - b)$ (d) $(x - a)^2 + (y - b)^2 = k$

50. The function $f(x) = \sin^3 x - m \sin x$ is defined on open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and if assumes only 1 maximum value and only 1 minimum value on this interval. Then, which one of the following must be correct ?

(a) $0 < m < 3$ (b) $-3 < m < 0$ (c) $m > 3$ (d) $m < -3$

51. The greatest of the numbers 1, $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $5^{1/5}$, $6^{1/6}$ and $7^{1/7}$ is :

(a) $2^{1/2}$ (b) $3^{1/3}$ (c) $7^{1/7}$ (d) $6^{1/6}$

52. Let l be the line through (0, 0) and tangent to the curve $y = x^3 + x + 16$. Then the slope of l equal to :

(a) 10 (b) 11 (c) 17 (d) 13

53. The slope of the tangent at the point of inflection of $y = x^3 - 3x^2 + 6x + 2009$ is equal to :

(a) 2 (b) 3 (c) 1 (d) 4

54. Let f be a real valued function with $(n + 1)$ derivatives at each point of R. For each pair of real numbers a, b, $a < b$, such that

$$ln\left[\frac{f(b) + f'(b) + ... + f^{(n)}(b)}{f(a) + f'(a) + ...+ f^{(n)}(a)}\right] = b - a$$

Statement-1 : There is a number $c \in (a, b)$ for which $f^{(n+1)}(c) = f(c)$

because

Statement-2 : If $h(x)$ be a derivable function such that $h(p) = h(q)$ then by Rolle's theorem $h'(d) = 0$; $d \in (p, q)$

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(b) Statement-1 is true, statement-2 is true and statement-2 is not correct explanation for statement-1

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true

55. If $g(x)$ is twice differentiable real valued function satisfying $g''(x) - 3g'(x) > 3 \ \forall \ x \geq 0$ and $g'(0) = -1$, then $h(x) = g(x) + x \ \forall x > 0$ is :

(a) strictly increasing (b) strictly decreasing

(c) non monotonic (d) data insufficient

56. If the straight line joining the points (0, 3) and (5, –2) is tangent to the curve $y = \frac{c}{x+1}$; then the value of c is :

(a) 2 (b) 3 (c) 4 (d) 5

57. Number of solutions(s) of $\ln|\sin x| = -x^2$ if $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is/are :

(a) 2 (b) 4 (c) 6 (d) 8

58. The equation $\sin^{-1} x = |x - a|$ will have atleast one solution then complete set of values of a be :

(a) $[-1, 1]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left[1-\frac{\pi}{2}, 1+\frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}-1, \frac{\pi}{2}+1\right]$

59. For any real number b, let $f(b)$ denotes the maximum of $\left|\sin x + \frac{2}{3+\sin x} + b\right| \forall \times x \in R$.

Then the minimum value of $f(b) \forall b \in R$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) 1

60. Which of the following are correct

(a) $x^4 + 2x^2 - 6x + 2 = 0$ has exactly four real solution

(b) $x^5 + 5x + 1 = 0$ has exactly three real solutions

(c) $x^n + ax + b = 0$ where n is an even natural number has atmost two real solution $a, b, \in R$.

(d) $x^3 - 3x + c = 0, c > 0$ has two real solution for $x \in (0, 1)$

61. For any real number b, let $f(b)$ denotes the maximum of $\left|\sin x + \frac{2}{3+\sin x} + b\right| \forall x \in R$. Then the minimum value of $f(b) \forall b \in R$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) 1

62. If p be a point on the graph of $y = \frac{x}{1+x^2}$, then coordinates of 'p' such that tangent drawn to curve at p has the greatest slope in magnitude is :

(a) $(0, 0)$ (b) $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$ (c) $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$ (d) $\left(1, \frac{1}{2}\right)$

63. Let $f:[0, 2\pi] \to [-3, 3]$ be a given function defined as $f(x) = 3\cos\frac{x}{2}$. The slope of the tangent to the curve $y = f^{-1}(x)$ at the point where the curve crosses the y-axis is :

(a) -1 (b) $-\frac{2}{3}$ (c) $-\frac{1}{6}$ (d) $-\frac{1}{3}$

64. Number of stationary points in $[0, \pi]$ for the function $f(x) = \sin x + \tan x - 2x$ is :

(a) 0 (b) 1 (c) 2 (d) 3

65. If $a, b, c, d \in R$ such that $\frac{a+2c}{b+3d} + \frac{4}{3} = 0$, then the equation $ax^3 + bx^2 + cx + d = 0$ has

(a) atleast one root in $(-1, 0)$ (b) atleast one root in $(0, 1)$

(c) no root in $(-1, 1)$ (d) no root in $(0, 2)$

66. If $f'(x) = \phi(x)(x-2)^2$. Where $\phi(2) \neq 0$ and $\phi(x)$ is continuous at $x = 2$, then in the neighbourhood of $x = 2$

(a) f is increasing if $\phi(2) < 0$ (b) f is decreasing if $\phi(2) > 0$

(c) f is neither increasing nor decreasing (d) f is increasing if $\phi(2) > 0$

67. If $f(x) = x^3 - 6x^2 + ax + b$ is defined on $[1, 3]$ satisfies Rolle's theorem for $c = \dfrac{2\sqrt{3}+1}{\sqrt{3}}$ then

(a) $a = -11, b = 6$ (b) $a = -11, b = -6$ (c) $a = 11, b \in R$ (d) $a = 22, b = -6$

68. For which of the following function(s) Lagrange's mean value theorem is not applicable in $[1, 2]$?

(a) $f(x) = \begin{cases} \dfrac{3}{2} - x & , \ x < \dfrac{3}{2} \\ \left(\dfrac{3}{2} - x\right)^2 & , \ x \geq \dfrac{3}{2} \end{cases}$ (b) $f(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1} & , \ x \neq 1 \\ 1 & , \ x = 1 \end{cases}$

(c) $f(x) = (x-1)|x-1|$ (d) $f(x) = |x-1|$

69. If the curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{4} = 1$ and $y^2 = 16x$ intersect at right angles, then :

(a) $a = \pm 1$ (b) $a = \pm\sqrt{3}$ (c) $a = \pm\dfrac{1}{\sqrt{3}}$ (d) $a = \pm\sqrt{2}$

70. If the line $x\cos\alpha + y\sin\alpha = P$ touches the curve $4x^3 = 27ay^2$, then $\dfrac{P}{a} =$

(a) $\cot^2\alpha\cos\alpha$ (b) $\cot^2\alpha\sin\alpha$ (c) $\tan^2\alpha\cos\alpha$ (d) $\tan^2\alpha\sin\alpha$

Answers

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(b)	**6.**	(c)	**7.**	(d)	**8.**	(d)	**9.**	(c)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(c)	**14.**	(c)	**15.**	(c)	**16.**	(a)	**17.**	(b)	**18.**	(a)	**19.**	(b)	**20.**	(b)
21.	(b)	**22.**	(d)	**23.**	(d)	**24.**	(a)	**25.**	(d)	**26.**	(c)	**27.**	(b)	**28.**	(d)	**29.**	(c)	**30.**	(d)
31.	(b)	**32.**	(c)	**33.**	(d)	**34.**	(d)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(a)	**39.**	(c)	**40.**	(a)
41.	(a)	**42.**	(b)	**43.**	(b)	**44.**	(a)	**45.**	(d)	**46.**	(b)	**47.**	(a)	**48.**	(a)	**49.**	(d)	**50.**	(a)
51.	(b)	**52.**	(d)	**53.**	(b)	**54.**	(a)	**55.**	(a)	**56.**	(c)	**57.**	(b)	**58.**	(c)	**59.**	(b)	**60.**	(c)
61.	(b)	**62.**	(a)	**63.**	(b)	**64.**	(c)	**65.**	(b)	**66.**	(d)	**67.**	(c)	**68.**	(a)	**69.**	(d)	**70.**	(a)

Exercise-2 : One or More than One Answer is/are Correct

1. Common tangent(s) to $y = x^3$ and $x = y^3$ is/are :

   (a) $x - y = \frac{1}{\sqrt{3}}$ (b) $x - y = -\frac{1}{\sqrt{3}}$ (c) $x - y = \frac{2}{3\sqrt{3}}$ (d) $x - y = \frac{-2}{3\sqrt{3}}$

2. Let $f:[0, 8] \to R$ be differentiable function such that $f(0) = 0$, $f(4) = 1$, $f(8) = 1$, then which of the following hold(s) good ?

   (a) There exist some $c_1 \in (0, 8)$ where $f'(c_1) = \frac{1}{4}$

   (b) There exist some $c \in (0, 8)$ where $f'(c) = \frac{1}{12}$

   (c) There exist $c_1, c_2 \in [0, 8]$ where $8f'(c_1)f(c_2) = 1$

   (d) There exist some $\alpha, \beta \in (0, 2)$ such that $\int_0^8 f(t)\,dt = 3(\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3))$

3. If $f(x) = \begin{cases} \sin^{-1}(\sin x) & x > 0 \\ \frac{\pi}{2} & x = 0 \\ \cos^{-1}(\cos x) & x < 0 \end{cases}$, then

   (a) $x = 0$ is a point of maxima

   (b) $f(x)$ is continuous $\forall\, x \in R$

   (c) global maximum value of $f(x)\ \forall\, x \in R$ is π

   (d) global minimum value of $f(x)\ \forall\, x \in R$ is 0

4. A function $f: R \to R$ is given by $f(x) = \begin{cases} x^4\left(2 + \sin\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, then

   (a) f has a continuous derivative $\forall\, x \in R$ (b) f is a bounded function

   (c) f has an global minimum at $x = 0$ (d) f'' is continuous $\forall\, x \in R$

5. If $|f''(x)| \le 1\ \forall\, x \in R$, and $f(0) = 0 = f'(0)$, then which of the following can not be true ?

   (a) $f\left(-\frac{1}{2}\right) = \frac{1}{6}$ (b) $f(2) = -4$ (c) $f(-2) = 3$ (d) $f\left(\frac{1}{2}\right) = \frac{1}{5}$

6. Let $f:[-3, 4] \to R$ such that $f''(x) > 0$ for all $x \in [-3, 4]$, then which of the following are always true ?

   (a) $f(x)$ has a relative minimum on $(-3, 4)$

   (b) $f(x)$ has a minimum on $[-3, 4]$

   (c) $f(x)$ has a maximum on $[-3, 4]$

   (d) if $f(3) = f(4)$, then $f(x)$ has a critical point on $[-3, 4]$

7. Let $f(x)$ be twice differentiable function such that $f''(x) > 0$ in $[0, 2]$. Then :

(a) $f(0) + f(2) = 2f(c)$, for atleast one $c, c \in (0, 2)$

(b) $f(0) + f(2) < 2f(1)$

(c) $f(0) + f(2) > 2f(1)$

(d) $2f(0) + f(2) > 3f\left(\frac{2}{3}\right)$

8. Let $g(x)$ be a cubic polynomial having local maximum at $x = -1$ and $g'(x)$ has a local minimum at $x = 1$. If $g(-1) = 10, g(3) = -22$, then :

(a) perpendicular distance between its two horizontal tangents is 12

(b) perpendicular distance between its two horizontal tangents is 32

(c) $g(x) = 0$ has atleast one real root lying in interval $(-1, 0)$

(d) $g(x) = 0$, has 3 distinct real roots

9. The function $f(x) = 2x^3 - 3(\lambda + 2)x^2 + 2\lambda x + 5$ has a maximum and a minimum for :

(a) $\lambda \in (-4, \infty)$ (b) $\lambda \in (-\infty, 0)$ (c) $\lambda \in (-3, 3)$ (d) $\lambda \in (1, \infty)$

10. The function $f(x) = 1 + x\ln\left(x + \sqrt{1 + x^2}\right) - \sqrt{1 - x^2}$ is :

(a) strictly increasing $\forall\, x \in (0, 1)$ (b) strictly decreasing $\forall\, x \in (-1, 0)$

(c) strictly decreasing for $x \in (-1, 0)$ (d) strictly decreasing for $x \in (0, 1)$

11. Let m and n be positive integers and $x, y > 0$ and $x + y = k$, where k is constant. Let $f(x, y) = x^m y^n$, then :

(a) $f(x, y)$ is maximum when $x = \dfrac{mk}{m + n}$

(b) $f(x, y)$ is maximum where $x = y$

(c) maximum value of $f(x, y)$ is $\dfrac{m^n n^m k^{m+n}}{(m + n)^{m+n}}$

(d) maximum value of $f(x, y)$ is $\dfrac{k^{m+n} m^m n^n}{(m + n)^{m+n}}$

12. The straight line which is both tangent and normal to the curve $x = 3t^2, y = 2t^3$ is :

(a) $y + \sqrt{3}(x - 1) = 0$ (b) $y - \sqrt{3}(x - 1) = 0$

(c) $y + \sqrt{2}(x - 2) = 0$ (d) $y - \sqrt{2}(x - 2) = 0$

13. A curve is such that the ratio of the subnormal at any point to the sum of its co-ordinates is equal to the ratio of the ordinate of this point to its abscissa. If the curve passes through (1, 0), then possible equation of the curve(s) is :

(a) $y = x\ln x$ (b) $y = \dfrac{\ln x}{x}$ (c) $y = \dfrac{2(x - 1)}{x^2}$ (d) $y = \dfrac{1 - x^2}{2x}$

14. A parabola of the form $y = ax^2 + bx + c\,(a > 0)$ intersects the graph of $f(x) = \dfrac{1}{x^2 - 4}$. The number of possible distinct intersection(s) of these graph can be :

(a) 0 (b) 2 (c) 3 (d) 4

15. Gradient of the line passing through the point (2, 8) and touching the curve $y = x^3$, can be :

(a) 3 (b) 6 (c) 9 (d) 12

16. The equation $x + \cos x = a$ has exactly one positive root, then :

(a) $a \in (0,1)$ (b) $a \in (2,3)$ (c) $a \in (1,\infty)$ (d) $a \in (-\infty,1)$

17. Given that $f(x)$ is a non-constant linear function. Then the curves :

(a) $y = f(x)$ and $y = f^{-1}(x)$ are orthogonal

(b) $y = f(x)$ and $y = f^{-1}(-x)$ are orthogonal

(c) $y = f(-x)$ and $y = f^{-1}(x)$ are orthogonal

(d) $y = f(-x)$ and $y = f^{-1}(-x)$ are orthogonal

18. Let $f(x) = \int_0^x e^{t^3}(t^2 - 1)t^2(t+1)^{2011}(t-2)^{2012}$ at $(x > 0)$ then :

(a) The number of point of inflections is atleast 1

(b) The number of point of inflections is 0

(c) The number of point of local maxima is 1

(d) The number of point of local minima is 1

19. Let $f(x) = \sin x + ax + b$. Then $f(x) = 0$ has :

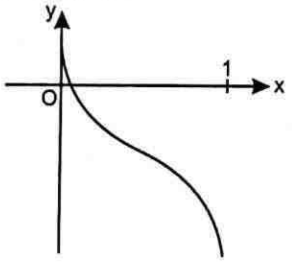
(a) only one real root which is positive if $a > 1, b < 0$

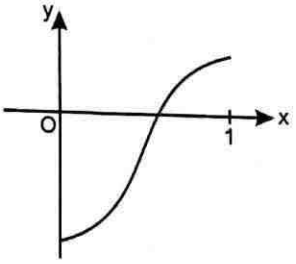
(b) only one real root which is negative if $a > 1, b > 0$

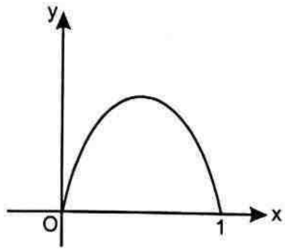
(c) only one real root which is negative if $a < -1, b < 0$

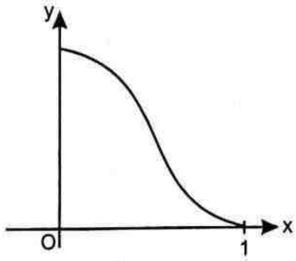
(d) only one real root which is positive if $a < -1, b < 0$

20. Which of the following graphs represent function whose derivatives have a maximum in the interval (0, 1) ?

(a)


(b)


(c) 

(d) 

21. Consider $f(x) = \sin^5 x + \cos^5 x - 1, x \in \left[0, \frac{\pi}{2}\right]$, which of the following is/are correct ?

(a) f is strictly decreasing in $\left[0, \frac{\pi}{4}\right]$

(b) f is strictly increasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(c) There exist a number 'c' in $\left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$

(d) The equation $f(x) = 0$ has only two roots in $\left[0, \frac{\pi}{2}\right]$

22. Let $f(x) = \begin{bmatrix} x^{2\alpha+1} \ln x & ; & x > 0 \\ 0 & ; & x = 0 \end{bmatrix}$

If $f(x)$ satisfies rolle's theorem in interval [0, 1], then α can be :

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $-\frac{1}{4}$ (d) -1

23. Which of the following is/are true for the function $f(x) = \int_0^x \frac{\cos t}{t} dt \ (x > 0)$?

(a) $f(x)$ is monotonically increasing in $\left((4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2}\right) \forall n \in N$

(b) $f(x)$ has a local minima at $x = (4n-1)\frac{\pi}{2} \ \forall n \in N$

(c) The points of inflection of the curve $y = f(x)$ lie on the curve $x \tan x + 1 = 0$

(d) Number of critical points of $y = f(x)$ in $(0, 10\pi)$ are 19

24. Let $F(x) = (f(x))^2 + (f'(x))^2$, $F(0) = 6$, where $f(x)$ is a thrice differentiable function such that $|f(x)| \le 1 \ \forall \ x \in [-1, 1]$, then choose the correct statement(s)

(a) there is atleast one point in each of the intervals (–1, 0) and (0, 1) where $|f'(x)| \le 2$

(b) there is atleast one point in each of the intervals (–1, 0) and (0, 1) where $F(x) \le 5$

(c) there is no point of local maxima of $F(x)$ in (–1, 1)

(d) for some $c \in (-1, 1)$, $F(c) \ge 6$, $F'(c) = 0$ and $F''(c) \le 0$

25. Let $f(x) = \begin{cases} x^3 + x^2 - 10x; & -1 \le x < 0 \\ \sin x; & 0 \le x < \dfrac{\pi}{2} \\ 1 + \cos x; & \dfrac{\pi}{2} \le x \le \pi \end{cases}$

then $f(x)$ has :

(a) local maximum at $x = \dfrac{\pi}{2}$ (b) local minimum at $x = \dfrac{\pi}{2}$

(c) absolute maximum at $x = 0$ (d) absolute maximum at $x = -1$

26. Minimum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is equal to :

(a) $\dfrac{\sqrt{2}}{4}$ (b) $\dfrac{3\sqrt{2}}{4}$ (c) $\dfrac{5\sqrt{2}}{4}$ (d) $\dfrac{7\sqrt{2}}{4}$

27. For the equation $\dfrac{e^{-x}}{1+x} = \lambda$ which of the following statement(s) is/are correct ?

(a) When $\lambda \in (0, \infty)$ equation has 2 real and distinct roots

(b) When $\lambda \in (-\infty, -e^2)$ equation has 2 real and distinct roots

(c) When $\lambda \in (0, \infty)$ equation has 1 real root

(d) When $\lambda \in (-e, 0)$ euqation has no real root

28. If $y = mx + 5$ is a tangent to the curve $x^3 y^3 = ax^3 + by^3$ at $P(1, 2)$, then

(a) $a + b = \dfrac{18}{5}$ (b) $a > b$ (c) $a < b$ (d) $a + b = \dfrac{19}{5}$

29. If $(f(x) - 1)(x^2 + x + 1)^2 - (f(x) + 1)(x^4 + x^2 + 1) = 0$

$\forall x \in R - \{0\}$ and $f(x) \neq \pm 1$, then which of the following statement(s) is/are correct ?

(a) $|f(x)| \ge 2 \, \forall x \in R - \{0\}$ (b) $f(x)$ has a local maximum at $x = -1$

(c) $f(x)$ has a local minimum at $x = 1$ (d) $\displaystyle\int_{-\pi}^{\pi} (\cos x) f(x)\, dx = 0$

Answers

1.	(c, d)	**2.**	(a, c, d)	**3.**	(a, c)	**4.**	(a, c)	**5.**	(a, b, c, d)	**6.**	(b, c, d)
7.	(c, d)	**8.**	(b, d)	**9.**	(a, b, c, d)	**10.**	(a, c)	**11.**	(a, d)	**12.**	(c, d)
13.	(a, d)	**14.**	(b, c, d)	**15.**	(a, d)	**16.**	(b, c)	**17.**	(b, c)	**18.**	(a, d)
19.	(a, b, c)	**20.**	(a, b)	**21.**	(a, b, c, d)	**22.**	(b, c)	**23.**	(a, b, c)	**24.**	(a, b, d)
25.	(a, d)	**26.**	(b)	**27.**	(b, c, d)	**28.**	(a, d)	**29.**	(a, b, c, d)		

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $y = f(x)$ such that $xy = x + y + 1$, $x \in R - \{1\}$ and $g(x) = xf(x)$

1. The minimum value of $g(x)$ is :

(a) $3-\sqrt{2}$ (b) $3+\sqrt{2}$ (c) $3-2\sqrt{2}$ (d) $3+2\sqrt{2}$

2. There exists two values of x, x_1 and x_2 where $g'(x) = \frac{1}{2}$, then $|x_1|+|x_2| =$

(a) 1 (b) 2 (c) 4 (d) 5

Paragraph for Question Nos. 3 to 5

Let $f(x) = \begin{cases} 1-x & ; \ 0 \le x \le 1 \\ 0 & ; \ 1 < x \le 2 \\ (2-x)^2 & ; \ 2 < x \le 3 \end{cases}$ and $g(x) = \int_0^x f(t)\,dt$.

Let the tangent to the curve $y = g(x)$ at point P whose abscissa is $\frac{5}{2}$ cuts x-axis in point Q.

Let the perpendicular from point Q on x-axis meets the curve $y = g(x)$ in point R.

3. $g(1) =$

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

4. Equation of tangent to the curve $y = g(x)$ at P is :

(a) $3y = 12x + 1$ (b) $3y = 12x - 1$ (c) $12y = 3x - 1$ (d) $12y = 3x + 1$

5. If 'θ' be the angle between tangents to the curve $y = g(x)$ at point P and R; then $\tan\theta$ equals to :

(a) $\frac{5}{6}$ (b) $\frac{5}{14}$ (c) $\frac{5}{7}$ (d) $\frac{5}{12}$

Paragraph for Question Nos. 6 to 8

Let $f(x) < 0 \ \forall \ x \in (-\infty, 0)$ and $f(x) > 0 \ \forall \ x \in (0, \infty)$ also $f(0) = 0$. Again $f'(x) < 0 \ \forall \ x \in (-\infty, -1)$ and $f'(x) > 0 \ \forall \ x \in (-1, \infty)$ also $f'(-1) = 0$ given $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$ and function is twice differentiable.

6. If $f''(x) > 0 \ \forall \ x \in (-1, \infty)$ and $f'(0) = 1$ then number of solutions of equation $f(x) = x$ is :

(a) 2 (b) 3 (c) 4 (d) None of these

7. If $f''(x) < 0 \ \forall \ x \in (0, \infty)$ and $f'(0) = 1$ then number of solutions of equation $f(x) = x^2$ is :

(a) 1 (b) 2 (c) 3 (d) 4

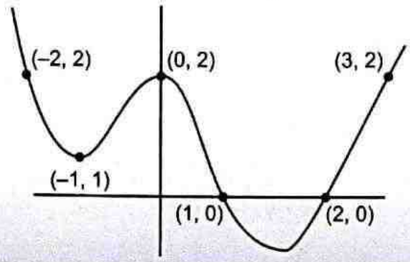
8. The minimum number of points where $f''(x)$ is zero is :

(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Question Nos. 9 to 11

In the given figure graph of :

$$y = p(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_n \text{ is given.}$$



9. The product of all imaginary roots of $p(x) = 0$ is :

(a) -2 (b) -1 (c) $-1/2$ (d) none of these

10. If $p(x) + k = 0$ has 4 distinct real roots $\alpha, \beta, \gamma, \delta$ then $[\alpha] + [\beta] + [\gamma] + [\delta]$, (where $[\cdot]$ denotes greatest integer function) is equal to :

(a) -1 (b) -2 (c) 0 (d) 1

11. The minimum number of real roots of equation $(p'(x))^2 + p(x)\,p''(x) = 0$ are :

(a) 3 (b) 4 (c) 5 (d) 6

Paragraph for Question Nos. 12 to 14

The differentiable function $y = f(x)$ has a property that the chord joining any two points $A(x_1, f(x_1))$ and $B(x_2, f(x_2))$ always intersects y-axis at $(0, 2x_1 x_2)$. Given that $f(1) = -1$, then :

12. $\int_0^{1/2} f(x)\,dx$ is equal to :

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$

13. The largest interval in which $f(x)$ is monotonically increasing, is :

(a) $\left(-\infty, \frac{1}{2}\right]$ (b) $\left[\frac{-1}{2}, \infty\right)$ (c) $\left(-\infty, \frac{1}{4}\right]$ (d) $\left[\frac{-1}{4}, \infty\right)$

14. In which of the following intervals, the Rolle's theorem is applicable to the function $F(x) = f(x) + x$?

(a) $[-1, 0]$ (b) $[0, 1]$ (c) $[-1, 1]$ (d) $[0, 2]$

Paragraph for Question Nos. 15 to 16

Let $f(x) = 1 + \int_0^1 (xe^y + ye^x)\, f(y)\, dy$ where x and y are independent variables.

15. If complete solution set of 'x' for which function $h(x) = f(x) + 3x$ is strictly increasing is $(-\infty, k)$ then $\left[\frac{4}{3}e^k\right]$ equals to : (where [·] denotes greatest integer function):

(a) 1 (b) 2 (c) 3 (d) 4

16. If acute angle of intersection of the curves $\frac{x}{2} + \frac{y}{3} + \frac{1}{3} = 0$ and $y = f(x)$ be θ then $\tan\theta$ equals to :

(a) $\frac{8}{25}$ (b) $\frac{16}{25}$ (c) $\frac{14}{25}$ (d) $\frac{4}{5}$

Answers

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(b)	**6.**	(d)	**7.**	(b)	**8.**	(a)	**9.**	(d)	**10.**	(a)
11.	(b)	**12.**	(d)	**13.**	(c)	**14.**	(b)	**15.**	(c)	**16.**	(a)								

Exercise-4 : Matching Type Problems

1. Column-I gives pair of curves and column-II gives the angle θ between the curves at their intersection point.

	Column-I		Column-II
(A)	$y = \sin x, y = \cos x$	(P)	$\frac{\pi}{4}$
(B)	$x^2 = 4y, y = \frac{8}{x^2+4}$	(Q)	$\frac{\pi}{2}$
(C)	$\frac{x^2}{18} + \frac{y^2}{8} = 1, x^2 - y^2 = 5$	(R)	$\tan^{-1} 3$
(D)	$xy = 1, x^2 - y^2 = 5$	(S)	$\tan^{-1} 5$
		(T)	$\tan^{-1}(2\sqrt{2})$

2.

	Column-I		Column-II
(A)	$(\sin^{-1} x)^{\cos^{-1} x} - (\cos^{-1} x)^{\sin^{-1} x} \; \forall \; x \in (\cos 1, \sin 1)$	(P)	Always positive
(B)	$(\cos x)^{\sin x} - (\sin x)^{\cos x} \; \forall \; x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$	(Q)	Always negative
(C)	$(\sin x)^{\sin x} - (\cos x)^{\sin x} \; \forall \; x \in \left(0, \frac{\pi}{2}\right)$	(R)	May be positive or negative for some values of x
(D)	$(\ln(\ln x))^{\ln(\ln x)} - (\ln x)^{\ln x} \; \forall \; x \in (e^e, \infty)$	(S)	May result in zero for some of values of x
		(T)	Indeterminate

3. Let $f(x) = \frac{x^3 - 4}{(x-1)^3} \forall x \neq 1$, $g(x) = \frac{x^4 - 2x^2}{4} \forall x \in R, h(x) \frac{x^3 + 4}{(x+1)^3} \forall x \neq -1$,

	Column-I		Column-II
(A)	The number of possible distinct real roots of equation $f(x) = c$ where $c \geq 4$ can be	(P)	0
(B)	The number of possible distinct real roots of equation $g(x) = c$, where $c \geq 0$ can be	(Q)	1
(C)	The number of possible distinct real roots of equation $h(x) = c$, where $c \geq 1$ can be	(R)	2

(D)	The number of possible distinct real roots of equation $g(x) = c$ where $-1 < c < 0$ can be	**(S)**	3
		(T)	4

4.

	Column-I		Column-II
(A)	If α, β, γ are roots of $x^3 - 3x^2 + 2x + 4 = 0$ and $y = 1 + \frac{\alpha}{x-\alpha} + \frac{\beta x}{(x-\alpha)(x-\beta)} + \frac{\gamma x^2}{(x-\alpha)(x-\beta)(x-\gamma)}$ then value of y at $x = 2$ is :	**(P)**	2
(B)	If $x^3 + ax + 1 = 0$ and $x^4 + ax + 1 = 0$ have a common roots then the value of $\lvert a \rvert$ can be equal to	**(Q)**	3
(C)	The number of local maximas of the function $x^2 + 4\cos x + 5$ is more than	**(R)**	4
(D)	If $f(x) = 2\lvert x\rvert^3 + 3x^2 - 12\lvert x\rvert + 1$, where $x \in [-1, 2]$ then greatest value of $f(x)$ is more than	**(S)**	5
		(T)	0

5.

	Column-I		Column-II
(A)	Maximum value of $f(x) = \log_2\left(\frac{4}{\sqrt{x+2} + \sqrt{2-x}}\right)$	**(P)**	0
(B)	The value of $\left[4\sum_{n=1}^{\infty} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right] =$ ([·] represent greatest integer function)	**(Q)**	1
(C)	Let $f(x) = x \sin \pi x, x > 0$ then number of points in $(0, 2)$ where $f'(x)$ vanishes, is	**(R)**	2
(D)	$\lim_{x \to 0^+} \left[\frac{x}{e^x - 1}\right] =$ ([·] represent greatest integer function)	**(S)**	3

6. Consider the function $f(x) = \frac{\ln x}{8} - ax + x^2$ and $a \geq 0$ is a real constant :

	Column-I		Column-II
(A)	$f(x)$ gives a local maxima at	**(P)**	$a = 1;\ x = \frac{1}{4}$
(B)	$f(x)$ gives a local minima at	**(Q)**	$a > 1;\ x = \frac{a - \sqrt{a^2 - 1}}{4}$
(C)	$f(x)$ gives a point of inflection for	**(R)**	$0 \leq a < 1$
(D)	$f(x)$ is strictly increasing for all $x \in R^+$	**(S)**	$a > 1;\ x = \frac{a + \sqrt{a^2 - 1}}{4}$

7. The function $f(x) = \sqrt{ax^3 + bx^2 + cx + d}$ has its non-zero local minimum and maximum values at $x = -2$ and $x = 2$ respectively. If 'a' is one of the root of $x^2 - x - 6 = 0$, then match the following :

	Column-I		Column-II
(A)	The value of 'a' is	**(P)**	0
(B)	The value of 'b' is	**(Q)**	24
(C)	The value of 'c' is	**(R)**	Greater than 32
(D)	The value of 'd' is	**(S)**	−2

8.

	Column-I		Column-II
(A)	The ratio of altitude to the radius of the cylinder of maximum volume that can be inscribed in a given sphere is	**(P)**	$\frac{1}{\sqrt{2}}$
(B)	The ratio of radius to the altitude of the cone of the greatest volume which can be inscribed in a given sphere is	**(Q)**	$\sqrt{2}$
(C)	The cone circumscribing the sphere of radius 'r' has the maximum volume if its semi vertical angle is θ, then $33 \sin\theta =$	**(R)**	$\frac{32}{3}$
(D)	The greatest value of x^3y^4 if $2x + 3y = 7$, $x \geq 0, y \geq 0$ is	**(S)**	11

Answers

1.	$A \to T$; $B \to R$; $C \to Q$; $D \to Q$
2.	$A \to R, S$; $B \to Q$; $C \to R, S$; $D \to Q$
3.	$A \to Q, R$; $B \to R, S$; $C \to Q, R, S$; $D \to P, R, T$
4.	$A \to P$; $B \to P$; $C \to T$; $D \to P, Q, R, T$
5.	$A \to Q$; $B \to S$; $C \to R$; $D \to P$
6.	$A \to Q$; $B \to S$; $C \to P$; $D \to R$
7.	$A \to S$; $B \to P$; $C \to Q$; $D \to R$
8.	$A \to Q$; $B \to P$; $C \to S$; $D \to R$

Exercise-5 : Subjective Type Problems

1. A conical vessel is to be prepared out of a circular sheet of metal of unit radius. In order that the vessel has maximum volume, the sectorial area that must be removed from the sheet is A_1 and the area of the given sheet is A_2. If $\frac{A_2}{A_1} = m + \sqrt{n}$, where $m, n \in N$, then $m + n$ is equal to.

2. On $[1, e]$, the least and greatest vlaues of $f(x) = x^2 \ln x$ are m and M respectively, then $[\sqrt{M + m}]$ is : (where [ ] denotes greatest integer function)

3. If $f(x) = \frac{px}{e^x} - \frac{x^2}{2} + x$ is a decreasing function for every $x \le 0$. Find the least value of p^2.

4. Let $f(x) = \begin{cases} xe^{ax} & , \; x \le 0 \\ x + ax^2 - x^3 & , \; x > 0 \end{cases}$. Where a is a positive constant. The interval in which $f'(x)$ is increasing is $\left[\frac{k}{a}, \frac{a}{l}\right]$. Then $k + l$ is equal to

5. Find sum of all possible values of the real parameter 'b' if the difference between the largest and smallest values of the function $f(x) = x^2 - 2bx + 1$ in the interval $[0, 1]$ is 4.

6. Let 'θ' be the angle in radians between the curves $\frac{x^2}{36} + \frac{y^2}{4} = 1$ and $x^2 + y^2 = 12$. If $\theta = \tan^{-1}\left(\frac{a}{\sqrt{3}}\right)$; Find the value of a.

7. Let set of all possible values of λ such that $f(x) = e^{2x} - (\lambda + 1)e^x + 2x$ is monotonically increasing for $\forall x \in R$ is $(-\infty, k]$. Find the value of k.

8. Let a, b, c and d be non-negative real number such that $a^5 + b^5 \le 1$ and $c^5 + d^5 \le 1$. Find the maximum value of $a^2c^3 + b^2d^3$.

9. There is a point (p, q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = -8/x$, where $p > 0$ and $r > 0$. If the line through (p, q) and (r, s) is also tangent to both the curves at these points respectively, then find the value of $(p + r)$.

10. $f(x) = \max|2\sin y - x|$ where $y \in R$ then determine the minimum value of $f(x)$.

11. Let $f(x) = \int_0^x ((a-1)(t^2 + t + 1)^2 - (a+1)(t^4 + t^2 + 1))\,dt$. Then the total number of integral values of 'a' for which $f'(x) = 0$ has no real roots is

12. The number of real roots of the equation $x^{2013} + e^{2014x} = 0$ is

13. Let the maximum value of expression $y = \frac{x^4 - x^2}{x^6 + 2x^3 - 1}$ for $x > 1$ is $\frac{p}{q}$ where p and q are relatively prime positive integers. Find the value of $(p + q)$.

14. The least positive value of the parameter 'a' for which there exists atleast one line that is tangent to the graph of the curve $y = x^3 - ax$, at one point and normal to the graph at another point is $\frac{p}{q}$; where p and q are relatively prime positive integers. Find product pq.

15. Let $f(x) = x^2 + 2x - t^2$ and $f(x) = 0$ has two roots $\alpha(t)$ and $\beta(t) (\alpha < \beta)$ where t is a real parameter. Let $I(t) = \int_{\alpha}^{\beta} f(x)\,dx$. If the maximum value of $I(t)$ be λ and $|\lambda| = \frac{p}{q}$ where p and q are relatively prime positive integers. Find the product (pq).

16. A tank contains 100 litres of fresh water. A solution containing 1 gm/litre of salt runs into the tank at the rate of 1 lit/min. The homogenised mixture is pumped out of the tank at the rate of 3 lit/min. If T be the time when the amount of salt in the tank is maximum.
Find $[T]$ (where $[\cdot]$ denotes greatest integer function)

17. If $f(x)$ is continuous and differentiable in $[-3, 9]$ and $f'(x) \in [-2, 8]\ \forall\ x \in (-3, 9)$. Let N be the number of divisors of the greatest possible value of $f(9) - f(-3)$, then find the sum of digits of N.

18. It is given that $f(x)$ is defined on R satisfying $f(1) = 1$ and for $\forall\ x \in R$,
$f(x+5) \geq f(x) + 5$ and $f(x+1) \leq f(x) + 1$. If $g(x) = f(x) + 1 - x$, then $g(2002) =$

19. The number of normals to the curve $3y^3 = 4x$ which passes through the point $(0, 1)$ is

20. Find the number of real root(s) of the equation $ae^x = 1 + x + \frac{x^2}{2}$; where a is positive constant.

21. Let $f(x) = ax + \cos 2x + \sin x + \cos x$ is defined for $\forall\ x \in R$ and $a \in R$ and is strictly increasing function. If the range of a is $\left[\frac{m}{n}, \infty\right)$, then find the minimum value of $(m - n)$.

22. If p_1 and p_2 are the lengths of the perpendiculars from origin on the tangent and normal drawn to the curve $x^{2/3} + y^{2/3} = 6^{2/3}$ respectively. Find the value of $\sqrt{4p_1^2 + p_2^2}$.

Answers

1.	9	**2.**	2	**3.**	1	**4.**	1	**5.**	1	**6.**	2	**7.**	3
8.	1	**9.**	5	**10.**	2	**11.**	3	**12.**	1	**13.**	7	**14.**	12
15.	12	**16.**	27	**17.**	3	**18.**	1	**19.**	1	**20.**	1	**21.**	9
22.	6												

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Chapter 5 – Indefinite and Definite Integration

5 INDEFINITE AND DEFINITE INTEGRATION

Exercise-1 : Single Choice Problems

1. $\int a^x \left(\ln x + \ln a \cdot \ln\left(\frac{x}{e}\right)^x \right) dx =$

(a) $a^x \ln\left(\frac{e}{x}\right)^{2x} + C$

(b) $a^x \ln\left(\frac{x}{e}\right)^x + C$

(c) $a^x + \ln\left(\frac{x}{e}\right)^x + C$

(d) None of these

2. The value of :

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}}+\frac{1}{\sqrt{n}\sqrt{n+2}}+\frac{1}{\sqrt{n}\sqrt{n+3}}+\ldots\ldots+\frac{1}{\sqrt{n}\sqrt{2n}}\right) \text{ is :}$$

(a) $\sqrt{2}-1$ (b) $2(\sqrt{2}-1)$ (c) $\sqrt{2}+1$ (d) $2(\sqrt{2}+1)$

3. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is :

(a) $(\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$ (c) $(-\sin\alpha, \cos\alpha)$ (d) $(-\cos\alpha, \sin\alpha)$

4. The value of the integral $\int_0^2 \frac{\log(x^2+2)}{(x+2)^2} dx$ is :

(a) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\frac{5}{12}\log 2-\frac{1}{4}\log 3$

(b) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\frac{5}{12}\log 2-\frac{1}{12}\log 3$

(c) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\frac{5}{12}\log 2+\frac{1}{12}\log 3$

(d) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\frac{5}{12}\log 2+\frac{1}{12}\log 3$

5. If $I_1 = \int_0^1 \frac{1+x^8}{1+x^4} dx$ and $I_2 = \int_0^1 \frac{1+x^9}{1+x^3} dx$, then :

(a) $I_1 > 1, I_2 < 1$ (b) $I_1 < 1, I_2 > 1$ (c) $1 < I_1 < I_2$ (d) $I_2 < I_1 < 1$

6. Let $f:(0,1)\to(0,1)$ be a differentiable function such that $f'(x)\neq 0$ for all $x\in(0,1)$ and $f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$. Suppose for all x, $\lim\limits_{t\to x}\left(\frac{\int_0^t \sqrt{1-(f(s))^2}\,ds-\int_0^x \sqrt{1-(f(s))^2}\,ds}{f(t)-f(x)}\right)=f(x)$. Then the value of $f\left(\frac{1}{4}\right)$ belongs to :

(a) $\left\{\frac{\sqrt{7}}{4},\frac{\sqrt{15}}{4}\right\}$ (b) $\left\{\frac{\sqrt{7}}{3},\frac{\sqrt{15}}{3}\right\}$ (c) $\left\{\frac{\sqrt{7}}{2},\frac{\sqrt{15}}{2}\right\}$ (d) $\{\sqrt{7},\sqrt{15}\}$

7. If $f(\theta)=\frac{4}{3}(1-\cos^6\theta-\sin^6\theta)$, then

$$\lim_{n\to\infty}\frac{1}{n}\left[\sqrt{f\left(\frac{1}{n}\right)}+\sqrt{f\left(\frac{2}{n}\right)}+\sqrt{f\left(\frac{3}{n}\right)}+\ldots\ldots+\sqrt{f\left(\frac{n}{n}\right)}\right]=$$

(a) $\frac{1-\cos 1}{2}$ (b) $1-\cos 2$ (c) $\frac{\sin 2}{2}$ (d) $\frac{1-\cos 2}{2}$

8. The value of $\int_0^1 \frac{(x^6-x^3)}{(2x^3+1)^3}dx$ is equal to :

(a) $-\frac{1}{6}$ (b) $-\frac{1}{12}$ (c) $-\frac{1}{18}$ (d) $-\frac{1}{36}$

9. $2\int_0^{\frac{1}{\sqrt{2}}}\frac{\sin^{-1}x}{x}dx-\int_0^1\frac{\tan^{-1}x}{x}dx=$

(a) $\frac{\pi}{8}\ln 2$ (b) $\frac{\pi}{4}\ln 2$ (c) $\frac{\pi}{2\sqrt{2}}\ln 2$ (d) $\frac{\pi}{2}\ln 2$

10. Let $f(x)$ be a differentiable function such that $f(x)=x^2+\int_0^x e^{-t}f(x-t)\,dt$, then $\int_0^1 f(x)\,dx=$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

11. If $f'(x)=f(x)+\int_0^1 f(x)\,dx$ and given $f(0)=1$, then $\int f(x)\,dx$ is equal to :

(a) $\frac{2}{3-e}e^x+\left(\frac{3-e}{1-e}\right)x+C$ (b) $\frac{2}{3-e}e^x+\left(\frac{1-e}{3-e}\right)x+C$

(c) $\frac{3}{2-e}e^x+\left(\frac{1+e}{3+e}\right)x+C$ (d) $\frac{2}{2-e}e^x+\left(\frac{1-e}{3+e}\right)x+C$

(where C is an arbitrary constant.)

12. For any $x \in R$, and f be a continuous function. Let $I_1 = \int_{\sin^2 x}^{1+\cos^2 x} tf(t(2-t))\,dt, I_2 = \int_{\sin^2 x}^{1+\cos^2 x} f(t(2-t))\,dt,$

then $I_1 =$

(a) I_2 (b) $\frac{1}{2}I_2$ (c) $2I_2$ (d) $3I_2$

13. If the integral $\int \frac{5\tan x\,dx}{\tan x - 2} = x + a\ln|\sin x - 2\cos x| + C$, then '$a$' is equal to :

(a) 1 (b) 2 (c) -1 (d) -2

14. $\int \frac{(2+\sqrt{x})\,dx}{(x+1+\sqrt{x})^2}$ is equal to :

(a) $\frac{x}{x+\sqrt{x}+1}+C$ (b) $\frac{2x}{x+\sqrt{x}+1}+C$

(c) $\frac{-2x}{x+\sqrt{x}+1}+C$ (d) $\frac{-x}{x+\sqrt{x}+1}+C$

(where C is an arbitrary constant.)

15. Evaluate $\int \frac{\left(\sqrt[3]{x+\sqrt{2-x^2}}\right)\left(\sqrt[6]{1-x\sqrt{2-x^2}}\right)dx}{\sqrt[3]{1-x^2}}$; $x \in (0,1)$:

(a) $2^{\frac{1}{6}}x+C$ (b) $2^{\frac{1}{12}}x+C$

(c) $2^{\frac{1}{3}}x+C$ (d) None of these

16. $\int \frac{dx}{\sqrt{1-\tan^2 x}} = \frac{1}{\lambda}\sin^{-1}(\lambda \sin x)+C$, then $\lambda =$

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{5}$

17. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$ is equal to :

(a) $-\left(\frac{x+1}{x}\right)^{1/6}+C$ (b) $6\left(\frac{x+1}{x}\right)^{-1/6}+C$

(c) $\left(\frac{x}{x+1}\right)^{5/6}+C$ (d) $-\left(\frac{x}{x+1}\right)^{5/6}+C$

18. If $I_n = \int (\sin x)^n\,dx; n \in N$, then $5I_4 - 6I_6$ is equal to :

(a) $\sin x\cdot(\cos x)^5+C$ (b) $\sin 2x\cos 2x+C$

(c) $\frac{\sin 2x}{8}[1+\cos^2 2x - 2\cos 2x]+C$ (d) $\frac{\sin 2x}{8}[1+\cos^2 2x+2\cos 2x]+C$

19. $\int \frac{x^2}{(a+bx)^2} dx$ equals to :

(a) $\frac{1}{b^3}\left(a+bx-a\ln|a+bx|-\frac{a^2}{a+bx}\right)+C$ (b) $\frac{1}{b^3}\left(a+bx-2a\ln|a+bx|-\frac{a^2}{a+bx}\right)+C$

(c) $\frac{1}{b^3}\left(a+bx+2a\ln|a+bx|-\frac{a^2}{a+bx}\right)+C$ (d) $\frac{1}{b^3}\left(a+bx-2a\ln|a+ax|-\frac{a^2}{a+bx}\right)+C$

20. $\int \frac{8x^{43}+13x^{38}}{(x^{13}+x^5+1)^4} dx =$

(a) $\frac{x^{39}}{3(x^{13}+x^5+1)^3}+C$ (b) $\frac{x^{39}}{(x^{13}+x^5+1)^3}+C$

(c) $\frac{x^{39}}{5(x^{13}+x^5+1)^5}+C$ (d) None of these

21. $\int\left(\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{10\cos^2 x+5\cos x\cos 3x+\cos x\cos 5x}\right)dx = f(x)+C$, then $f(10)$ is equal to :

(a) 20 (b) 10 (c) $2\sin 10$ (d) $2\cos 10$

22. $\int (1+x-x^{-1})e^{x+x^{-1}} dx =$

(a) $(x+1)e^{x+x^{-1}}+C$ (b) $(x-1)e^{x+x^{-1}}+C$

(c) $-xe^{x+x^{-1}}+C$ (d) $xe^{x+x^{-1}}+C$

23. If $\int e^x\left(\frac{2\tan x}{1+\tan x}+\operatorname{cosec}^2\left(x+\frac{\pi}{4}\right)\right)dx = e^x\cdot g(x)+K$, then $g\left(\frac{5\pi}{4}\right)=$

(a) 0 (b) 1 (c) -1 (d) 2

24. $\int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x-x\sin x+\cos x}{x^2\cos^2 x}\right)dx =$

(a) $e^{x\sin x+\cos x}\left(x-\frac{1}{\cos x}\right)+C$ (b) $e^{x\sin x+\cos x}\left(x-\frac{1}{x\cos x}\right)+C$

(c) $e^{x\sin x+\cos x}\left(1-\frac{1}{x\cos x}\right)+C$ (d) $e^{x\sin x+\cos x}\left(1-\frac{x}{\cos x}\right)+C$

25. The value of the definite integral $\int_0^1 \frac{1+x+\sqrt{x+x^2}dx}{\sqrt{x}+\sqrt{1+x}} dx$ is :

(a) $\frac{1}{3}(2^{1/2}-1)$ (b) $\frac{2}{3}(2^{1/2}-1)$

(c) $\frac{2}{3}(2^{3/2}-1)$ (d) $\frac{1}{3}(2^{3/2}-1)$

26. $\int x^{x^2+1}(2\ln x+1)\,dx$

(a) $x^{2x}+C$ (b) $x^2\ln x+C$ (c) $x^{(x^x)}+C$ (d) $(x^x)^x+C$

27. If $\int \frac{\operatorname{cosec}^2 x-2010}{\cos^{2010} x}dx=-\frac{f(x)}{(g(x))^{2010}}+C$; where $f\left(\frac{\pi}{4}\right)=1$; then the number of solutions of the equation $\frac{f(x)}{g(x)}=\{x\}$ in $[0, 2\pi]$ is/are : (where $\{\cdot\}$ represents fractional part function)

(a) 0 (b) 1 (c) 2 (d) 3

28. $\int x^x\left((\ln x)^2+\ln x+\frac{1}{x}\right)dx$ is equal to :

(a) $x^x\left((\ln x)^2-\frac{1}{x}\right)+C$ (b) $x^x(\ln x-x)+C$

(c) $x^x\frac{(\ln x)^2}{2}+C$ (d) $x^x\ln x+C$

29. If $I=\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}$ is equal to :

(a) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+C$ (b) $\frac{\sqrt{2x^4-2x^2+1}}{x}+C$

(c) $\frac{\sqrt{2x^4-2x^2+1}}{x}+C$ (d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+C$

30. $I=\int\left(\frac{\ln x-1}{(\ln x)^2+1}\right)^2 dx$ is equal to :

(a) $\frac{x}{x^2+1}+C$ (b) $\frac{\ln x}{(\ln x)^2+1}+C$ (c) $\frac{x}{1+(\ln x)^2}+C$ (d) $e^x\left(\frac{x}{x^2+1}\right)+C$

31. $I=\int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}}=k\sqrt[4]{\frac{x-1}{x+2}}+C$, then '$k$' is equal to :

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

32. $\int \frac{1-x^7}{x(1+x^7)}dx=P\log|x|+Q\log|x^7+1|+C$, then :

(a) $2P-7Q=0$ (b) $2P+7Q=0$ (c) $7P+2Q=0$ (d) $7P-2Q=1$

33. $I=\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$ is equal to :

(a) $\sin 2x+C$ (b) $\frac{\sin 2x}{2}+C$ (c) $\frac{-\sin 2x}{2}+C$ (d) $-2\sin 2x+C$

34. $I = \int \dfrac{(\sin 2x)^{1/3}\, d(\tan^{1/3} x)}{\sin^{2/3} x + \cos^{2/3} x} =$

(a) $\dfrac{1}{2^{2/3}} \ln(1+\tan^{1/3} x) + C$

(b) $\ln(1+\tan^{2/3} x) + C$

(c) $2^{1/3} \ln(1+\tan^{2/3} x) + C$

(d) $\dfrac{1}{2^{2/3}} \ln(1+\tan^{2/3} x) + C$

35. $\int \sqrt{\dfrac{(2012)^{2x}}{1-(2012)^{2x}}}\,(2012)^{\sin^{-1}(2012)^x}\, dx =$

(a) $(\log_{2012} e)^2 (2012)^{\sin^{-1}(2012)^x} + C$

(b) $(\log_{2012} e)^2 (2012)^{x+\sin^{-1}(2012)^x} + C$

(c) $(\log_{2012} e)^2 (2012)^{\sin^{-1}(2012)^x} + C$

(d) $\dfrac{(2012)^{\sin^{-1}(2012)^x}}{(\log_{2012} e)^2} + C$

(where C denotes arbitrary constant.)

36. $\int \dfrac{(x+2)\,dx}{(x^2+3x+3)\sqrt{x+1}}$ is equal to :

(a) $\dfrac{2}{\sqrt{3}} \tan^{-1}\left(\dfrac{x}{\sqrt{3(x+1)}}\right) + C$

(b) $\dfrac{2}{\sqrt{3}} \tan^{-1}\left(\sqrt{\dfrac{x}{3(x+1)}}\right) + C$

(c) $\dfrac{1}{\sqrt{3}} \tan^{-1}\left(\dfrac{\sqrt{x}}{3(x+1)}\right) + C$

(d) $\dfrac{2}{\sqrt{3}} \tan^{-1}\left(\dfrac{x}{\sqrt{3}(x+1)}\right) + C$

(where C is arbitrary constant.)

37. $\int \left(\dfrac{f(x)g'(x) - f'(x)g(x)}{f(x)g(x)}\right)(\log(g(x)) - \log(f(x)))\, dx$ is equal to :

(a) $\log\left(\dfrac{g(x)}{f(x)}\right) + C$

(b) $\dfrac{1}{2}\left(\dfrac{g(x)}{f(x)}\right)^2 + C$

(c) $\dfrac{1}{2}\left(\log\left(\dfrac{g(x)}{f(x)}\right)\right)^2 + C$

(d) $\log\left(\left(\dfrac{g(x)}{f(x)}\right)^2\right) + C$

38. $\int \left(\int e^x \left(\ln x + \dfrac{2}{x} - \dfrac{1}{x^2}\right) dx\right) dx =$

(a) $e^x \ln x + C_1 x + C_2$

(b) $e^x \ln x + \dfrac{1}{x} + C_1 x + C_2$

(c) $\dfrac{\ln x}{x} + C_1 x + C_2$

(d) None of these

39. Maximum value of the function $f(x) = \pi^2 \int_0^1 t \sin(x + \pi t)\,dt$ over all real number x :

(a) $\sqrt{\pi^2 + 1}$ (b) $\sqrt{\pi^2 + 2}$ (c) $\sqrt{\pi^2 + 3}$ (d) $\sqrt{\pi^2 + 4}$

40. Let 'f' is a function, continuous on [0, 1] such that $f(x) \le \sqrt{5}\ \ \forall x \in [0,1]$ and $f(x) \le \frac{2}{x} \forall x \in \left[\frac{1}{2}, 1\right]$ then the smallest 'a' for which $\int_0^1 f(x)\,dx \le a$ holds for all 'f' is :

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2} + 2\ln 2$ (c) $2 + \ln\left(\frac{\sqrt{5}}{2}\right)$ (d) $2 + 2\ln\left(\frac{\sqrt{5}}{2}\right)$

41. Let $I_n = \int_1^{e^2} (\ln x)^n\, d(x^2)$, then the value of $2I_n + nI_{n-1}$ equals to :

(a) 0 (b) $2e^2$ (c) e^2 (d) 1

42. Let a function $f: R \to R$ be defined as $f(x) = x + \sin x$. The value of $\int_0^{2\pi} f^{-1}(x)\,dx$ will be :

(a) $2\pi^2$ (b) $2\pi^2 - 2$ (c) $2\pi^2 + 2$ (d) π^2

43. The value of the definite integral $\int_{-1}^{1} e^{-x^4}\left(2 + \ln\left(x + \sqrt{x^2 + 1}\right) + 5x^3 - 8x^4\right) dx$ is equal to :

(a) $4e$ (b) $\frac{4}{e}$ (c) $2e$ (d) $\frac{2}{e}$

44. $\displaystyle\int_{-10}^{0} \frac{\left|\frac{2[x]}{3x - [x]}\right|}{\frac{2[x]}{3x - [x]}}\,dx$ is equal to (where [*] denotes greatest integer function.)

(a) $\frac{28}{3}$ (b) $\frac{1}{3}$ (c) 0 (d) None of these

45. If $f(x) = \dfrac{x}{1 + (\ln x)(\ln x)\ldots\infty}\ \forall x \in [1, \infty)$ then $\int_1^{2e} f(x)\,dx$ equals is :

(a) $\frac{e^2 - 1}{2}$ (b) $\frac{e^2 + 1}{2}$ (c) $\frac{e^2 - 2e}{2}$ (d) None of these

46. $\displaystyle\int_0^4 \frac{(y^2 - 4y + 5)\sin(y - 2)}{(2y^2 - 8y + 11)}\,dy$ is equal to :

(a) 0 (b) 2 (c) −2 (d) None of these

47. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right)$, $x>0$. If $\int_1^4 \frac{3}{x}e^{\sin x^3}dx=F(k)-F(1)$, then one of the possible values of k, is:

(a) 15 (b) 16 (c) 63 (d) 64

48. Value of $\lim_{h\to 0}\frac{\int_0^{\pi+he^{-1/h}} x^2e^{-x^2}dx-\int_0^{\pi} x^2e^{-x^2}dx}{he^{-1/h}}$ is equal to :

(a) $\pi(1-\pi^2)e^{-\pi^2}$ (b) $2\pi(1-\pi^2)e^{-\pi^2}$ (c) $\pi(1-\pi)e^{-\pi}$ (d) $\pi^2e^{-\pi^2}$

49. Let $f:R^+\to R$ be a differentiable function with $f(1)=3$ and satisfying :

$\int_1^{xy} f(t)\,dt=y\int_1^x f(t)\,dt+x\int_1^y f(t)\,dt\ \forall\, x,y\in R^+$, then $f(e)=$

(a) 3 (b) 4 (c) 1 (d) None of these

50. If [·] denotes the greatest integer function, then the integral $\int_0^{\pi/2}\frac{e^{\sin x-[\sin x]}d(\sin^2 x-[\sin^2 x])}{\sin x-[\sin x]}$ is λ, then $[\lambda-1]$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 3

51. Calculate the reciprocal of the limit $\lim_{x\to\infty}\int_0^x xe^{t^2-x^2}dt$

(a) 0 (b) 1 (c) 2 (d) 3

52. Let $L=\lim_{n\to\infty}\left(\frac{(2\cdot 1+n)}{1^2+n\cdot 1+n^2}+\frac{(2\cdot 2+n)}{2^2+n\cdot 2+n^2}+\frac{(2\cdot 3+n)}{3^2+n\cdot 3+n^2}+......+\frac{(2\cdot n+n)}{3n^2}\right)$ then value of e^L is :

(a) 2 (b) 3 (c) 4 (d) $\frac{3}{2}$

53. The value of the definite integral $\int_0^2\left(\sqrt{1+x^3}+\sqrt[3]{x^2+2x}\right)dx$ is :

(a) 4 (b) 5 (c) 6 (d) 7

54. The value of the definite integral $\int_0^\infty \frac{\ln x}{x^2+4}dx$ is :

(a) $\frac{\pi\ln 3}{2}$ (b) $\frac{\pi\ln 2}{3}$

(c) $\frac{\pi\ln 2}{4}$ (d) $\frac{\pi\ln 4}{3}$

55. The value of the definite integral $\int_0^{10} ((x-5)+(x-5)^2+(x-5)^3)\,dx$ is :

(a) $\frac{125}{3}$ (b) $\frac{250}{3}$ (c) $\frac{125}{6}$ (d) $\frac{250}{4}$

56. The value of definite integral $\int_0^{\infty} \frac{dx}{(1+x^9)(1+x^2)}$ equals to :

(a) $\frac{\pi}{16}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

57. The value of the definite integral $\int_0^{\pi/2} \left(\frac{1+\sin 3x}{1+2\sin x}\right) dx$ equals to :

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$

58. The value of $\lim_{x\to\infty} \frac{\int_0^x (\tan^{-1} x)^2\,dx}{\sqrt{x^2+1}} =$

(a) $\frac{\pi^2}{16}$ (b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{2}$ (d) None of these

59. If $\int_0^1 \left(\sum_{r=1}^{2013} \frac{x}{x^2+r^2}\right)\left(\prod_{r=1}^{2013}(x^2+r^2)\right) dx = \frac{1}{2}\left[\left(\prod_{r=1}^{2013}(1+r^2)\right) - k^2\right]$

then $k =$

(a) 2013 (b) 2013! (c) 2013^2 (d) 2013^{2013}

60. $f(x) = 2x - \tan^{-1} x - \ln(x+\sqrt{1+x^2})$

(a) strictly increases $\forall\, x \in R$

(b) strictly increases only in $(0, \infty)$

(c) strictly decreases $\forall\, x \in R$

(d) strictly decreases in $(0, \infty)$ and strictly increases in $(-\infty, 0)$

61. The value of the definite integral $\int_0^{\pi/2} \frac{dx}{\tan x + \cot x + \operatorname{cosec} x + \sec x}$ is :

(a) $1-\frac{\pi}{4}$ (b) $\frac{\pi}{4}+1$ (c) $\pi+\frac{1}{4}$ (d) None of these

62. The value of the definite integral $\int_3^7 \frac{\cos x^2}{\cos x^2 + \cos(10-x)^2} dx$ is :

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

63. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\ln x}{x}\right| dx$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5

64. The value of $\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\operatorname{cosec}^2 x} tg(t)\,dt}{x^2 - \frac{\pi^2}{16}}$ is :

(a) $\frac{2}{\pi}g(2)$ (b) $-\frac{4}{\pi}g(2)$ (c) $-\frac{16}{\pi}g(2)$ (d) $-4g(2)$

65. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n-k}{n^2} \cos\frac{4k}{n}$ equals :

(a) $\frac{1}{4}\sin 4 + \frac{1}{16}\cos 4 - \frac{1}{16}$ (b) $\frac{1}{4}\sin 4 - \frac{1}{16}\cos 4 + \frac{1}{16}$

(c) $\frac{1}{16}(1-\sin 4)$ (d) $\frac{1}{16}(1-\cos 4)$

66. For each positive integer n, define a function f_n on [0, 1] as follows :

$$f_n(x) = \begin{cases} 0 & \text{if} \quad x = 0 \\ \sin\frac{\pi}{2n} & \text{if} \quad 0 < x \le \frac{1}{n} \\ \sin\frac{2\pi}{2n} & \text{if} \quad \frac{1}{n} < x \le \frac{2}{n} \\ \sin\frac{3\pi}{2n} & \text{if} \quad \frac{2}{n} < x \le \frac{3}{n} \\ \sin\frac{n\pi}{2n} & \text{if} \quad \frac{n-1}{n} < x \le 1 \end{cases}$$

Then the value of $\lim_{n\to\infty} \int_0^1 f_n(x)\,dx$ is :

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{1}{\pi}$ (d) $\frac{2}{\pi}$

67. Let n be a positive integer, then

$$\int_0^{n+1} \min\{|x-1|,|x-2|,|x-3|,\ldots\ldots|x-n|\}\,dx \text{ equals}$$

(a) $\frac{(n+1)}{4}$ (b) $\frac{(n+2)}{4}$ (c) $\frac{(n+3)}{4}$ (d) $\frac{(n+4)}{4}$

68. For positive integers $k = 1, 2, 3\ldots\ldots, n$, let S_k denotes the area of ΔAOB_k (where 'O' is origin) such that $\angle AOB_k = \frac{k\pi}{2n}$, $OA = 1$ and $OB_k = k$. The value of the $\lim\limits_{n\to\infty} \frac{1}{n^2}\sum\limits_{k=1}^{n} S_k$ is :

(a) $\frac{2}{\pi^2}$ (b) $\frac{4}{\pi^2}$ (c) $\frac{8}{\pi^2}$ (d) $\frac{1}{2\pi^2}$

69. If $A = \int_0^1 \prod\limits_{r=1}^{2014}(r-x)\,dx$ and $B = \int_0^1 \prod\limits_{r=0}^{2013}(r+x)\,dx$, then :

(a) $A = 2B$ (b) $2A = B$ (c) $A + B = 0$ (d) $A = B$

70. If $f(x) = \left[\frac{x}{120} + \frac{x^3}{30}\right]$ defined in [0, 3], then $\int_0^1 (f(x)+2)\,dx =$

(where [·] denotes greatest integer function)

(a) 0 (b) 1 (c) 2 (d) 4

71. If $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$, $g(x) = \int_0^{\cos x} (1+\sin t)^2\,dt$, then the value of $f'\left(\frac{\pi}{2}\right)$ is equal to :

(a) 1 (b) -1 (c) 0 (d) $\frac{1}{2}$

72. Let $f(x) = \frac{1}{x^2}\int_0^x (4t^2 - 2f'(t))\,dt$, find $9f'(4)$

(a) 16 (b) 4 (c) 8 (d) 32

73. Evaluate $\lim\limits_{n\to\infty}\left(\frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \frac{3^2}{n^3+3^3} + \ldots\ldots + \frac{4}{9n}\right)$.

(a) $\frac{1}{3}\ln 3$ (b) $\frac{\ln 9}{3}$ (c) $\frac{\ln 4}{3}$ (d) $\frac{\ln 6}{3}$

74. The value of $\int_0^{2\pi} \cos^{-1}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)dx$ is :

(a) π^2 (b) $\frac{\pi^2}{2}$ (c) $2\pi^2$ (d) π^3

75. Given a function 'g' continuous everywhere such that $\int_0^1 g(t)\,dt = 2$ and $g(1) = 5$.

If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)\,dt$, then the value of $f'''(1) - f''(1)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

76. If $\int_0^{\pi} \frac{x^3 \cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2}\,dx = \lambda \int_0^{\pi/2} \sin^2 x\,dx$, then the value of λ is :

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

77. $\int_0^{\sqrt{3}} \left(\frac{1}{2}\frac{d}{dx}\left(\tan^{-1}\frac{2x}{1-x^2}\right)\right)dx$ equals to :

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) None of these

78. Let $y = \{x\}^{[x]}$ then the value of $\int_0^3 y\,dx$ equals to :

(where $\{\cdot\}$ and $[\cdot]$ denote fractional part and greatest integer function respectively.)

(a) 1 (b) $\frac{11}{6}$ (c) 3 (d) $\frac{5}{6}$

79. $\int_0^1 \frac{\tan^{-1} x}{x}\,dx =$

(a) $\int_0^{\pi/4} \frac{\sin x}{x}\,dx$ (b) $\int_0^{\pi/2} \frac{x}{\sin x}\,dx$ (c) $\frac{1}{2}\int_0^{\pi/2} \frac{x}{\sin x}\,dx$ (d) $\frac{1}{2}\int_0^{\pi/4} \frac{x}{\sin x}\,dx$

80. The value of $\int_0^{4/\pi} \left(3x^2 \sin\frac{1}{x} - x\cos\frac{1}{x}\right)dx$ is :

(a) $\frac{8\sqrt{2}}{\pi^3}$ (b) $\frac{24\sqrt{2}}{\pi^3}$

(c) $\frac{32\sqrt{2}}{\pi^3}$ (d) None of these

81. The number of values of x satisfying the equation :

$$\int_{-1}^{x}\left(8t^2 + \frac{28t}{3} + 4\right)dt = \frac{\frac{3}{2}x + 1}{\log_{(x+1)}\sqrt{x+1}}, \text{ is :}$$

(a) 0 (b) 1 (c) 2 (d) 3

82. $\lim_{n\to\infty} \frac{1+2^4+3^4+\ldots\ldots+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+\ldots\ldots+n^3}{n^5}$ is :

(a) $\frac{1}{30}$ (b) zero (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

83. The value of $\lim_{x\to 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t)\, dt}{2x - \sin 2x}$ is equal to :

(a) 0 (b) -1 (c) $\frac{2}{3}$ (d) $-\frac{1}{4}$

84. Consider a parabola $y = \frac{x^2}{4}$ and the point $F(0,1)$.

Let $A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3), \ldots\ldots, A_n(x_n, y_n)$ are 'n' points on the parabola such $x_k > 0$ and $\angle OFA_k = \frac{k\pi}{2n} (k = 1, 2, 3, \ldots\ldots, n)$. Then the value of $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} FA_k$, is equal to :

(a) $\frac{2}{\pi}$ (b) $\frac{4}{\pi}$ (c) $\frac{8}{\pi}$ (d) None of these

85. The minimum value of $f(x) = \int_0^4 e^{|x-t|}\, dt$ where $x \in [0,3]$ is :

(a) $2e^2 - 1$ (b) $e^4 - 1$ (c) $2(e^2 - 1)$ (d) $e^2 - 1$

86. If $\int_0^\infty \frac{\cos x}{x} dx = \frac{\pi}{2}$, then $\int_0^\infty \frac{\cos^3 x}{x} dx$ is equals to :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) π (d) $\frac{3\pi}{2}$

87. $\int \sqrt{1+\sin x} \left(\cos\frac{x}{2} - \sin\frac{x}{2} \right) dx =$:

(a) $\frac{1+\sin x}{2} + C$ (b) $(1+\sin x)^2 + C$ (c) $\frac{1}{\sqrt{1+\sin x}} + C$ (d) $\sin x + C$

88. If $I_n = \int_0^\pi \frac{\sin(2nx)}{\sin 2x} dx$, then the value of $I_{n+\frac{1}{2}}$ is equal to $(n \in I)$:

(a) $\frac{n\pi}{2}$ (b) π (c) $\frac{\pi}{2}$ (d) 0

89. The value of function $f(x) = 1 + x + \int_1^x (ln^2 t + 2\, ln\, t)\, dt$ where $f'(x)$ vanishes is :

(a) $\frac{1}{e}$ (b) 0 (c) $\frac{2}{e}$ (d) $1 + \frac{2}{e}$

90. Let f be a differentiable function on R and satisfies $f(x) = x^2 + \int_0^x e^{-t} f(x-t)dt$; then $\int_0^1 f(x)\, dx$ is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

91. The value of the definite integral $\int_{-(\pi/2)}^{\pi/2} \frac{\cos^2 x}{1+5^x}$ equals to :

(a) $\frac{3\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

92. $\int \left(\frac{x^2 - x + 1}{x^2 + 1} \right) e^{\cot^{-1}(x)} dx = f(x) \cdot e^{\cot^{-1}(x)} + C$

where C is constant of integration. Then $f(x)$ is equal to :

(a) $-x$ (b) $\sqrt{1-x}$ (c) x (d) $\sqrt{1+x}$

93. $\lim_{n\to\infty} \frac{1}{n^3} (\sqrt{n^2+1} + 2\sqrt{n^2+2^2} + \ldots\ldots + n\sqrt{(n^2+n^2)}) =$:

(a) $\frac{3\sqrt{2}-1}{2}$ (b) $\frac{2\sqrt{2}-1}{3}$ (c) $\frac{3\sqrt{3}-1}{3}$ (d) $\frac{4\sqrt{2}-1}{2}$

94. $\int \frac{(x^3-1)}{(x^4+1)(x+1)} dx$, is :

(a) $\frac{1}{4}\ln(1+x^4) + \frac{1}{3}\ln(1+x^3) + c$ (b) $\frac{1}{4}\ln(1+x^4) - \frac{1}{3}\ln(1+x^3) + c$

(c) $\frac{1}{4}\ln(1+x^4) - \ln(1+x) + c$ (d) $\frac{1}{4}\ln(1+x^4) + \ln(1+x) + c$

95. The value of $\underset{x\to 0^+}{\text{Limit}} \frac{\int_1^{\cos x} (\cos^{-1} t)\, dt}{2x - \sin 2x}$ is equal to :

(a) 0 (b) -1 (c) $\frac{2}{3}$ (d) $\frac{-1}{4}$

96. Let $f(x) = \lim_{n\to\infty} \frac{\cos x}{1 + (\tan^{-1} x)^n}$, then $\int_0^\infty f(x)\, dx =$

(a) $\tan(\sin 1)$ (b) $\sin(\tan 1)$ (c) 0 (d) $\sin\left(\frac{\tan 1}{2}\right)$

97. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{k}{n^2 + n + 2k} \right) =$

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

98. The value of $\lim\limits_{y \to 1^+} \frac{\int_1^y |t-1|\, dt}{\tan(y-1)}$ is :

(a) 0 (b) 1 (c) 2 (d) does not exist

99. Given that $\int \frac{dx}{(1+x^2)^n} = \frac{x}{2(n-1)(1+x^2)^{n-1}} + \frac{(2n-3)}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}}$. Find the value of $\int_0^1 \frac{dx}{(1+x^2)^4}$: (you may or may not use reduction formula given)

(a) $\frac{11}{48} + \frac{5\pi}{64}$ (b) $\frac{11}{48} + \frac{5\pi}{32}$ (c) $\frac{1}{24} + \frac{5\pi}{64}$ (d) $\frac{1}{96} + \frac{5\pi}{32}$

100. Find the value of $\int_0^{\pi/4} (\sin x)^4\, dx$:

(a) $\frac{3\pi}{16}$ (b) $\frac{3\pi}{32} - \frac{1}{4}$ (c) $\frac{3\pi}{32} - \frac{3}{4}$ (d) $\frac{3\pi}{16} - \frac{7}{8}$

101. $\int \frac{\cos 9x + \cos 6x}{2\cos 5x - 1} dx = A \sin 4x + B \sin + C$, then $A + B$ is equal to :

(Where C is constant of integration)

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) 2 (d) $\frac{5}{4}$

102. $\int \frac{dx}{x^{2014} + x} = \frac{1}{p} \ln\left(\frac{x^q}{1+x^r}\right) + C$ where $p, q, r \in N$ then the value of $(p+q+r)$ equals

(Where C is constant of integration)

(a) 6039 (b) 6048 (c) 6047 (d) 6021

103. If $\int_0^1 e^{-x^2} dx = a$, then $\int_0^1 x^2 e^{-x^2} dx$ is equal to

(a) $\frac{1}{2e}(ea-1)$ (b) $\frac{1}{2e}(ea+1)$ (c) $\frac{1}{e}(ea-1)$ (d) $\frac{1}{e}(ea+1)$

104. If $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)\, dx = \frac{n^2}{2}\ \forall\, n \in I$, then $\int_{-3}^{5} f(|x|)\, dx$ is equal to :

(a) $\frac{19}{2}$ (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) $\frac{37}{2}$

105. If $\int \frac{dx}{x^4(1+x^3)^2} = a\ln\left|\frac{1+x^3}{x^3}\right| + \frac{b}{x^3} + \frac{c}{1+x^3} + d$, then

(where d is arbitrary constant)

(a) $a=\frac{1}{3}, b=\frac{1}{3}, x=\frac{1}{3}$ (b) $a=\frac{2}{3}, b=-\frac{1}{3}, c=\frac{1}{3}$

(c) $a=\frac{2}{3}, b=-\frac{1}{3}, c=-\frac{1}{3}$ (d) $a=\frac{2}{3}, b=\frac{1}{3}, c=-\frac{1}{3}$

106. $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \ldots\ldots + \frac{1}{\sqrt{n}\sqrt{4n}}$ is equal to :

(a) 2 (b) 4 (c) $2(\sqrt{2}-1)$ (d) $2\sqrt{2}-1$

107. Let $f(x) = \int_x^2 \frac{dy}{\sqrt{1+y^3}}$. The value of the integral $\int_0^2 xf(x)\,dx$ is equal to :

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

108. The value of the definite integral $\int_0^{\pi/3} \ln(1+\sqrt{3}\tan x)\,dx$ equals

(a) $\frac{\pi}{3}\ln 2$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi^2}{6}\ln 2$ (d) $\frac{\pi}{2}\ln 2$

109. If $\int_0^{100} f(x)\,dx = a$, then $\sum_{r=1}^{100}\int_0^1 (f(r-1+x)\,dx) =$

(a) $100a$ (b) a (c) 0 (d) $10a$

110. The value of $\int_0^1 \lim\limits_{n\to\infty} \sum_{k=0}^{n} \frac{x^{k+2}2^k}{k!}\,dx$ is :

(a) e^2-1 (b) 2 (c) $\frac{e^2-1}{2}$ (d) $\frac{e^2-1}{4}$

111. Evaluate : $\int x^5\sqrt{1+x^3}\,dx$.

(a) $\frac{1}{15}(1+x^3)^{5/2} - \frac{1}{9}(1+x)^3)^{3/2} + c$

(b) $\frac{2}{15}(1+x^3)^{5/2} - \frac{1}{9}(1+x^3)^{3/2} + c$

(c) $\frac{2}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

(d) $\frac{1}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

112. If $f(x)=\int_0^x \frac{\sin t}{t}\,dt$, which of the following is true ?

(a) $f(0)>f(1\cdot 1)$

(b) $f(0)<f(1\cdot 1)>f(2\cdot 1)$

(c) $f(0)<f(1\cdot 1)<f(2\cdot 1)>f(3\cdot 1)$

(d) $f(0)<f(1\cdot 1)<f(2\cdot 1)<f(3\cdot 1)>f(4\cdot 1)$

113. Evaluate : $\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}dx.$

(a) $\ln|x^2+3|+3\tan^{-1}x+c$

(b) $\frac{1}{2}\ln|x^2+3|+\tan^{-1}x+c$

(c) $\frac{1}{2}\ln|x^2+3|+3\tan^{-1}x+c$

(d) $\ln|x^2+3|-\tan^{-1}x+c$

114. $\int \frac{\sqrt{\sec^5 x}}{\sqrt{\sin^3 x}}dx$ equals to :

(a) $(\tan x)^{3/2}-\sqrt{\tan x}+C$

(b) $2\left(\frac{1}{3}(\tan x)^{3/2}-\frac{1}{\sqrt{\tan x}}\right)+C$

(c) $\frac{1}{3}(\tan x)^{3/2}-\sqrt{\tan x}+C$

(d) $\sqrt{\sin x}+\sqrt{\cos x}+C$

115. $\lim\limits_{x\to 0}\int_0^x \frac{e^{\sin(tx)}}{x}dt$ equals to :

(a) 1 (b) 2 (c) e (d) Does not exist

116. If $A=\int_0^\pi \frac{\sin x}{x^2}dx$, then $\int_0^{\pi/2}\frac{\cos 2x}{x}dx$ is equal to :

(a) $1-A$ (b) $\frac{3}{2}-A$ (c) $A-1$ (d) $1+A$

Answers

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (d)
11. (b)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (a)
21. (a)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (c)	**26.** (d)	**27.** (a)	**28.** (d)	**29.** (d)	**30.** (c)
31. (d)	**32.** (b)	**33.** (c)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (d)	**40.** (d)
41. (b)	**42.** (a)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (d)	**48.** (d)	**49.** (d)	**50.** (c)
51. (c)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (b)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (b)	**60.** (a)
61. (a)	**62.** (a)	**63.** (b)	**64.** (c)	**65.** (d)	**66.** (d)	**67.** (a)	**68.** (d)	**69.** (d)	**70.** (b)
71. (d)	**72.** (b)	**73.** (a)	**74.** (d)	**75.** (b)	**76.** (a)	**77.** (b)	**78.** (c)	**79.** (c)	**80.** (c)
81. (b)	**82.** (d)	**83.** (d)	**84.** (b)	**85.** (c)	**86.** (a)	**87.** (d)	**88.** (d)	**89.** (d)	**90.** (d)
91. (d)	**92.** (c)	**93.** (b)	**94.** (c)	**95.** (d)	**96.** (b)	**97.** (c)	**98.** (a)	**99.** (a)	**100.** (b)
101. (d)	**102.** (a)	**103.** (a)	**104.** (b)	**105.** (c)	**106.** (a)	**107.** (d)	**108.** (a)	**109.** (b)	**110.** (d)
111. (c)	**112.** (d)	**113.** (c)	**114.** (b)	**115.** (a)	**116.** (c)				

Exercise-2 : One or More than One Answer is/are Correct

1. $\int \frac{dx}{(1+\sqrt{x})^8} = -\frac{1}{3(1+\sqrt{x})^{k_1}} + \frac{2}{7(1+\sqrt{x})^{k_2}} + C$, then :

(a) $k_1 = 5$ (b) $k_1 = 6$ (c) $k_2 = 7$ (d) $k_2 = 8$

2. If $\int_{-\alpha}^{\alpha} \left(e^x + \cos x \ln\left(x + \sqrt{1+x^2}\right)\right) dx > \frac{3}{2}$, then possible value of α can be :

(a) 1 (b) 2 (c) 3 (d) 4

3. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}}\, dx = A \sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C$, where C is any arbitrary constant, then :

(a) $A = \frac{2}{3}$ (b) $B = a^{3/2}$ (c) $A = \frac{1}{3}$ (d) $B = a^{1/2}$

4. Let $\int x \sin x \cdot \sec^3 x\, dx = \frac{1}{2}(x \cdot f(x) - g(x)) + k$, then :

(a) $f(x) \notin (-1, 1)$

(b) $g(x) = \sin x$ has 6 solution for $x \in [-\pi, 2\pi]$

(c) $g'(x) = f(x), \forall\, x \in R$

(d) $f(x) = g(x)$ has no solution

5. If $\int (\sin 3\theta + \sin\theta) \cos\theta\, e^{\sin\theta} d\theta = (A \sin^3\theta + B\cos^2\theta + C\sin\theta + D\cos\theta + E) e^{\sin\theta} + F$, then :

(a) $A = -4$ (b) $B = -12$ (c) $C = -20$ (d) None of these

6. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}}\, dx = A \sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C$, where C is any arbitrary constant, then :

(a) $A = \frac{2}{3}$ (b) $B = a^{3/2}$ (c) $A = \frac{1}{3}$ (d) $B = a^{1/2}$

7. If $f(\theta) = \lim_{n\to\infty} \sum_{r=0}^{n\theta} \frac{2r}{n\sqrt{(3\theta n - 2r)(n\theta + 2r)}}$ then :

(a) $f(1) = \frac{\pi}{6}$

(b) $f(\theta) = \frac{\theta}{2} \int_0^{\theta} \frac{dx}{\sqrt{\theta^2 - \left(x - \frac{\theta}{2}\right)^2}}$

(c) $f(\theta)$ is a constant function

(d) $y = f(\theta)$ is invertible

8. If $f(x+y) = f(x)f(y)$ for all x, y and $f(0) \neq 0$, and $F(x) = \frac{f(x)}{1 + (f(x))^2}$ then :

(a) $\int_{-2010}^{2011} F(x)\,dx = \int_0^{2011} F(x)\,dx$

(b) $\int_{-2010}^{2011} F(x)\,dx - \int_0^{2010} F(x)\,dx = \int_0^{2011} F(x)\,dx$

(c) $\int_{-2010}^{2011} F(x)\,dx = 0$

(d) $\int_{-2010}^{2010} (2F(-x) - F(x))\,dx = 2\int_0^{2010} F(x)\,dx$

9. Let $J = \int_{-1}^{2}\left(\cot^{-1}\frac{1}{x} + \cot^{-1}x\right)dx, K = \int_{-2\pi}^{7\pi}\frac{\sin x}{|\sin x|}dx$. Then which of the following alternative(s) is/are correct ?

(a) $2J + 3K = 8\pi$ (b) $4J^2 + K^2 = 26\pi^2$ (c) $2J - K = 3\pi$ (d) $\frac{J}{K} = \frac{2}{5}$

10. Which of the following function(s) is/are even ?

(a) $f(x) = \int_0^x \ln\left(t + \sqrt{1+t^2}\right)dt$ (b) $g(x) = \int_0^x \frac{(2^t+1)t}{2^t - 1}dt$

(c) $h(x) = \int_0^x \left(\sqrt{1+t+t^2} - \sqrt{1-t+t^2}\right)dt$ (d) $l(x) = \int_0^x \ln\left(\frac{1-t}{1+t}\right)dt$

11. Let $l_1 = \lim_{x\to\infty}\sqrt{\frac{x-\cos^2 x}{x+\sin x}}$ and $l_2 = \lim_{h\to 0^+}\int_{-1}^{1}\frac{h\,dx}{h^2+x^2}$. Then :

(a) Both l_1 and l_2 are less than 22/7

(b) One of the two limits is rational and other irrational

(c) $l_2 > l_1$

(d) l_2 is greater than 3 times of l_1

12. For $a > 0$, if $I = \int\sqrt{\frac{x}{a^3 - x^3}}\,dx = A\sin^{-1}\left(\frac{x^{3/2}}{B}\right) + C$, where C is any arbitrary constant, then :

(a) $A = \frac{2}{3}$ (b) $B = a^{3/2}$ (c) $A = \frac{1}{3}$ (d) $B = a^{1/2}$

13. If $\int\frac{dx}{1-\sin^4 x} = a\tan x + b\tan^{-1}(c\tan x) + D$, then :

(a) $a = \frac{1}{2}$ (b) $b = \sqrt{2}$ (c) $c = \sqrt{2}$ (d) $b = \frac{1}{2\sqrt{2}}$

14. The value of definite integral :

$$\int_{-2014}^{2014}\frac{dx}{1+\sin^{2015}(x) + \sqrt{1+\sin^{4030}(x)}} \text{ equals :}$$

(a) 0 (b) 2014 (c) $(2014)^2$ (d) 4028

15. Let $L = \lim_{n\to\infty}\int_a^{\infty}\frac{n\,dx}{1+n^2x^2}$ where $a \in R$ then L can be :

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{3}$

16. Let $I = \int_0^1 \sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}\,dx$ and $J = \int_0^1 \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ then correct statement(s) is/are :

(a) $I + J = 2$ (b) $I - J = \pi$ (c) $I = \frac{2+\pi}{2}$ (d) $J = \frac{4-\pi}{2}$

Answers

1.	(b, c)	**2.**	(a, b, c, d)	**3.**	(a, b)	**4.**	(a, c, d)	**5.**	(a, b, c)	**6.**	(a, b)
7.	(a, b, d)	**8.**	(b, d)	**9.**	(a, b)	**10.**	(a, b, c, d)	**11.**	(a, b, c, d)	**12.**	(a, b)
13.	(a, c)	**14.**	(b)	**15.**	(a, b, c)	**16.**	(b, c)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $f(x)=\int x^2\cos^2 x(2x+6\tan x-2x\tan^2 x)\,dx$ and $f(x)$ passes through the point $(\pi,0)$

1. If $f:R-(2n+1)\frac{\pi}{2}\longrightarrow R$ then $f(x)$ be a :

(a) even function (b) odd function

(c) neither even nor odd (d) even as well as odd both

2. The number of solution(s) of the equation $f(x)=x^3$ in $[0,2\pi]$ be :

(a) 0 (b) 3 (c) 2 (d) None of these

Paragraph for Question Nos. 3 to 4

Let $f(x)$ be a twice differentiable function defined on $(-\infty,\infty)$ such that $f(x)=f(2-x)$ and $f'\left(\frac{1}{2}\right)=f'\left(\frac{1}{4}\right)=0$. Then

3. The minimum number of values where $f''(x)$ vanishes on [0, 2] is :

(a) 2 (b) 3 (c) 4 (d) 5

4. $\int_{-1}^{1} f'(1+x)\,x^2e^{x^2}\,dx$ is equal to :

(a) 1 (b) π (c) 2 (d) 0

5. $\int_0^1 f(1-t)\,e^{-\cos\pi t}\,dt-\int_1^2 f(2-t)\,e^{\cos\pi t}\,dt$ is equal to :

(a) $\int_0^2 f'(t)\,e^{\cos\pi t}\,dt$ (b) 1 (c) 2 (d) π

Paragraph for Question Nos. 6 to 8

Consider the function $f(x)$ and $g(x)$, both defined from $R\to R$

$$f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)\,dt \text{ and } g(x)=x-\int_0^1 f(t)\,dt, \text{ then}$$

6. Minimum value of $f(x)$ is :

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) Does not exist

7. The number of points of intersection of $f(x)$ and $g(x)$ is/are :

(a) 0 (b) 1 (c) 2 (d) 3

8. The area bounded by $g(x)$ with co-ordinate axes is (in square units) :

(a) $\frac{9}{4}$ (b) $\frac{9}{2}$ (c) $\frac{9}{8}$ (d) None of these

Paragraph for Question Nos. 9 to 11

Let $f(x)$ be function defined on $[0, 1]$ such that $f(1) = 0$ and for any $a \in (0, 1]$,

$\int_0^a f(x)\,dx - \int_a^1 f(x)\,dx = 2f(a) + 3a + b$ where b is constant.

9. $b =$

(a) $\frac{3}{2e} - 3$ (b) $\frac{3}{2e} - \frac{3}{2}$ (c) $\frac{3}{2e} + 3$ (d) $\frac{3}{2e} + \frac{3}{2}$

10. The length of the subtangent of the curve $y = f(x)$ at $x = 1/2$ is :

(a) $\sqrt{e} - 1$ (b) $\frac{\sqrt{e} - 1}{2}$ (c) $\sqrt{e} + 1$ (d) $\frac{\sqrt{e} + 1}{2}$

11. $\int_0^1 f(x)\,dx =$

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{3}{2e}$ (d) $\frac{2}{e}$

Paragraph for Question Nos. 12 to 13

Let $f_0(x) = \ln x$ and for $n \geq 0$ and $x > 0$

Let $f_{n+1}(x) = \int_0^x f_n(t)\,dt$ then :

12. $f_3(x)$ equals :

(a) $\frac{x^3}{3}\left(\ln x - \frac{5}{6}\right)$ (b) $\frac{x^3}{3}\left(\ln x - \frac{11}{6}\right)$ (c) $\frac{x^3}{\lfloor 3}\left(\ln x - \frac{11}{6}\right)$ (d) $\frac{x^3}{\lfloor 3}\left(\ln x - \frac{5}{6}\right)$

13. Value of $\lim_{n\to\infty} \frac{(\lfloor n\,)\, f_n(1)}{\ln(n)}$:

(a) 0 (b) 1 (c) -1 (d) $-e$

Paragraph for Question Nos. 14 to 15

Let $f:R\to\left[\frac{3}{4},\infty\right)$ be a surjective quadratic function with line of symmetry $2x-1=0$ and $f(1)=1$

14. If $g(x)=\dfrac{f(x)+f(-x)}{2}$ then $\displaystyle\int\frac{dx}{\sqrt{g(e^x)-2}}$ is equal to :

(a) $\sec^{-1}(e^{-x})+C$ (b) $\sec^{-1}(e^{x})+C$ (c) $\sin^{-1}(e^{-x})+C$ (d) $\sin^{-1}(e^{x})+C$

(Where C is constant of integration)

15. $\displaystyle\int\frac{e^x}{f(e^x)}dx$

(a) $\cot^{-1}\left(\dfrac{2e^x-1}{\sqrt{3}}\right)+C$ (b) $\dfrac{2}{\sqrt{3}}\cot^{-1}\left(\dfrac{2e^x+1}{\sqrt{3}}\right)+C$

(c) $\tan^{-1}\left(\dfrac{2e^x+1}{\sqrt{3}}\right)+C$ (d) $\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2e^x-1}{\sqrt{3}}\right)+C$

Paragraph for Question Nos. 16 to 17

Let $g(x)=x^Ce^{Cx}$ and $f(x)=\displaystyle\int_0^x te^{2t}(1+3t^2)^{1/2}dt$. If $L=\displaystyle\lim_{x\to\infty}\frac{f'(x)}{g'(x)}$ is non-zero finite number then :

16. The value of C is :

(a) 7 (b) $\dfrac{3}{2}$ (c) 2 (d) 3

17. The value of L is :

(a) $\dfrac{2}{7}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{\sqrt{3}}{4}$ (d) $\dfrac{\sqrt{3}}{2}$

Answers

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(b)	**7.**	(a)	**8.**	(c)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(c)	**13.**	(c)	**14.**	(b)	**15.**	(d)	**16.**	(c)	**17.**	(d)						

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	$\lim\limits_{n\to\infty} 4\left[\frac{e^{\frac{1}{n}}}{n^2}+\frac{2}{n^2}e^{\frac{2}{n}}+\frac{3}{n^2}e^{\frac{3}{n}}+\ldots\frac{1}{n}e\right]=$	**(P)**	0
(B)	$\int\limits_0^1 \ln\left(\frac{1}{x}-1\right)dx=$	**(Q)**	1
(C)	$\int\limits_0^{10\pi}\left(\lim\limits_{x\to y}\left(\frac{\sin x-\sin y}{x-y}\right)\right)dy=$	**(R)**	2
(D)	$\int\limits_0^{\infty}\frac{\ln\left(x+\frac{1}{x}\right)dx}{(1+x^2)}=\frac{\pi}{2}\ln a$, then $a=$	**(S)**	4
		(T)	5

2. Match the following $\int f(x)\,dx$ is equal to, if

	Column-I		Column-II
(A)	$f(x)=\frac{1}{(x^2+1)\sqrt{x^2+2}}$	**(P)**	$\frac{x^5}{5(1-x^4)^{5/2}}+C$
(B)	$f(x)=\frac{1}{(x+2)\sqrt{x^2+6x+7}}$	**(Q)**	$\sin^{-1}\left(\frac{x+1}{(x+2)\sqrt{2}}\right)+C$
(C)	$f(x)=\frac{x^4+x^8}{(1-x^4)^{7/2}}$	**(R)**	$(\sqrt{x}-2)\sqrt{1-x}+\cos^{-1}\sqrt{x}+C$
(D)	$f(x)=\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$	**(S)**	$-\tan^{-1}\sqrt{1+\frac{2}{x^2}}+C$
		(T)	$\frac{x^6}{6(1-x^4)^{5/2}}+C$

3.

	Column-I		Column-II
(A)	$\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx =$	(P)	$\frac{\pi}{6}$
(B)	$\int_0^{\frac{41\pi}{4}} \lvert\cos x\rvert\, dx =$	(Q)	$20+\frac{1}{\sqrt{2}}$
(C)	$\int_{-1/2}^{1/2} \left([x]+\ln\left(\frac{1+x}{1-x}\right)\right)dx =$ where [·] greatest integer function	(R)	$\ln 4 - \ln 3$
(D)	$\int_0^{\pi/2} \frac{2\sqrt{\cos\theta}}{3(\sqrt{\sin\theta}+\sqrt{\cos\theta})} d\theta =$	(S)	$-\frac{1}{2}$

4.

	Column-I		Column-II
(A)	If quardratic equation $3x^2+ax+1=0$ and $2x^2+bx+1=0$ have a common root then value of $5ab-2a^2-3b^2 =$	(P)	6
(B)	Number of solution of $x^4-2x^2\sin^2\frac{\pi x}{2}+1=0$ is/are	(Q)	1
(C)	Number of points of discontinuity $y=\frac{1}{u^2+u-2}$ where $u=\frac{1}{x-1}$ is/are	(R)	2
(D)	$\int \frac{dx}{\sqrt[3]{x^{5/2}(1+x)^{7/2}}} = A\left(\frac{x+1}{x}\right)^{-1/A}+C$ (Where C is integration constant), then $A =$	(S)	3

5. :

	Column-I		Column-II
(A)	$\int_0^{1.5} [x^2] dx$	**(P)**	$-\pi$
(B)	$\int_0^4 \{\sqrt{x}\}\, dx$ where $\{x\}$ denotes the fractional part of x	**(Q)**	$4(\sqrt{2}-1)$
(C)	$\int_0^{2\pi} [\sin x + \cos x]\, dx$	**(R)**	$\frac{7}{3}$
(D)	$\int_0^{\pi} \lvert\, \lvert \sin x \rvert - \lvert \cos x \rvert\, \rvert\, dx$	**(S)**	$2-\sqrt{2}$

Answers

1. A → S; B → P; C → P; D → S

2. A → S; B → Q; C → P; D → R

3. A → R; B → Q; C → S; D → P

4. A → Q; B → R; C → S; D → P

5. A → S; B → R; C → P; D → Q

Exercise-5 : Subjective Type Problems

1. $\int \frac{x+(\arccos 3x)^2}{\sqrt{1-9x^2}}dx = \frac{1}{k_1}\left(\sqrt{1-9x^2}+(\cos^{-1}3x)^{k_2}\right)+C$, then $k_1^2+k_2^2=$

(where C is an arbitrary constant.)

2. If $\int_0^{\infty}\frac{x^3}{(a^2+x^2)^5}dx = \frac{1}{ka^6}$, then find the value of $\frac{k}{8}$.

3. Let $f(x) = x\cos x$; $x \in \left[\frac{3\pi}{2}, 2\pi\right]$ and $g(x)$ be its inverse. If $\int_0^{2\pi} g(x)\,dx = \alpha\pi^2+\beta\pi+\gamma$, where α, β and $\gamma \in R$, then find the value of $2(\alpha+\beta+\gamma)$.

4. If $\int (x^6+x^4+x^2)\sqrt{2x^4+3x^2+6}\;dx = \frac{(\alpha x^6+\beta x^4+\gamma x^2)^{3/2}}{18}+C$ where C is constant, then find the value of $(\beta+\gamma-\alpha)$.

5. If the value of the definite integral $\int_{-1}^{1}\cot^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)\cdot\left(\cot^{-1}\frac{x}{\sqrt{1-(x^2)^{|x|}}}\right)dx = \frac{\pi^2(\sqrt{a}-\sqrt{b})}{\sqrt{c}}$

where $a, b, c, \in N$ in their lowest from, then find the value of $(a+b+c)$.

6. The value of $\int \frac{\tan x}{\tan^2 x+\tan x+1}dx = x - \frac{2}{\sqrt{A}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{A}}\right)+C$

Then the value of A is :

7. Let $\int_0^1 \frac{4x^3\left(1+(x^4)^{2010}\right)}{(1+x^4)^{2012}}dx = \frac{\lambda}{\mu}$

where λ and μ are relatively prime positive integers. Find unit digit of μ.

8. Let $\int_1^{\sqrt{3}}\left(x^{2x^2+1}+\ln\left(x^{2x^{2x^2+1}}\right)\right)dx = N$. Find the value of $(N-6)$.

9. If $\int \frac{dx}{\cos^3 x-\sin^3 x} = A\tan^{-1}(f(x))+B\ln\left|\frac{\sqrt{2}+f(x)}{\sqrt{2}-f(x)}\right|+C$ where $f(x) = \sin x+\cos x$ find the value of $(12A+9\sqrt{2}B)-3$.

10. Find the value of $|a|$ for which the area of triangle included between the coordinate axes and any tangent to the curve $x^a y = \lambda^a$ is constant (where λ is constant.)

11. Let $I = \int_0^{\pi} x^6(\pi-x)^8\,dx$, then $\frac{\pi^{15}}{({}^{15}C_9)I} =$

12. If maximum value of $\int_0^1 (f(x))^3\, dx$ under the condition $-1 \le f(x) \le 1$; $\int_0^1 f(x)\, dx = 0$ is $\frac{p}{q}$ (where p and q are relatively prime positive integers.). Find $p+q$.

13. Let a differentiable function $f(x)$ satisfies $f(x)\cdot f'(-x) = f(-x)\cdot f'(x)$ and $f(0)=1$. Find the value of $\int_{-2}^{2} \frac{dx}{1+f(x)}$.

14. If $\{x\}$ denotes the fractional part of x, then $I = \int_0^{100} \{\sqrt{x}\}\, dx$, then the value of $\frac{9I}{155}$ is :

15. Let $I_n = \int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx$ where $n \in W$. If $I_1^2 + I_2^2 + I_3^2 + \ldots\ldots + I_{20}^2 = m\pi^2$, then find the largest prime factor of m.

16. If M be the maximum value of $72\int_0^y \sqrt{x^4 + (y-y^2)^2}\, dx$ for $y \in [0,1]$, then find $\frac{M}{6}$.

17. Find the number of points where $f(\theta) = \int_{-1}^{1} \frac{\sin\theta\, dx}{1-2x\cos\theta + x^2}$ is discontinuous where $\theta \in [0, 2\pi]$.

18. Find the value of $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\ldots+\frac{1}{\sqrt{n}}\right)$.

19. The maximum value of $\int_{-\pi/2}^{3\pi/2} \sin x\cdot f(x)\, dx$, subject to the condition $|f(x)| \le 5$ is M, then $\frac{M}{10}$ is equal to :

20. Given a function g, continuous everywhere such that $g(1)=5$ and $\int_0^1 g(t)\, dt = 2$. If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)\, dt$, then find the value of $f'''(1) + f''(1)$.

21. If $f(n) = \frac{1}{\pi}\int_0^{\pi/2} \frac{\sin^2(n\theta)\, d\theta}{\sin^2\theta}$, $n \in N$, then evaluate $\frac{f(15)+f(3)}{f(12)-f(10)}$.

22. Let $f(2-x) = f(2+x)$ and $f(4-x) = f(4+x)$. Function $f(x)$ satisfies $\int_0^2 f(x)\, dx = 5$. If $\int_0^{50} f(x)\, dx = I$. Find $[\sqrt{I} - 3]$. (where $[\cdot]$ denotes greatest integer function.)

23. Let $I_n = \int_{-1}^{1} |x|\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots + \frac{x^{2n}}{2n}\right)dx$. If $\lim_{n\to\infty} I_n$ can be expressed as rational $\frac{p}{q}$ in its lowest form, then find the value of $\frac{pq(p+q)}{10}$.

24. Let $\lim_{n\to\infty} n^{-\frac{1}{2}\left(1+\frac{1}{n}\right)} \cdot (1^1 \cdot 2^2 \cdot 3^3 \cdot \ldots\ldots n^n)^{\frac{1}{n^2}} = e^{\frac{-p}{q}}$

where p and q are relative prime positive integers. Find the value of $|p+q|$.

25. If $\int_a^b |\sin x|\, dx = 8$ and $\int_0^{a+b} |\cos x|\, dx = 9$ then the value of $\frac{1}{\sqrt{2}\,x}\left|\int_a^b x\sin x\, dx\right|$ is :

26. If $f(x)$, $g(x)$, $h(x)$ and $\phi(x)$ are polynomial in x,

$$\left(\int_1^x f(x)\, h(x)\, dx\right)\left(\int_1^x g(x)\, \phi(x)\, dx\right) - \left(\int_1^x f(x)\, \phi(x)\, dx\right)\left(\int_1^x g(x)\, h(x)\, dx\right)$$

is divisible by $(x-1)^{\lambda}$. Find maximum value of λ.

27. If $\int_0^2 (3x^2 - 3x + 1)\cos(x^3 - 3x^2 + 4x - 2)dx = a\sin(b)$, where a and b are positive integers. Find the value of $(a+b)$.

28. let $f(x) = \int_0^x e^{x-y} f'(y)\, dy - (x^2 - x + 1)e^x$

Find the number of roots of the equation $f(x) = 0$.

29. For a positive integer n, let $I_n = \int_{-\pi}^{\pi} \left(\frac{\pi}{2} - |x|\right)\cos nx\, dx$

Find the value of $[I_1 + I_2 + I_3 + I_4]$ where $[\cdot]$ denotes greatest integer function.

Answers

1.	90	**2.**	3	**3.**	3	**4.**	7	**5.**	7	**6.**	3	**7.**	1
8.	7	**9.**	8	**10.**	1	**11.**	9	**12.**	5	**13.**	2	**14.**	3
15.	5	**16.**	4	**17.**	3	**18.**	2	**19.**	2	**20.**	7	**21.**	9
22.	8	**23.**	3	**24.**	5	**25.**	2	**26.**	4	**27.**	2	**28.**	1
29.	4												

Chapter 6 – Area Under Curves

6

AREA UNDER CURVES

Exercise-1 : Single Choice Problems

1. The area enclosed by the curve
$[x + 3y] = [x - 2]$ where $x \in [3, 4)$ is :
(where [·] denotes greatest integer function.)
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

2. The area of region enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is :
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{16}{3}$

3. Area enclosed by the figure described by the equation $x^4 + 1 = 2x^2 + y^2$, is :
(a) 2 (b) $\frac{16}{3}$ (c) $\frac{8}{3}$ (d) $\frac{4}{3}$

4. The area defined by $|y| \le e^{-|x|} - \frac{1}{2}$ in cartesian co-ordinate system, is :
(a) $(4 - 2\ln 2)$ (b) $(4 - \ln 2)$ (c) $(2 - \ln 2)$ (d) $(2 - 2\ln 2)$

5. For each positive integer $n > 1$; A_n represents the area of the region restricted to the following two inequalities : $\frac{x^2}{n^2} + y^2 \le 1$ and $x^2 + \frac{y^2}{n^2} \le 1$. Find $\lim\limits_{n\to\infty} A_n$.
(a) 4 (b) 1 (c) 2 (d) 3

6. The ratio in which the area bounded by curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$ is :
(a) 7 : 15 (b) 15 : 49 (c) 1 : 3 (d) 17 : 49

7. The value of positive real parameter 'a' such that area of region bounded by parabolas $y = x - ax^2$, $ay = x^2$ attains its maximum value is equal to :
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) 1

8. For $0 < r < 1$, let n_r denotes the line that is normal to the curve $y = x^r$ at the point $(1, 1)$. Let S_r denotes the region in the first quadrant bounded by the curve $y = x^r$; the x-axis and the line n_r. Then the value of r that minimizes the area of S_r is :

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}-1$ (c) $\frac{\sqrt{2}-1}{2}$ (d) $\sqrt{2}-\frac{1}{2}$

9. The area bounded by $|x| = 1 - y^2$ and $|x| + |y| = 1$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

10. Point A lies on curve $y = e^{-x^2}$ and has the coordinate (x, e^{-x^2}) where $x > 0$. Point B has coordinates $(x, 0)$. If 'O' is the origin, then the maximum area of ΔAOB is :

(a) $\frac{1}{\sqrt{8e}}$ (b) $\frac{1}{\sqrt{4e}}$ (c) $\frac{1}{\sqrt{2e}}$ (d) $\frac{1}{\sqrt{e}}$

11. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq. unit, then the value of a is :

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$

12. Let $f(x) = x^3 - 3x^2 + 3x + 1$ and g be the inverse of it ; then area bounded by the curve $y = g(x)$ with x-axis between $x = 1$ to $x = 2$ is (in square units) :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 1

13. Area bounded by $x^2y^2 + y^4 - x^2 - 5y^2 + 4 = 0$ is equal to :

(a) $\frac{4\pi}{3} + \sqrt{2}$ (b) $\frac{4\pi}{3} - \sqrt{2}$ (c) $\frac{4\pi}{3} + 2\sqrt{3}$ (d) none of these

14. Let $f : R^+ \to R^+$ is an invertible function such that $f'(x) > 0$ and $f''(x) > 0 \;\forall\; x \in [1, 5]$. If $f(1) = 1$ and $f(5) = 5$ and area bounded by $y = f(x)$, x-axis, $x = 1$ and $x = 5$ is 8 sq. units. Then the area bounded by $y = f^{-1}(x)$, x-axis, $x = 1$ and $x = 5$ is :

(a) 12 (b) 16 (c) 18 (d) 20

15. A circle centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of the upper part equals to :

(a) $\frac{\pi^2}{2}$ (b) $\frac{\pi^3}{4}$ (c) $\frac{\pi^3}{2}$ (d) $\frac{\pi^3}{8}$

16. The area of the loop formed by $y^2 = x(1 - x^3)dx$ is :

(a) $\int_0^1 \sqrt{x - x^4}\, dx$ (b) $2\int_0^1 \sqrt{x - x^4}\, dx$

(c) $\int_{-1}^1 \sqrt{x - x^4}\, dx$ (d) $4\int_0^{1/2} \sqrt{x - x^4}\, dx$

17. If $f(x) = \min\left[x^2, \sin\frac{x}{2}, (x-2\pi)^2\right]$, the area bounded by the curve $y = f(x)$, x-axis, $x = 0$ and $x = 2\pi$ is given by

(**Note :** x_1 is the point of intersection of the curves x^2 and $\sin\frac{x}{2}$; x_2 is the point of intersection of the curves $\sin\frac{x}{2}$ and $(x-2\pi)^2$)

(a) $\int_0^{x_1}\left(\sin\frac{x}{2}\right)dx + \int_{x_1}^{\pi} x^2 dx + \int_{\pi}^{x_2}(x-2\pi)^2 dx + \int_{x_2}^{2\pi}\left(\sin\frac{x}{2}\right)dx$

(b) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\left(\sin\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(0, \frac{\pi}{3}\right)$ and $x_2 \in \left(\frac{5\pi}{3}, 2\pi\right)$

(c) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\sin\left(\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ and $x_2 \in \left(\frac{3\pi}{2}, 2\pi\right)$

(d) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\sin\left(\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $x_2 \in (\pi, 2\pi)$

18. The area enclosed between the curves $|x|+|y| \ge 2$ and $y^2 = 4\left(1-\frac{x^2}{9}\right)$ is :

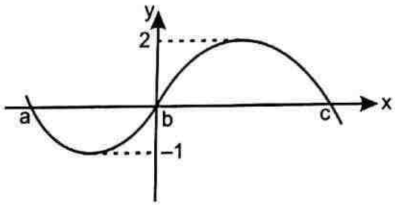
(a) $(6\pi - 4)$ *sq.* units (b) $(6\pi - 8)$ *sq.* units (c) $(3\pi - 4)$ *sq.* units (d) $(3\pi - 2)$ *sq.* units

Answers

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(b)	**7.**	(d)	**8.**	(b)	**9.**	(c)	**10.**	(a)
11.	(d)	**12.**	(b)	**13.**	(c)	**14.**	(b)	**15.**	(c)	**16.**	(b)	**17.**	(b)	**18.**	(b)				

Exercise-2 : One or More than One Answer is/are Correct

1. Let $f(x)$ be a polynomial function of degree 3 where $a < b < c$ and $f(a) = f(b) = f(c)$. If the graph of $f(x)$ is as shown, which of the following statements are **INCORRECT** ? (Where $c > |a|$)



(a) $\int_a^c f(x)\,dx = \int_b^c f(x)\,dx + \int_a^b f(x)\,dx$

(b) $\int_a^c f(x)\,dx < 0$

(c) $\int_a^b f(x)\,dx < \int_c^b f(x)\,dx$

(d) $\dfrac{1}{b-a}\int_a^b f(x)\,dx > \dfrac{1}{c-b}\int_b^c f(x)\,dx$

2. $T_n = \sum\limits_{r=2n}^{3n-1} \dfrac{r}{r^2+n^2}$, $S_n = \sum\limits_{r=2n+1}^{3n} \dfrac{r}{r^2+n^2}$, then $\forall\, n \in \{1, 2, 3, \ldots\}$:

(a) $T_n > \frac{1}{2}\ln 2$ (b) $S_n < \frac{1}{2}\ln 2$ (c) $T_n < \frac{1}{2}\ln 2$ (d) $S_n > \frac{1}{2}\ln 2$

3. If a curve $y = a\sqrt{x} + bx$ passes through point (1, 2) and the area bounded by curve, line $x = 4$ and x-axis is 8, then :

(a) $a = 3$ (b) $b = 3$ (c) $a = -1$ (d) $b = -1$

4. Area enclosed by the curves $y = x^2 + 1$ and a normal drawn to it with gradient -1; is equal to :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

Answers

1.	(b, c, d)	**2.**	(a, b)	**3.**	(a, d)	**4.**	(d)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Let $f : A \to B$ $f(x) = \dfrac{x+a}{bx^2+cx+2}$, where A represent domain set and B represent range set of function $f(x)$, $a, b, c \in R$, $f(-1) = 0$ and $y = 1$ is an asymptote of $y = f(x)$ and $y = g(x)$ is the inverse of $f(x)$.

1. $g(0)$ is equal to :

(a) -1 (b) -3 (c) $-\dfrac{5}{2}$ (d) $-\dfrac{3}{2}$

2. Area bounded between the curves $y = f(x)$ and $y = g(x)$ is :

(a) $2\sqrt{5} + \ln\left(\dfrac{3-\sqrt{5}}{3+\sqrt{5}}\right)$ (b) $3\sqrt{5} + 2\ln\left(\dfrac{3+\sqrt{5}}{3-\sqrt{5}}\right)$

(c) $3\sqrt{5} + 4\ln\left(\dfrac{3-\sqrt{5}}{3+\sqrt{5}}\right)$ (d) $3\sqrt{5} + 2\ln\left(\dfrac{3-\sqrt{5}}{3+\sqrt{5}}\right)$

3. Area of region enclosed by asymptotes of curves $y = f(x)$ and $y = g(x)$ is :

(a) 4 (b) 9 (c) 12 (d) 25

Paragraph for Question Nos. 4 to 6

For $j = 0, 1, 2, \dots n$ let S_j be the area of region bounded by the x-axis and the curve $ye^x = \sin x$ for $j\pi \le x \le (j+1)\pi$

4. The value of S_0 is :

(a) $\dfrac{1}{2}(1+e^{\pi})$ (b) $\dfrac{1}{2}(1+e^{-\pi})$ (c) $\dfrac{1}{2}(1-e^{-\pi})$ (d) $\dfrac{1}{2}(e^{\pi}-1)$

5. The ratio $\dfrac{S_{2009}}{S_{2010}}$ equals :

(a) $e^{-\pi}$ (b) e^{π} (c) $\dfrac{1}{2}e^{\pi}$ (d) $2e^{\pi}$

6. The value of $\sum_{j=0}^{\infty} S_j$ equals to :

(a) $\dfrac{e^{\pi}(1+e^{\pi})}{2(e^{\pi}-1)}$ (b) $\dfrac{1+e^{\pi}}{2(e^{\pi}-1)}$ (c) $\dfrac{1+e^{\pi}}{e^{\pi}-1}$ (d) $\dfrac{e^{\pi}(1+e^{\pi})}{(e^{\pi}-1)}$

Answers

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(b)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	Area of region formed by points (x, y) satisfying $[x]^2 = [y]^2$ for $0 \le x \le 4$ is equal to (where [ ] denotes greatest integer function)	**(P)**	48
(B)	The area of region formed by points (x, y) satisfying $x + y \le 6$, $x^2 + y^2 \le 6y$ and $y^2 \le 8x$ is $\dfrac{k\pi - 2}{12}$, then $k =$	**(Q)**	27
(C)	The area in the first quardant bounded by the curve $y = \sin x$ and the line $\dfrac{2y-1}{\sqrt{2}-1} = \dfrac{2}{\pi}(6x - \pi)$ is $\left[\dfrac{\sqrt{3}-\sqrt{2}}{2} - \dfrac{(\sqrt{2}+1)\pi}{k}\right]$, then $k =$	**(R)**	7
(D)	If the area bounded by the graph of $y = xe^{-ax}$ $(a > 0)$ and the abscissa axis is $\dfrac{1}{9}$ then the value of 'a' is equal to	**(S)**	4
		(T)	3

Answers

1. $A \to R$; $B \to Q$; $C \to P$; $D \to T$

Exercise-5 : Subjective Type Problems

1. Let f be a differentiable function satisfying the condition $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}(y \neq 0, f(y) \neq 0)$ $\forall x, y \in R$ and $f'(1)=2$. If the smaller area enclosed by $y=f(x)$, $x^2+y^2=2$ is A, then find $[A]$, where [·] represents the greatest integer function.
2. Let $f(x)$ be a function which satisfy the equation $f(xy)=f(x)+f(y)$ for all $x>0, y>0$ such that $f'(1)=2$. Let A be the area of the region bounded by the curves $y=f(x)$, $y=|x^3-6x^2+11x-6|$ and $x=0$, then find value of $\frac{28}{17}A$.
3. If the area bounded by circle $x^2+y^2=4$, the parabola $y=x^2+x+1$ and the curve $y=\left[\sin^2\frac{x}{4}+\cos\frac{x}{4}\right]$, (where [ ] denotes the greates integer function) and x-axis is $\left(\sqrt{3}+\frac{2\pi}{3}-\frac{1}{k}\right)$, then the numerical quantity k should be :
4. Let the function $f:[-4,4]\to[-1,1]$ be defined implicitly by the equation $x+5y-y^5=0$. If the area of triangle formed by tangent and normal to $f(x)$ at $x=0$ and the line $y=5$ is A, find $\frac{A}{13}$.
5. Area of the region bounded by $[x]^2=[y]^2$, if $x\in[1,5]$, where [ ] denotes the greatest integer function, is :
6. Consider $y=x^2$ and $f(x)$ where $f(x)$, is a differentiable function satisfying $f(x+1)+f(z-1)=f(x+z)\ \forall x, z\in R$ and $f(0)=0$; $f'(0)=4$. If area bounded by curve $y=x^2$ and $y=f(x)$ is Δ, find the value of $\left(\frac{3}{16}\Delta\right)$.
7. The least integer which is greater than or equal to the area of region in $x-y$ plane satisfying $x^6-x^2+y^2\le 0$ is :
8. The set of points (x, y) in the plane statisfying $x^{2/5}+|y|=1$ form a curve enclosing a region of area $\frac{p}{q}$ square units, where p and q are relatively prime positive integers. Find $p-q$.

Answers

1.	1	2.	7	3.	6	4.	5	5.	8	6.	2	7.	2
8.	1												

Chapter 7 – Differential Equations

7

DIFFERENTIAL EQUATIONS

Exercise-1 : Single Choice Problems

1. $\frac{dy}{dx}\left(\frac{1+\cos x}{y}\right)=-\sin x$ and $f\left(\frac{\pi}{2}\right)=-1$, then $f(0)$ is :

(a) 2 (b) 1 (c) 3 (d) 4

2. The differential equation satisfied by family of curves $y=Ae^{x}+Be^{3x}+Ce^{5x}$ where A, B, C are arbitrary constants is :

(a) $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}+15y=0$ (b) $\frac{d^3y}{dx^3}+9\frac{d^2y}{dx^2}-23\frac{dy}{dx}-15y=0$

(c) $\frac{d^3y}{dx^3}+9\frac{d^2y}{dx^2}-23\frac{dy}{dx}+15y=0$ (d) $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$

3. If $y=y(x)$ and it follows the relation $e^{xy^2}+y\cos(x^2)=5$ then $y'(0)$ is equal to :

(a) 4 (b) -16 (c) -4 (d) 16

4. $(x^2+y^2)\,dy=xy\,dx$. If $y(x_0)=e$, $y(1)=1$, then the value of x_0 is equal to :

(a) $\sqrt{3}\,e$ (b) $\sqrt{e^2-\frac{1}{2}}$ (c) $\sqrt{\frac{e^2-1}{2}}$ (d) $\sqrt{e^2+\frac{1}{2}}$

5. The differential equation $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circles with :

(a) Variable radii and fixed centre at $(0,1)$

(b) Variable radii and fixed centre at $(0,-1)$

(c) Fixed radius 1 and variable centres along x-axis

(d) Fixed radius 1 and variable centres along y-axis

6. Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2y'+y=0$ is :

(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ (c) $\left(\frac{\pi}{8},\frac{5\pi}{4}\right)$ (d) $(-\pi,\pi)$

7. A function $y = f(x)$ satisfies the differential equation $(x+1)f'(x) - 2(x^2+x)f(x) = \dfrac{e^{x^2}}{(x+1)}$; $\forall\, x > -1$. If $f(0) = 5$, then $f(x)$ is :

(a) $\left(\dfrac{3x+5}{x+1}\right)\cdot e^{x^2}$ (b) $\left(\dfrac{6x+5}{x+1}\right)\cdot e^{x^2}$

(c) $\left(\dfrac{6x+5}{(x+1)^2}\right)\cdot e^{x^2}$ (d) $\left(\dfrac{5-6x}{x+1}\right)\cdot e^{x^2}$

8. The solution of the differential equation $2x^2y\dfrac{dy}{dx} = \tan(x^2y^2) - 2xy^2$ given $y(1) = \sqrt{\dfrac{\pi}{2}}$ is :

(a) $\sin(x^2y^2) - 1 = 0$ (b) $\cos\left(\dfrac{\pi}{2} + x^2y^2\right) + x = 0$

(c) $\sin(x^2y^2) = e^{x-1}$ (d) $\sin(x^2y^2) = e^{2(x-1)}$

9. The differential equation whose general solution is given by $y = C_1\cos(x + C_2) - C_3e^{-x+C_4} + C_5\sin x$, where $C_1, C_2, \ldots\ldots, C_5$ are constants is :

(a) $\dfrac{d^4y}{dx^4} - \dfrac{d^2y}{dx^2} + y = 0$ (b) $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - y = 0$

(c) $\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 0$ (d) $\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$

10. If $y = e^{(\alpha+1)x}$ be solution of differential equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$; then α is :

(a) 0 (b) 1 (c) -1 (d) 2

11. The order and degree of the differential equation $\left(\dfrac{dy}{dx}\right)^{1/3} - 4\dfrac{d^2y}{dx^2} - 7x = 0$ are α and β, then the value of $(\alpha + \beta)$ is :

(a) 3 (b) 4 (c) 2 (d) 5

12. General solution of differential equation of $f(x)\dfrac{dy}{dx} = f^2(x) + f(x)y + f'(x)y$ is :

(c being arbitary constant.)

(a) $y = f(x) + ce^x$ (b) $y = -f(x) + ce^x$

(c) $y = -f(x) + ce^x f(x)$ (d) $y = c\,f(x) + e^x$

13. The order and degree respectively of the differential equation of all tangent lines to parabola $x^2 = 2y$ is :

(a) 1, 2 (b) 2, 1

(c) 1, 1 (d) 1, 3

14. The general solution of the differential equation $\frac{dy}{dx} + x(x+y) = x(x+y)^3 - 1$ is :

(a) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^4}\right| = x^2 + C$ (b) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x^2 + C$

(c) $2\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x^2 + C$ (d) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x + C$

(where C is arbitrary constant.)

15. The general solution of $\frac{dy}{dx} = 2y\tan x + \tan^2 x$ is :

(a) $y\cos^2 x = \frac{x}{2} - \frac{\sin 2x}{4} + C$ (b) $y\sec^2 x = \frac{x}{2} - \frac{\sin 2x}{4} + C$

(c) $y\cos^2 x = \frac{x}{2} - \frac{\cos 2x}{4} + C$ (d) $y\cos^2 x = \frac{x}{2} - \frac{\sin 2x}{2} + C$

(where C is an arbitrary constant.)

16. The solution of differential equation $\frac{d^2y}{dx^2} = \frac{dy}{dx}$, $y(0) = 3$ and $y'(0) = 2$:

(a) is a periodic function (b) approaches to zero as $x \to -\infty$

(c) has an asymptote parallel to x-axis (d) has an asymptote parallel to y-axis

17. The solution of the differential equation $(x^2+1)\frac{d^2y}{dx^2} = 2x\left(\frac{dy}{dx}\right)$ under the conditions $y(0) = 1$ and $y'(0) = 3$, is :

(a) $y = x^2 + 3x + 1$ (b) $y = x^3 + 3x + 1$

(c) $y = x^4 + 3x + 1$ (d) $y = 3\tan^{-1} x + x^2 + 1$

18. The differential equation of the family of curves $cy^2 = 2x + c$ (where c is an arbitrary constant.) is :

(a) $\frac{xdy}{dx} = 1$ (b) $\left(\frac{dy}{dx}\right)^2 = \frac{2xdy}{dx} + 1$ (c) $y^2 = 2xy\frac{dy}{dx} + 1$ (d) $y^2 = \frac{2ydy}{dx} + 1$

19. The solution of the equation $\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\tan y \sin y$ is :

(a) $2y = \sin y(1 - 2cx^2)$ (b) $2x = \cot y(1 + 2cx^2)$

(c) $2x = \sin y(1 - 2cx^2)$ (d) $2x\sin y = 1 - 2cx^2$

20. Solution of the differential equation $xdy - ydx - \sqrt{x^2+y^2}\,dx = 0$ is :

(a) $y - \sqrt{x^2+y^2} = cx^2$ (b) $y + \sqrt{x^2+y^2} = cx$

(c) $x - \sqrt{x^2+y^2} = cx^2$ (d) $y + \sqrt{x^2+y^2} = cx^2$

21. Let $f(x)$ be differentiable function on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \left(\frac{t^3 f(x) - x^3 f(t)}{t^2 - x^2} \right) = \frac{1}{2} \; \forall \, x > 0$, then $f(x)$ is :

(a) $\frac{1}{4x} + \frac{3x^2}{4}$ (b) $\frac{3}{4x} + \frac{x^3}{4}$ (c) $\frac{1}{4x} + \frac{3x^3}{4}$ (d) $\frac{1}{4x^3} + \frac{3x}{4}$

22. The population $p(t)$ at time 't' of a certain mouse species satisfies the differential equation $\frac{d}{dt} p(t) = 0.5\, p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is :

(a) $\frac{1}{2} \ln 18$ (b) $\ln 18$ (c) $2 \ln 18$ (d) $\ln 9$

23. The solution of the differential equation $\sin 2y \frac{dy}{dx} + 2 \tan x \cos^2 y = 2 \sec x \cos^3 y$ is :

(where C is arbitrary constant)

(a) $\cos y \sec x = \tan x + C$ (b) $\sec y \cos x = \tan x + C$

(c) $\sec y \sec x = \tan x + C$ (d) $\tan y \sec x = \sec x + C$

24. The solution of the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$ is :

(where C is arbitrary constant)

(a) $4x + y + 1 = 2 \tan (2x + y + C)$ (b) $4x + y + 1 = 2 \tan (x + 2y + C)$

(c) $4x + y + 1 = 2 \tan (2y + C)$ (d) $4x + y + 1 = 2 \tan (2x + C)$

25. If a curve is such that line joining origin to any point $P(x, y)$ on the curve and the line parallel to y-axis through P are equally inclined to tangent to curve at P, then the differential equation of the curve is :

(a) $x\left(\frac{dy}{dx}\right)^2 - 2y \frac{dy}{dx} = x$ (b) $x\left(\frac{dy}{dx}\right)^2 + 2y \frac{dy}{dx} = x$

(c) $y\left(\frac{dy}{dx}\right)^2 - 2x \frac{dy}{dx} = x$ (d) $y\left(\frac{dy}{dx}\right)^2 - 2y \frac{dy}{dx} = x$

26. If $y = f(x)$ satisfy the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$; $f(1) = 1$; then value of $f(3)$ equals :

(a) 7 (b) 5 (c) 9 (d) 27

27. Let $y = f(x)$ and $\frac{x}{y} \frac{dy}{dx} = \frac{3x^2 - y}{2y - x^2}$; $f(1) = 1$ then the possible value of $\frac{1}{3} f(3)$ equals :

(a) 9 (b) 4 (c) 3 (d) 2

Answers

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(b)	**8.**	(c)	**9.**	(c)	**10.**	(b)
11.	(d)	**12.**	(c)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(c)	**17.**	(b)	**18.**	(c)	**19.**	(c)	**20.**	(d)
21.	(c)	**22.**	(c)	**23.**	(c)	**24.**	(d)	**25.**	(a)	**26.**	(a)	**27.**	(c)						

Exercise-2 : One or More than One Answer is/are Correct

1. Let $y = f(x)$ be a real valued function satisfying $x\frac{dy}{dx} = x^2 + y - 2$, $f(1) = 1$, then :
   (a) $f(x)$ is minimum at $x = 1$
   (b) $f(x)$ is maximum at $x = 1$
   (c) $f(3) = 5$
   (d) $f(2) = 3$

2. Solution of differential equation $x\cos x\left(\frac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$ is :
   (a) $xy = \sin x + c\cos x$
   (b) $xy\sec x = \tan x + c$
   (c) $xy + \sin x + c\cos x = 0$
   (d) None of these
   (where C is an arbitrary constant.)

3. If a differentiable function satisfies $(x-y)f(x+y) - (x+y)f(x-y) = 2(x^2y - y^3)\ \forall\ x, y \in R$ and $f(1) = 2$, then :
   (a) $f(x)$ must be polynomial function
   (b) $f(3) = 12$
   (c) $f(0) = 0$
   (d) $f(3) = 13$

4. A function $y = f(x)$ satisfies the differential equation
$$f(x)\sin 2x - \cos x + (1 + \sin^2 x)\ f'(x) = 0$$
   with $f(0) = 0$. The value of $f\left(\frac{\pi}{6}\right)$ equals to :
   (a) $\frac{2}{5}$
   (b) $\frac{3}{5}$
   (c) $\frac{1}{5}$
   (d) $\frac{4}{5}$

5. Solution of the differential equation $(2 + 2x^2\sqrt{y})\,y dx + (x^2\sqrt{y} + 2)x\,dy = 0$ is/are :
   (a) $xy(x^2\sqrt{y} + 5) = c$
   (b) $xy(x^2\sqrt{y} + 3) = c$
   (c) $xy(y^2\sqrt{x} + 3) = c$
   (d) $xy(y^2\sqrt{x} + 5) = c$

6. If $y(x)$ satisfies the differential equation $\frac{dy}{dx} = \sin 2x + 3y\cot x$ and $y\left(\frac{\pi}{2}\right) = 2$ then which of the following statement(s) is/are correct ?
   (a) $y\left(\frac{\pi}{6}\right) = 0$
   (b) $y'\left(\frac{\pi}{3}\right) = \frac{9 - 3\sqrt{2}}{2}$
   (c) $y(x)$ increases in the interval
   (d) $\int_{-\pi/2}^{\pi/2} y(x)\,dx = x$

Answers

1.	(a, c)	2.	(a, b)	3.	(a, b, c)	4.	(a)	5.	(b)	6.	(a, c)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

A differentiable function $y = g(x)$ satisfies $\int_0^x (x-t+1)g(t)\,dt = x^4 + x^2;\ \forall\, x \geq 0.$

1. $y = g(x)$ satisfies the differential equation :

(a) $\frac{dy}{dx} - y = 12x^2 + 2$ (b) $\frac{dy}{dx} + 2y = 12x^2 + 2$

(c) $\frac{dy}{dx} + y = 12x^2 + 2$ (d) $\frac{dy}{dx} + y = 12x + 2$

2. The value of $g(0)$ equals to :

(a) 0 (b) 1 (c) e^2 (d) Data insufficient

Paragraph for Question Nos. 3 to 5

Suppose f and g are differentiable functions such that $xg(f(x))f'(g(x))g'(x) = f(g(x))$ $g'(f(x))f'(x)\ \forall\, x \in R$ and f is positive, g is positive $\forall\, x \in R$. Also $\int_0^x f(g(t))\,dt = \frac{1}{2}(1-e^{-2x})$ $\forall\, x \in R$, $g(f(0)) = 1$ and $h(x) = \frac{g(f(x))}{f(g(x))}\ \forall\, x \in R$.

3. The graph of $y = h(x)$ is symmetric with respect to line :

(a) $x = -1$ (b) $x = 0$ (c) $x = 1$ (d) $x = 2$

4. The value of $f(g(0)) + g(f(0))$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

5. The largest possible value of $h(x)\ \forall\, x \in R$ is :

(a) 1 (b) $e^{1/3}$ (c) e (d) e^2

Paragraph for Question Nos. 6 to 8

Given a function 'g' which has a derivative $g'(x)$ for every real x and which satisfy $g'(0) = 2$ and $g(x+y) = e^y g(x) + e^x g(y)$ for all x and y.

6. The function $g(x)$ is :

(a) $x(2 + xe^x)$ (b) $x(e^x + 1)$ (c) $2xe^x$ (d) $x + \ln(x+1)$

7. The range of function $g(x)$ is :

(a) R (b) $\left[-\frac{2}{e}, \infty\right)$ (c) $\left[-\frac{1}{e}, \infty\right)$ (d) $[0, \infty)$

8. The value of $\lim_{x \to -\infty} g(x)$ is :

(a) 0 (b) 1 (c) 2 (d) Does not exist

Answers

1.	(c)	**2.**	(a)	**3.**	(c)	**4.**	(b)	**5.**	(c)	**6.**	(c)	**7.**	(b)	**8.**	(a)				

Exercise-4 : Matching Type Problems

1.

	Column-I (Differential equation)		Column-II Solution (Integral curves)
(A)	$y - x\dfrac{dy}{dx} = y^2 + \dfrac{dy}{dx}$	(P)	$y = A_1x^2 + A_2x + A_3$
(B)	$(2x - 10y^3)\dfrac{dy}{dx} + y = 0$	(Q)	$x^2y^2 + 1 = cy$
(C)	$\left(\dfrac{dy}{dx}\right)\left(\dfrac{d^3y}{dx^3}\right) - 3\left(\dfrac{d^2y}{dx^2}\right)^2 = 0$	(R)	$(x+1)(1-y) = cy$
(D)	$(x^2y^2 - 1)dy + 2xy^3dx = 0$	(S)	$x = A_1y^2 + A_2y + A_3$
		(T)	$xy^2 = 2y^5 + c$

2.

	Column-I		Column-II
(A)	Solution of differential equation $[3x^2y + 2xy - e^x(1+x)]dx + (x^3 + x^2)dy = 0$ is :	(P)	$y^2(x^2 + 1 + ce^{x^2}) = 1$
(B)	Solution of differential equation $ydx - xdy - 3xy^2e^{x^2}dx = 0$ is :	(Q)	$(x^2 + x^3)y - xe^x = c$
(C)	Solution of differential equation $\dfrac{dy}{dx} = xy(x^2y^2 - 1)$ is :	(R)	$\dfrac{x}{y} - \dfrac{3}{2}e^{x^2} = c$
(D)	Solution of differential equation $\dfrac{dy}{dx}(x^2y^3 + xy) = 1$ is :	(S)	$\dfrac{1}{x} = 2 - y^2 + ce^{-y^2/2}$
		(T)	$\dfrac{2}{x} = 1 - y^2 + ce^{-y/2}$
	(where c is arbitrary constant).		

Answers

1. A → R; B → T; C → S; D → Q

2. A → Q; B → R; C → P; D → S

Exercise-5 : Subjective Type Problems

1. Find the value of $|a|$ for which the area of triangle included between the coordinate axes and any tangent to the curve $x^a y = \lambda^a$ is constant (where λ is constant.).

2. Let $y = f(x)$ satisfies the differential equation $xy(1+y)dx = dy$. If $f(0) = 1$ and $f(2) = \dfrac{e^2}{k - e^2}$, then find the value of k.

3. If $y^2 = 3\cos^2 x + 2\sin^2 x$, then the value of $y^4 + y^3 \dfrac{d^2y}{dx^2}$ is

4. Let $f(x)$ be a differentiable function in $[-1, \infty)$ and $f(0) = 1$ such that $\displaystyle\lim_{t \to x+1} \frac{t^2 f(x+1) - (x+1)^2 f(t)}{f(t) - f(x+1)} = 1$. Find the value of $\displaystyle\lim_{x \to 1} \frac{\ln(f(x)) - \ln 2}{x - 1}$.

5. Let $y = (a\sin x + (b+c)\cos x)e^{x+d}$, where a, b, c and d are parameters represent a family of curves, then differential equation for the given family of curves is given by $y'' - \alpha y' + \beta y = 0$, then $\alpha + \beta =$

6. Let $y = f(x)$ satisfies the differential equation $xy(1+y)dx = dy$. If $f(0) = 1$ and $f(2) = \dfrac{e^2}{k - e^2}$, then find the value of k.

Answers

1.	1	2.	2	3.	6	4.	1	5.	4	6.	2		

□□□

Algebra

Chapter 8 – Quadratic Equations

8

QUADRATIC EQUATIONS

Exercise-1 : Single Choice Problems

1. Sum of values of x and y satisfying the equation $3^x - 4^y = 77;\ 3^{x/2} - 2^y = 7$ is :
(a) 2 (b) 3 (c) 4 (d) 5

2. If $f(x) = \prod_{i=1}^{3}(x - a_i) + \sum_{i=1}^{3} a_i - 3x$ where $a_i < a_{i+1}$ for $i = 1, 2$, then $f(x) = 0$ has :
(a) only one distinct real root (b) exactly two distinct real roots
(c) exactly 3 distinct real roots (d) 3 equal real roots

3. Complete set of real values of 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all its roots real :
(a) $\left[\frac{3}{4}, \infty\right)$ (b) $[1, \infty)$ (c) $[2, \infty)$ (d) $[0, \infty)$

4. The cubic polynomial with leading coefficient unity all whose roots are 3 units less than the roots of the equation $x^3 - 3x^2 - 4x + 12 = 0$ is denoted as $f(x)$, then $f'(x)$ is equal to:
(a) $3x^2 - 12x + 5$ (b) $3x^2 + 12x + 5$ (c) $3x^2 + 12x - 5$ (d) $3x^2 - 12x - 5$

5. The set of values of $k\ (k \in R)$ for which the equation $x^2 - 4|x| + 3 - |k - 1| = 0$ will have exactly four real roots, is :
(a) $(-2, 4)$ (b) $(-4, 4)$ (c) $(-4, 2)$ (d) $(-1, 0)$

6. The number of integers satisfying the inequality $\frac{x}{x+6} \le \frac{1}{x}$ is :
(a) 7 (b) 8 (c) 9 (d) 3

7. The product of uncommon real roots of the two polynomials $p(x) = x^4 + 2x^3 - 8x^2 - 6x + 15$ and $q(x) = x^3 + 4x^2 - x - 10$ is:
(a) 4 (b) 6 (c) 8 (d) 12

8. If $\lambda_1, \lambda_2\ (\lambda_1 > \lambda_2)$ are two values of λ for which the expression $f(x, y) = x^2 + \lambda xy + y^2 - 5x - 7y + 6$ can be resolved as a product of two linear factors, then the value of $3\lambda_1 + 2\lambda_2$ is :
(a) 5 (b) 10 (c) 15 (d) 20

9. Let α, β be the roots of the quadratic equation $ax^2 + bx + c = 0$, then the roots of the equation $a(x+1)^2 + b(x+1)(x-2) + c(x-2)^2 = 0$ are :

(a) $\frac{2\alpha+1}{\alpha-1}, \frac{2\beta+1}{\beta-1}$
(b) $\frac{2\alpha-1}{\alpha+1}, \frac{2\beta-1}{\beta+1}$
(c) $\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$
(d) $\frac{2\alpha+3}{\alpha-1}, \frac{2\beta+3}{\beta-1}$

10. If $a, b \in R$ distinct numbers satisfying $|a-1|+|b-1|=|a|+|b|=|a+1|+|b+1|$, then the minimum value of $|a-b|$ is :

(a) 3 (b) 0 (c) 1 (d) 2

11. The smallest positive integer p for which expression $x^2 - 2px + 3p + 4$ is negative for atleast one real x is :

(a) 3 (b) 4 (c) 5 (d) 6

12. For $x \in R$, the expression $\frac{x^2 + 2x + c}{x^2 + 4x + 3c}$ can take all real values if $c \in$:

(a) (1, 2)
(b) [0, 1]
(c) (0, 1)
(d) (–1, 0)

13. If 2 lies between the roots of the equation $t^2 - mt + 2 = 0$, $(m \in R)$ then the value of $\left[\left(\frac{3|x|}{9+x^2}\right)^m\right]$ is :

(where [·] denotes greatest integer function)

(a) 0 (b) 1 (c) 8 (d) 27

14. The number of integral roots of the equation $x^8 - 24x^7 - 18x^5 + 39x^2 + 1155 = 0$ is :

(a) 0 (b) 2 (c) 4 (d) 6

15. If the value of $m^4 + \frac{1}{m^4} = 119$, then the value of $\left|m^3 - \frac{1}{m^3}\right| =$

(a) 11 (b) 18 (c) 24 (d) 36

16. If the equation $ax^2 + 2bx + c = 0$ and $ax^2 + 2cx + b = 0$, $a \neq 0$, $b \neq c$, have a common root, then their other roots are the roots of the quadratic equation:

(a) $a^2x(x+1) + 4bc = 0$
(b) $a^2x(x+1) + 8bc = 0$
(c) $a^2x(x+2) + 8bc = 0$
(d) $a^2x(1+2x) + 8bc = 0$

17. If $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the roots of the equation $9x^3 - 9x^2 - x + 1 = 0$; $\alpha, \beta, \gamma \in [0, \pi]$ then the radius of the circle whose centre is $(\Sigma\alpha, \Sigma\cos\alpha)$ and passing through $(2\sin^{-1}(\tan\pi/4), 4)$ is:

(a) 2 (b) 3 (c) 4 (d) 5

18. For real values of x, the value of expression $\frac{11x^2 - 12x - 6}{x^2 + 4x + 2}$:

(a) lies between -17 and -3
(b) does not lie between -17 and -3
(c) lies between 3 and 17
(d) does not lie between 3 and 17

19. $\dfrac{x+3}{x^2-x-2} \geq \dfrac{1}{x-4}$ holds for all x satisfying:
(a) $-2 < x < 1$ or $x > 4$
(b) $-1 < x < 2$ or $x > 4$
(c) $x < -1$ or $2 < x < 4$
(d) $x > -1$ or $2 < x < 4$

20. If $x = 4 + 3i$ (where $i = \sqrt{-1}$), then the value of $x^3 - 4x^2 - 7x + 12$ equals:
(a) -88 (b) $48 + 36i$ (c) $-256 + 12i$ (d) -84

21. Let $f(x) = \dfrac{x^2+x-1}{x^2-x+1}$, then the largest value of $f(x)\ \forall x \in [-1, 3]$ is:
(a) $\dfrac{3}{5}$ (b) $\dfrac{5}{3}$ (c) 1 (d) $\dfrac{4}{3}$

22. In above problem, the range of $f(x) \forall x \in [-1, 1]$ is:
(a) $\left[-1, \dfrac{3}{5}\right]$ (b) $\left[-1, \dfrac{5}{3}\right]$ (c) $\left[-\dfrac{1}{3}, 1\right]$ (d) $[-1, 1]$

23. If the roots of the equation $\dfrac{1}{x+p} + \dfrac{1}{x+q} = \dfrac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots is:
(a) $-2(p^2+q^2)$ (b) $-(p^2+q^2)$ (c) $-\dfrac{(p^2+q^2)}{2}$ (d) $-pq$

24. If a root of the equation $a_1x^2 + b_1x + c_1 = 0$ is the reciprocal of a root of the equation $a_2x^2 + b_2x + c_2 = 0$, then :
(a) $(a_1a_2 - c_1c_2)^2 = (a_1b_2 - b_1c_2)(a_2b_1 - b_2c_1)$
(b) $(a_1a_2 - b_1b_2)^2 = (a_1b_2 - b_1c_2)(a_2b_1 - b_2c_1)$
(c) $(b_1c_2 - b_2c_1)^2 = (a_1b_2 - b_1c_2)(a_2b_1 + b_2c_1)$
(d) $(b_1c_2 - b_2c_1)^2 = (a_1b_2 + b_1c_2)(a_2b_1 - b_2c_1)$

25. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$, then the equation with roots $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$ is:
(a) $3x^2 - 25x + 3 = 0$
(b) $x^2 + 5x - 3 = 0$
(c) $x^2 - 5x + 3 = 0$
(d) $3x^2 - 19x + 3 = 0$

26. If the difference between the roots of $x^2 + ax + b = 0$ is same as that of $x^2 + bx + a = 0$, $a \neq b$, then:
(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$ (c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

27. If $\tan\theta_i$; $i = 1, 2, 3, 4$ are the roots of equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4) =$
(a) $\sin\beta$ (b) $\cos\beta$ (c) $\tan\beta$ (d) $\cot\beta$

28. Let a, b, c, d are positive real numbers such that $\frac{a}{b} \neq \frac{c}{d}$, then the roots of the equation: $(a^2 + b^2)x^2 + 2x(ac + bd) + (c^2 + d^2) = 0$ are:

(a) real and distinct (b) real and equal
(c) imaginary (d) nothing can be said

29. If α, β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are $2 + \alpha$, $2 + \beta$, is :

(a) $ax^2 + x(4a - b) + 4a - 2b + c = 0$ (b) $ax^2 + x(4a - b) + 4a + 2b + c = 0$
(c) $ax^2 + x(b - 4a) + 4a + 2b + c = 0$ (d) $ax^2 + x(b - 4a) + 4a - 2b + c = 0$

30. Minimum possible number of positive root of the quadratic equation $x^2 - (1 + \lambda)x + \lambda - 2 = 0$, $\lambda \in R$:

(a) 2 (b) 0
(c) 1 (d) can not be determined

31. Let α, β be real roots of the quadratic equation $x^2 + kx + (k^2 + 2k - 4) = 0$, then the minimum value of $\alpha^2 + \beta^2$ is equal to :

(a) 12 (b) $\frac{4}{9}$ (c) $\frac{16}{9}$ (d) $\frac{8}{9}$

32. Polynomial $P(x) = x^2 - ax + 5$ and $Q(x) = 2x^3 + 5x - (a - 3)$ when divided by $x - 2$ have same remainders, then 'a' is equal to:

(a) 10 (b) -10 (c) 20 (d) -20

33. If a and b are non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{9}{4}$ (c) $-\frac{9}{4}$ (d) 1

34. Let α, β be the roots of the equation $ax^2 + bx + c = 0$. A root of the equation $a^3x^2 + abcx + c^3 = 0$ is :

(a) $\alpha + \beta$ (b) $\alpha^2 + \beta$ (c) $\alpha^2 - \beta$ (d) $\alpha^2\beta$

35. Let a, b, c be the lengths of the sides of a triangle (no two of them are equal) and $k \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 6k(ab + bc + ca) = 0$ are real, then:

(a) $k < \frac{2}{3}$ (b) $k > \frac{2}{3}$ (c) $k > 1$ (d) $k < \frac{1}{4}$

36. Root(s) of the equation $9x^2 - 18|x| + 5 = 0$ belonging to the domain of definition of the function $f(x) = \log(x^2 - x - 2)$ is/are:

(a) $\frac{-5}{3}, \frac{-1}{3}$ (b) $\frac{5}{3}, \frac{1}{3}$ (c) $\frac{-5}{3}$ (d) $\frac{-1}{3}$

37. If $\beta + \cos^2\alpha, \beta + \sin^2\alpha$ are the roots of $x^2 + 2bx + c = 0$ and $\gamma + \cos^4\alpha$, $\gamma + \sin^4\alpha$ are the roots of $x^2 + 2Bx + C = 0$, then:

(a) $b - B = c - C$ (b) $b^2 - B^2 = c - C$ (c) $b^2 - B^2 = 4(c - C)$ (d) $4(b^2 - B^2) = c - C$

38. Minimum value of $|x-p|+|x-15|+|x-p-15|$. If $p \le x \le 15$ and $0 < p < 15$:

(a) 30 (b) 15 (c) 10 (d) 0

39. If the quadratic equation $4x^2 - 2x - m = 0$ and $4p(q-r)x^2 - 2q(r-p)x + r(p-q) = 0$ have a common root such that second equation has equal roots then the value of m will be :

(a) 0 (b) 1 (c) 2 (d) 3

40. The range of k for which the inequality $k\cos^2 x - k\cos x + 1 \ge 0 \forall x \in (-\infty, \infty)$ is :

(a) $k > -\frac{1}{2}$ (b) $k > 4$ (c) $-\frac{1}{2} \le k \le 4$ (d) $\frac{1}{2} \le k \le 5$

41. If $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$ are roots of the cubic equation $f(x) = 0$ where α, β, γ are the roots of the cubic equation $3x^3 - 2x + 5 = 0$, then the number of negative real roots of the equation $f(x) = 0$ is:

(a) 0 (b) 1 (c) 2 (d) 3

42. The sum of all integral values of λ for which $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1 \, \forall x \in R$, is :

(a) -1 (b) -3 (c) 0 (d) -2

43. If $\alpha, \beta, \gamma, \delta \in R$ satisfy $\frac{(\alpha+1)^2 + (\beta+1)^2 + (\gamma+1)^2 + (\delta+1)^2}{\alpha+\beta+\gamma+\delta} = 4$

If biquadratic equation $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$ has the roots $\left(\alpha + \frac{1}{\beta} - 1\right), \left(\beta + \frac{1}{\gamma} - 1\right), \left(\gamma + \frac{1}{\delta} - 1\right), \left(\delta + \frac{1}{\alpha} - 1\right)$. Then the value of a_2/a_0 is :

(a) 4 (b) -4 (c) 6 (d) none of these

44. If the complete set of value of x satisfying $|x-1|+|x-2|+|x-3| \ge 6$ is $(-\infty, a] \cup [b, \infty)$, then $a + b =$:

(a) 2 (b) 3 (c) 6 (d) 4

45. If exactly one root of the quadratic equation $x^2 - (a+1)x + 2a = 0$ lies in the interval $(0, 3)$, then the set of value 'a' is given by:

(a) $(-\infty, 0) \cup (6, \infty)$ (b) $(-\infty, 0] \cup (6, \infty)$

(c) $(-\infty, 0] \cup [6, \infty)$ (d) $(0, 6)$

46. The condition that the root of $x^3 + 3px^2 + 3qx + r = 0$ are in H.P. is :

(a) $2p^3 - 3pqr + r^2 = 0$ (b) $3p^3 - 2pqr + p^2 = 0$

(c) $2q^3 - 3pqr + r^2 = 0$ (d) $r^3 - 3pqr + 2q^3 = 0$

47. If x is real and $4y^2 + 4xy + x + 6 = 0$, then the complete set of values of x for which y is real, is:

(a) $x \le -2$ or $x \ge 3$ (b) $x \le 2$ or $x \ge 3$ (c) $x \le -3$ or $x \ge 2$ (d) $-3 \le x \le 2$

48. The solution of the equation $\log_{\cos x^2}(3-2x) < \log_{\cos x^2}(2x-1)$ is :

(a) $(1/2, 1)$ (b) $(-\infty, 1)$

(c) $(1/2, 3)$ (d) $(1, \infty) - \sqrt{2n\pi}, \, n \in N$

49. If the roots α, β of the equation $px^2 + qx + r = 0$ are real and of opposite sign (where p, q, r are real coefficient), then the roots of the equation $\alpha(x-\beta)^2 + \beta(x-\alpha)^2 = 0$ are:

(a) positive (b) negative
(c) real and of opposite sign (d) imaginary

50. Let a, b and c be three distinct real roots of the cubic $x^3 + 2x^2 - 4x - 4 = 0$.
If the equation $x^3 + qx^2 + rx + s = 0$ has roots $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$, then the value of $(q + r + s)$ is equal to :

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

51. Solution set of the inequality, $2 - \log_2(x^2 + 3x) \geq 0$ is :

(a) $[-4, 1]$ (b) $[-4, -3) \cup (0, 1]$ (c) $(-\infty, -3) \cup (1, \infty)$ (d) $(-\infty, -4) \cup [1, \infty)$

52. For what least integral 'k' is the quadratic trinomial $(k-2)x^2 + 8x + (k+4)$ is positive for all real values of x?

(a) $k = 4$ (b) $k = 5$ (c) $k = 3$ (d) $k = 6$

53. If roots of the equation $(m-2)x^2 - (8-2m)x - (8-3m) = 0$ are opposite in sign, then number of integral values(s) of m is/are :

(a) 0 (b) 1 (c) 2 (d) more than 2

54. If $\log_{0.6}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0$, then complete set of value of 'x' is :

(a) $(-4, -3) \cup (8, \infty)$ (b) $(-\infty, -3) \cup (8, \infty)$
(c) $(8, \infty)$ (d) None of these

55. Two different real numbers α and β are the roots of the quadratic equation $ax^2 + c = 0$ with $a, c \neq 0$, then $\alpha^3 + \beta^3$ is :

(a) a (b) $-c$ (c) 0 (d) -1

56. The least integral value of 'k' for which $(k-1)x^2 - (k+1)x + (k+1)$ is positive for all real value of x is:

(a) 1 (b) 2 (c) 3 (d) 4

57. If $(-2, 7)$ is the highest point on the graph of $y = -2x^2 - 4ax + k$, then k equals:

(a) 31 (b) 11 (c) -1 (d) $-1/3$

58. If $a + b + c = 0$, $a, b, c \in Q$ then roots of the equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are :

(a) rational (b) irrational (c) imaginary (d) none of these

59. If two roots of $x^3 - ax^2 + bx - c = 0$ are equal in magnitude but opposite in sign. Then:

(a) $a + bc = 0$ (b) $a^2 = bc$
(c) $ab = c$ (d) $a - b + c = 0$

60. If α and β are the real roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$. Then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always ($\alpha \neq \beta, p \neq 0, p, q, r, s \in R$):

(a) one positive and one negative root (b) two positive roots

(c) two negative roots (d) can't say anything

61. If $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, then:

(a) $a^2 + c^2 = -ab$ (b) $a^2 + c^2 = ab$ (c) $a^2 - c^2 = ab$ (d) $a^2 - c^2 = -ab$

62. In a ΔABC $\tan\frac{A}{2}$, $\tan\frac{B}{2}$, $\tan\frac{C}{2}$ are in H.P., then the value of $\cot\frac{A}{2}\cot\frac{C}{2}$ is :

(a) 3 (b) 2 (c) 1 (d) $\sqrt{3}$

63. Let $f(x) = 10 - |x - 10|\ \forall x \in [-9, 9]$, if M and m be the maximum and minimum value of $f(x)$ respectively, then :

(a) $M + m = 0$ (b) $2M + m = -9$ (c) $2M + m = 7$ (d) $M + m = 7$

64. Solution of the quadratic equation $(3|x|-3)^2 = |x| + 7$, which belongs to the domain of the function $y = \sqrt{(x-4)x}$ is :

(a) $\pm\frac{1}{9}, \pm 2$ (b) $\frac{1}{9}, 8$ (c) $-2, -\frac{1}{9}$ (d) $-\frac{1}{9}, 8$

65. Number of real solutions of the equation $x^2 + 3|x| + 2 = 0$ is :

(a) 0 (b) 1 (c) 2 (d) 4

66. If the roots of equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c =$

(a) 3 (b) -2 (c) 1 (d) 2

67. If x is real, then maximum value of $\dfrac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is :

(a) 41 (b) 1 (c) $\frac{17}{7}$ (d) $\frac{1}{4}$

68. If $\dfrac{x^2 + 2x + 7}{2x + 3} < 6,\ x \in R$ then :

(a) $x \in \left(-\infty, -\frac{3}{2}\right) \cup (11, \infty)$ (b) $x \in (-\infty, -1) \cup (11, \infty)$

(c) $x \in \left(-\frac{3}{2}, -1\right)$ (d) $x \in \left(-\infty, -\frac{3}{2}\right) \cup (-1, 11)$

69. If x is real, then range of $\dfrac{3x - 2}{7x + 5}$ is :

(a) $R - \left\{\frac{2}{5}\right\}$ (b) $R - \left\{\frac{3}{7}\right\}$ (c) $(-\infty, \infty)$ (d) $R - \left\{\frac{-2}{5}\right\}$

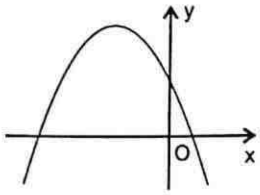
70. Let A denotes the set of values of x for which $\dfrac{x+2}{x-4} \leq 0$ and B denotes the set of values of x for which $x^2 - ax - 4 \leq 0$. If B is the subset of A, then a **CAN NOT** take the integral value :

(a) 0 (b) 1 (c) 2 (d) 3

71. If the quadratic polynomial $P(x) = (p-3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be :

(a) 3 (b) 4 (c) 6 (d) 7

72. If graph of the quadratic $y = ax^2 + bx + c$ is given below :



then :

(a) $a < 0, b > 0, c > 0$ (b) $a < 0, b > 0, c < 0$

(c) $a < 0, b < 0, c > 0$ (d) $a < 0, b < 0, c < 0$

73. If quadratic equation $ax^2 + bx + c = 0$ does not have real roots, then which of the following may be false :

(a) $a(a-b+c) > 0$ (b) $c(a-b+c) > 0$

(c) $b(a-b+c) > 0$ (d) $(a+b+c)(a-b+c) > 0$

74. Minimum value of $y = x^2 - 3x + 5,\ x \in [-4, 1]$ is :

(a) 3 (b) $\frac{11}{4}$ (c) 0 (d) 9

75. If $3x^2 - 17x + 10 = 0$ and $x^2 - 5x + m = 0$ has a common root, then sum of all possible real values of 'm' is :

(a) 0 (b) $-\frac{26}{9}$ (c) $\frac{29}{9}$ (d) $\frac{26}{3}$

76. For real numbers x and y, if $x^2 + xy - y^2 + 2x - y + 1 = 0$, then :

(a) y can not be between 0 and $\frac{8}{5}$ (b) y can not be between $-\frac{8}{5}$ and $\frac{8}{5}$

(c) y can not be between $-\frac{8}{5}$ and 0 (d) y can not be between $-\frac{16}{5}$ and 0

77. If $3x^4 - 6x^3 + kx^2 - 8x - 12$ is divisible by $x - 3$, then it is also divisible by :

(a) $3x^2 - 4$ (b) $3x^2 + 4$

(c) $3x^2 + x$ (d) $3x^2 - x$

78. The complete set of values of a so that equation $\sin^4 x + a\sin^2 x + 4 = 0$ has at least one real root is :

(a) $(\infty, -5]$ (b) $(-\infty, 4] \cup [4, \infty)$

(c) $(-\infty, -4]$ (d) $[4, \infty)$

79. Let r, s, t be the roots of the equation $x^3 + ax^2 + bx + c = 0$, such that $(rs)^2 + (st)^2 + (rt)^2 = b^2 - kac$, then $k =$

(a) 1 (b) 2 (c) 3 (d) 4

80. If the roots of the cubic $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers, then the value of $\frac{a^2}{b+1} =$

(a) 1 (b) 2 (c) 3 (d) 4

81. Let 'k' be a real number. The minimum number of distinct real roots possible of the equation $(3x^2 + kx + 3)(x^2 + kx - 1) = 0$ is :

(a) 0 (b) 2 (c) 3 (d) 4

82. If r and s are variables satisfying the equation $\frac{1}{r+s} = \frac{1}{r} + \frac{1}{s}$. The value of $\left(\frac{r}{s}\right)^3$ is equal to :

(a) 1 (b) -1
(c) 3 (d) not possible to determine

83. Let $f(x) = x^2 + ax + b$. If the maximum and the minimum values of $f(x)$ are 3 and 2 respectively for $0 \le x \le 2$, then the possible ordered pair of (a, b) is :

(a) $(-2, 3)$ (b) $(-3/2, 2)$ (c) $(-5/2, 3)$ (d) $(-5/2, 2)$

84. The roots of the equation $|x^2 - x - 6| = x + 2$ are given by :

(a) $-2, 2, 4$ (b) $0, 1, 4$ (c) $-2, 1, 4$ (d) $0, 2, 4$

85. If a, b, c be the sides of ΔABC and equations $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ have a common root, then $\angle C$ is :

(a) 60° (b) 90° (c) 120° (d) 45°

86. If α, β and γ are three real roots of the equation $x^3 - 6x^2 + 5x - 1 = 0$, then the value of $\alpha^4 + \beta^4 + \gamma^4$ is :

(a) 250 (b) 650 (c) 150 (d) 950

87. If one of the roots of the equation $2x^2 - 6x + k = 0$ is $\frac{\alpha + 5i}{2}$, then the value of α and k are :

(a) $\alpha = 3, k = 8$ (b) $\alpha = \frac{3}{2}, k = 17$ (c) $\alpha = -3, k = -17$ (d) $\alpha = 3, k = 17$

88. Let x_1 and x_2 be the real roots of the equation $x^2 - (k-2)x + (k^2 + 3k + 5) = 0$, then the maximum value of $x_1^2 + x_2^2$ is :

(a) 19 (b) 18 (c) $\frac{50}{9}$ (d) non-existent

89. The complete set of values of 'a' for which the inequality $(a-1)x^2 - (a+1)x + (a-1) \ge 0$ is true for all $x \ge 2$.

(a) $\left(\frac{3}{7}, 1\right]$ (b) $(-\infty, 1)$ (c) $\left(-\infty, \frac{7}{3}\right]$ (d) $\left[\frac{7}{3}, \infty\right)$

90. If α, β be the roots of $4x^2 - 17x + \lambda = 0$, $\lambda \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral values of λ is :

(a) 1 (b) 2 (c) 3 (d) 4

91. Assume that p is a real number. In order of $\sqrt[3]{x+3p+1}-\sqrt[3]{x}=1$ to have real solutions, it is necessary that:

(a) $p \geq 1/4$ (b) $p \geq -1/4$ (c) $p \geq 1/3$ (d) $p \geq -1/3$

92. If α, β are the roots of the quadratic equation $x^2-\left(3+2^{\sqrt{\log_2 3}}-3^{\sqrt{\log_3 2}}\right)x-2(3^{\log_3 2}-2^{\log_2 3})=0$, then the value of $\alpha^2+\alpha\beta+\beta^2$ is equal to :

(a) 3 (b) 5 (c) 7 (d) 11

93. The minimum value of $f(x, y)=x^2-4x+y^2+6y$ when x and y are subjected to the restrictions $0 \leq x \leq 1$ and $0 \leq y \leq 1$, is :

(a) -1 (b) -2 (c) -3 (d) -5

94. The expression $ax^2+2bx+c$, where 'a' is non-zero real number, has same sign as that of 'a' for every real value of x, then roots of quadratic equation $ax^2+(b-c)\ x-2b-c-a=0$, are :

(a) real and equal
(b) real and unequal
(c) non-real having positive real part
(d) non-real having negative real part

95. Let a, b and c be the roots of $x^3-x+1=0$, then the value of $\left(\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}\right)$ equals to :

(a) 1 (b) -1 (c) 2 (d) -2

96. The number of integral values of k for which the inequality $x^2-2(4k-1)x+15k^2-2k-7 \geq 0$ holds for all $x \in R$ is :

(a) 2 (b) 3 (c) 4 (d) infinite

97. The number of integral values which can be taken by the expression, $f(x)=\dfrac{x^3-1}{(x-1)\ (x^2-x+1)}$ for $x \in R$, is :

(a) 1 (b) 2 (c) 3 (d) infinite

98. The complete set of values of m for which the inequality $\dfrac{x^2-mx-2}{x^2+mx+4}>-1$ is satisfied $\forall x \in R$, is :

(a) $m=0$ (b) $-1<m<1$ (c) $-2<m<2$ (d) $-4<m<4$

99. The complete set of values of a for which the roots of the equation $x^2-2|a+1|x+1=0$ are real is given by :

(a) $(-\infty,-2] \cup [0, \infty)$
(b) $(-\infty, -1] \cup [0, \infty)$
(c) $(-\infty, -1] \cup [1, \infty)$
(d) $(-\infty, -2] \cup [1, \infty)$

100. The quadratic polynomials defined on real coefficients $P(x)=a_1x^2+2b_1x+c_1$, $Q(x)=a_2x^2+2b_2x+c_2 \cdot P(x)$ and $Q(x)$ both take positive values $\forall x \in R$. If $f(x)=a_1a_2x^2+b_1b_2x+c_1c_2$, then :

(a) $f(x)<0\ \forall\ x \in R$
(b) $f(x)>0\ \forall\ x \in R$

(c) $f(x)$ takes both positive and negative values

(d) Nothing can be said about $f(x)$

101. If the equation $x^2 + 4 + 3\cos(ax+b) = 2x$ has a solution then a possible value of $(a+b)$ equals

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

102. Let α, β be the roots of $x^2 - 4x + A = 0$ and γ, δ be the roots of $x^2 - 36x + B = 0$. If $\alpha, \beta, \gamma, \delta$ form an increasing G.P. and $A^t = B$ then the value of 't' equals

(a) 4 (b) 5 (c) 6 (d) 8

103. How many roots does the following equation possess $3^{|x|}(|2-|x||) = 1$?

(a) 2 (b) 3 (c) 4 (d) 6

104. If $\cot\alpha$ equals the integral solution of inequality $4x^2 - 16x + 15 < 0$ and $\sin\beta$ equals to the slope of the bisector of the first quadrant, then $\sin(\alpha+\beta)\sin(\alpha-\beta)$ is equal to :

(a) $-\frac{3}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{2}{\sqrt{2}}$ (d) 3

105. Consider the functions $f_1(x) = x$ and $f_2(x) = 2 + \log_e x$, $x > 0$, where e is the base of natural logarithm. The graphs of the functions intersect :

(a) once in (0, 1) and never in $(1, \infty)$ (b) once in (0, 1) and once in (e^2, ∞)

(c) once in (0, 1) and once in (e, e^2) (d) more than twice in $(0, \infty)$

106. The sum of all the real roots of equation

$$x^4 - 3x^3 - 2x^2 - 3x + 1 = 0 \text{ is :}$$

(a) 1 (b) 2 (c) 3 (d) 4

107. If α, β $(\alpha < \beta)$ are the real roots of the equation $x^2 - (k+4)x + k^2 - 12 = 0$ such that $4 \in (\alpha, \beta)$; then the number of integral values of k equal to :

(a) 4 (b) 5 (c) 6 (d) 7

108. Let α, β be real roots of the quadratic equation $x^2 + kx + (k^2 + 2k - 4) = 0$, then the maximum value of $(\alpha^2 + \beta^2)$ is equal to :

(a) 9 (b) 10 (c) 11 (d) 12

109. Let $f(x) = a^x - x\ln a$, $a > 1$. Then the complete set of real values of x for which $f'(x) > 0$ is :

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(0, \infty)$ (d) (0, 1)

110. If a, b and c are the roots of the equation $x^3 + 2x^2 + 1 = 0$, find $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$:

(a) 8 (b) –8 (c) 0 (d) 2

111. Let α, β are the two real roots of equation $x^2 + px + q = 0$, $p, q \in R$, $q \neq 0$. If the quadratic equation $g(x) = 0$ has two roots $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$ such that sum of roots is equal to product of roots, then the complete range of q is :

(a) $\left[\frac{1}{3}, 3\right]$ (b) $\left(\frac{1}{3}, 3\right]$ (c) $\left[\frac{1}{3}, 3\right)$ (d) $\left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$

112. If the equation $\ln(x^2 + 5x) - \ln(x + a + 3) = 0$ has exactly one solution for x, then number of integers in the range of a is :

(a) 4 (b) 5 (c) 6 (d) 7

113. Let $f(x) = x^2 + \frac{1}{x^2} - 6x - \frac{6}{x} + 2$, then minimum value of $f(x)$ is :

(a) -2 (b) -8 (c) -9 (d) -12

114. If $x^2 + bx + b$ is a factor of $x^3 + 2x^2 + 2x + c\,(c \neq 0)$, then $b - c$ is :

(a) 2 (b) -1 (c) 0 (d) -2

115. If roots of $x^3 + 2x^2 + 1 = 0$ are α, β and γ, then the value of $(\alpha\beta)^3 + (\beta\gamma)^3 + (\alpha\gamma)^3$, is :

(a) -11 (b) 3 (c) 0 (d) -2

116. How many roots does the following equation possess $3^{|x|}(|2 - |x||) = 1$?

(a) 2 (b) 3 (c) 4 (d) 6

117. The sum of all the real roots of equation $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$ is :

(a) 1 (b) 2 (c) 3 (d) 4

118. If α and β are the roots of the quadratic equation $4x^2 + 2x - 1 = 0$ then the value of $\sum_{r=1}^{\infty}(\alpha^r + \beta^r)$ is :

(a) 2 (b) 3 (c) 6 (d) 0

119. The number of value(s) of x satisfying the equation $(2011)^x + (2012)^x + (2013)^x - (2014)^x = 0$ is/are :

(a) exactly 2 (b) exactly 1 (c) more than one (d) 0

120. If $\alpha, \beta\,(\alpha < \beta)$ are the real roots of the equation $x^2 - (k+4)x + k^2 - 12 = 0$ such that $4 \in (\alpha, \beta)$; then the number of integral values of k equals to :

(a) 4 (b) 5 (c) 6 (d) 7

121. Let α, β be real roots of the quadratic equation $x^2 + kx + (k^2 + 2k - 4) = 0$, then the maximum value of $(\alpha^2 + \beta^2)$ is equal to :

(a) 9 (b) 10 (c) 11 (d) 12

122. The exhaustive set of values of a for which inequation $(a-1)x^2 - (a+1)x + a - 1 \geq 0$ is true $\forall\, x \geq 2$.

(a) $(-\infty, 1)$ (b) $\left[\frac{7}{3}, \infty\right)$ (c) $\left[\frac{3}{7}, \infty\right)$ (d) None of these

123. If the equation $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a+b)x + 36 = 0$ have a common positive root, then $b - 2a$ is equal to.

(a) -6 (b) 22 (c) 6 (d) -22

124. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(a) infinite number of real roots (b) no real root

(c) exactly one real root (d) exactly four real roots

125. The difference between the maximum and minimum value of the function $f(x) = 3\sin^4 x - \cos^6 x$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 4

126. If α, β are the roots of $x^2 - 3x + \lambda = 0$ $(\lambda \in R)$ and $\alpha < 1 < \beta$, then the true set of values of λ equals :

(a) $\lambda \in \left(2, \frac{9}{4}\right]$ (b) $\lambda \in \left(-\infty, \frac{9}{4}\right]$ (c) $\lambda \in (2, \infty)$ (d) $\lambda \in (-\infty, 2)$

127. If $2x^2 + 5x + 7 = 0$ and $ax^2 + bx + c = 0$ have at least one root common such that $a, b, c \in \{1, 2, \ldots\ldots, 100\}$, then the difference between the maximum and minimum values of $a + b + c$ is :

(a) 196 (b) 284 (c) 182 (d) 126

128. Two particles, A and B, are in motion in the xy-plane. Their co-ordinates at each instant of time $t(t \geq 0)$ are given by $x_A = t$, $y_A = 2t$, $x_B = 1 - t$ and $y_B = t$. The minimum distance between particles A and B is :

(a) $\frac{1}{5}$ (b) $\frac{1}{\sqrt{5}}$ (c) 1 (d) $\sqrt{\frac{2}{3}}$

129. If $a \neq 0$ and the equation $ax^2 + bx + c = 0$ has two roots α and β such that $\alpha < -3$ and $\beta > 2$, which of the following is always true ?

(a) $a(a + |b| + c) > 0$ (b) $a(a + |b| + c) < 0$

(c) $9a - 3b + c > 0$ (d) $(9a - 3b + c)(4a + 2b + c) < 0$

130. The number of negative real roots of the equation $(x^2 + 5x)^2 - 24 = 2(x^2 + 5x)$ is :

(a) 4 (b) 3 (c) 2 (d) 1

131. The number of real values of x satisfying the equation $3|x - 2| + |1 - 5x| + 4|3x + 1| = 13$ is :

(a) 1 (b) 4 (c) 2 (d) 3

132. If $\log_{\cos x} \sin x \geq 2$ and $0 \leq x \leq 3\pi$ then $\sin x$ lies in the interval

(a) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$ (b) $\left[0, \frac{\sqrt{5}-1}{2}\right]$ (c) $\left[\frac{1}{2}, 1\right]$ (d) none of these

133. Let $f(x) = x^2 + bx + c$, minimum value of $f(x)$ is -5, then absolute value of the difference of the roots of $f(x)$ is :

(a) 5 (b) $\sqrt{20}$

(c) $\sqrt{15}$ (d) Can't be determined

134. Sum of all the solutions of the equation $|x - 3| + |x + 5| = 7x$, is :

(a) $\frac{6}{7}$ (b) $\frac{8}{7}$ (c) $\frac{58}{63}$ (d) $\frac{8}{45}$

135. Let $f(x) = x^2 + \frac{1}{x^2} - 6x - \frac{6}{x} + 2$, then minimum value of $f(x)$ is :

(a) -2 (b) -8 (c) -9 (d) -12

136. If $a+b+c=1, a^2+b^2+c^2=9$ and $a^3+b^3+c^3=1$, then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is :

(a) $\frac{2}{3}$ (b) 5 (c) 6 (d) 1

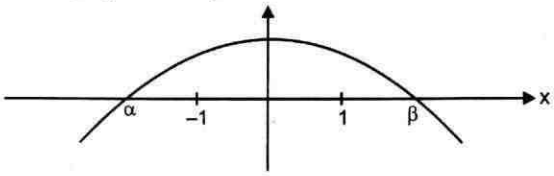
137. If roots of $x^3+2x^2+1=0$ are α, β and γ, then the value of $(\alpha\beta)^3+(\beta\gamma)^3+(\alpha\gamma)^3$, is :

(a) -11 (b) 3 (c) 0 (d) -2

138. If x^2+bx+b is a factor of x^3+2x^2+2x+c $(c \neq 0)$, then $b-c$ is :

(a) 2 (b) -1 (c) 0 (d) -2

139. The graph of quadratic polynomical $f(x) = ax^2+bx+c$ is shown below



(a) $\frac{c}{a}|\beta-\alpha| < -2$ (b) $f(x) > 0 \ \forall \ x > \beta$ (c) $ac > 0$ (d) $\frac{c}{a} > -1$

140. If $f(x) = \frac{x^2-3x+4}{x^2+3x+4}$, then complete solution of $0 < f(x) < 1$, is :

(a) $(-\infty, \infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(0,1) \cup (2, \infty)$

141. If α, β, γ are the roots of the equation $x^3+2x^2-x+1=0$, then value of $\frac{(2-\alpha)(2-\beta)(2-\gamma)}{(2+\alpha)(2+\beta)(2+\gamma)}$, is:

(a) 5 (b) -5 (c) 10 (d) $\frac{5}{3}$

142. If α and β are roots of the quadratic equation $x^2+4x+3=0$, then the equation whose roots are $2\alpha+\beta$ and $\alpha+2\beta$ is :

(a) $x^2-12x+35=0$ (b) $x^2+12x-33=0$ (c) $x^2-12x-33=0$ (d) $x^2+12x+35=0$

143. If a, b, c are real distinct numbers such that $a^3+b^3+c^3=3abc$, then the quadratic equation $ax^2+bx+c=0$ has

(a) Real roots (b) At least one negative root

(c) Both roots are negative (d) Non real roots

144. If the equation $x^2+ax+12=0$, $x^2+bx+15=0$ and $x^2+(a+b)x+36=0$ have a common positive root, then $b-2a$ is equal to.

(a) -6 (b) 22 (c) 6 (d) -22

145. Consider the equation $x^3 - ax^2 + bx - c = 0$, where a, b, c are rational number, $a \neq 1$. It is given that x_1, x_2 and $x_1 x_2$ are the real roots of the equation. Then $x_1 x_2 \left(\frac{a+1}{b+c}\right) =$

(a) 1 (b) 2 (c) 3 (d) 4

146. The exhaustive set of values of a for which inequation $(a-1)x^2 - (a+1)x + a - 1 \geq 0$ is true $\forall x \geq 2$.

(a) $(-\infty, 1)$ (b) $\left[\frac{7}{3}, \infty\right)$ (c) $\left[\frac{3}{7}, \infty\right)$ (d) None of these

147. The number of real solutions of the equation

$$x^2 - 3|x| + 2 = 0$$

(a) 2 (b) 4 (c) 1 (d) 3

148. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(a) infinite number of real roots (b) no real root

(c) exactly one real root (d) exactly four real roots

149. If α, β are the roots of the quadratic equation $x^2 - 2(1 - \sin 2\theta)x - 2\cos^2 2\theta = 0$, $(\theta \in R)$ then the minimum value of $(\alpha^2 + \beta^2)$ is equal to :

(a) -4 (b) 8 (c) 0 (d) 2

150. If the equation $|\sin x|^2 + |\sin x| + b = 0$ has two distinct roots in $[0, \pi]$; then the number of integers in the range of b is equals to :

(a) 0 (b) 1 (c) 2 (d) 3

151. If $a \neq 0$ and the equation $ax^2 + bx + c = 0$ has two roots α and β such that $\alpha < -3$ and $\beta > 2$. Which of the following is always true ?

(a) $a(a + |b| + c) > 0$ (b) $a(a + |b| + c) < 0$

(c) $9a - 3b + c > 0$ (d) $(9a - 3b + c)(4a + 2b + c) < 0$

152. If α, β are the roots of the quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$ then $(\alpha - \gamma)(\alpha - \delta)$ is equal to :

(a) $q + r$ (b) $q - r$ (c) $-(q + r)$ (d) $-(p + q + r)$

153. Complete set of solution of $\log_{1/3}(2^{x+2} - 4^x) \geq -2$ is :

(a) $(-\infty, 2)$ (b) $(-\infty, 2 + \sqrt{13})$ (c) $(2, \infty)$ (d) None of these

Answers

1. (d)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (d)
11. (c)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (a)
21. (b)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (c)
31. (d)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (c)
41. (b)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (b)	**46.** (c)	**47.** (a)	**48.** (a)	**49.** (c)	**50.** (c)
51. (b)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (c)	**56.** (b)	**57.** (c)	**58.** (a)	**59.** (c)	**60.** (a)
61. (c)	**62.** (a)	**63.** (a)	**64.** (c)	**65.** (a)	**66.** (c)	**67.** (a)	**68.** (d)	**69.** (b)	**70.** (d)
71. (c)	**72.** (c)	**73.** (c)	**74.** (a)	**75.** (c)	**76.** (c)	**77.** (b)	**78.** (a)	**79.** (b)	**80.** (c)
81. (b)	**82.** (a)	**83.** (a)	**84.** (a)	**85.** (b)	**86.** (b)	**87.** (d)	**88.** (b)	**89.** (d)	**90.** (b)
91. (b)	**92.** (c)	**93.** (c)	**94.** (b)	**95.** (d)	**96.** (b)	**97.** (b)	**98.** (d)	**99.** (a)	**100.** (b)
101. (d)	**102.** (b)	**103.** (c)	**104.** (b)	**105.** (c)	**106.** (d)	**107.** (d)	**108.** (d)	**109.** (c)	**110.** (a)
111. (a)	**112.** (b)	**113.** (c)	**114.** (c)	**115.** (b)	**116.** (c)	**117.** (d)	**118.** (d)	**119.** (b)	**120.** (d)
121. (d)	**122.** (b)	**123.** (c)	**124.** (b)	**125.** (d)	**126.** (d)	**127.** (c)	**128.** (b)	**129.** (b)	**130.** (b)
131. (c)	**132.** (b)	**133.** (b)	**134.** (b)	**135.** (c)	**136.** (d)	**137.** (b)	**138.** (c)	**139.** (a)	**140.** (b)
141. (b)	**142.** (d)	**143.** (a)	**144.** (c)	**145.** (a)	**146.** (b)	**147.** (b)	**148.** (b)	**149.** (c)	**150.** (c)
151. (b)	**152.** (c)	**153.** (a)							

Exercise-2 : One or More than One Answer Is/are Correct

1. Let S is the set of all real x such that $\dfrac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains :

(a) $\left(-\infty,-\dfrac{3}{2}\right)$ (b) $\left(-\dfrac{3}{2},-\dfrac{1}{2}\right)$ (c) $\left(-\dfrac{1}{2},0\right)$ (d) $\left(\dfrac{1}{2},2\right)$

2. If $kx^2-4x+3k+1>0$ for atleast one $x>0$, then if $k\in S$, then S contains:

(a) $(1,\infty)$ (b) $(0,\infty)$ (c) $(-1,\infty)$ (d) $\left(-\dfrac{1}{4},\infty\right)$

3. The equation $|x^2-x-6|=x+2$ has:

(a) two positive roots (b) two real roots
(c) three real roots (d) four real roots

4. If the roots of the equation $x^2-ax-b=0$ $(a,\ b\in R)$ are both lying between -2 and 2, then :

(a) $|a|<2-\dfrac{b}{2}$ (b) $|a|>2-\dfrac{b}{2}$
(c) $|a|<4$ (d) $|a|>\dfrac{b}{2}-2$

5. Consider the equation in real number x and a real parameter λ, $|x-1|-|x-2|+|x-4|=\lambda$ Then for $\lambda\geq 1$, the number of solutions, the equation can have is/are :

(a) 1 (b) 2 (c) 3 (d) 4

6. If a and b are two distinct non-zero real numbers such that $a-b=\dfrac{a}{b}=\dfrac{1}{b}-\dfrac{1}{a}$, then :

(a) $a>0$ (b) $a<0$ (c) $b<0$ (d) $b>0$

7. Let $f(x)=ax^2+bx+c$, $a>0$ and $f(2-x)=f(2+x)\forall x\in R$ and $f(x)=0$ has 2 distinct real roots, then which of the following is true ?

(a) Atleast one root must be positive
(b) $f(2)<f(0)>f(1)$
(c) Minimum value of $f(x)$ is negative
(d) Vertex of graph of $y=f(x)$ lies in 3rd quadrat

8. In the above problem, if roots of equation $f(x)=0$ are non-real complex, then which of the following is false ?

(a) $f(x)=\sin\dfrac{\pi x}{4}$ must have 2 solutions
(b) $4a-2b+c<0$
(c) If $\log_{f(2)} f(3)$ is not defined, then $f(x)\geq 1\ \forall\ x\in R$
(d) All a,b,c are positive

9. If exactly two integers lie between the roots of equation $x^2+ax-1=0$. Then integral value(s) of 'a' is/are :

(a) -1 (b) -2 (c) 1 (d) 2

10. If the minimum value of the quadratic expression $y = ax^2 + bx + c$ is negative attained at negative value of x, then :

(a) $a > 0$ (b) $b > 0$ (c) $c > 0$ (d) $D > 0$

(where D is discriminant)

11. The quadratic expression $ax^2 + bx + c > 0 \; \forall \; x \in R$, then:

(a) $13a - 5b + 2c > 0$ (b) $13a - b + 2c > 0$

(c) $c > 0, \; D < 0$ (d) $a + c > b, \; D < 0$

(where D is discriminant)

12. The possible positive integral value of 'k' for which $5x^2 - 2kx + 1 < 0$ has exactly one integral solution may be divisible by :

(a) 2 (b) 3 (c) 5 (d) 7

13. If the equation $x^2 + px + q = 0$, the coefficient of x was incorrectly written as 17 instead of 13. Then roots were found to be -2 and -15. The correct roots are :

(a) -1 (b) -3 (c) -5 (d) -10

14. If $x^2 - 3x + 2 > 0$ and $x^2 - 3x - 4 \le 0$, then :

(a) $|x| \le 2$ (b) $2 \le x \le 4$ (c) $-1 \le x < 1$ (d) $2 < x \le 4$

15. If $5^x + (2\sqrt{3})^{2x} - 169 \le 0$ is true for x lying in the interval :

(a) $(-\infty, 2)$ (b) $(0, 2]$ (c) $(2, \infty)$ (d) $(0, 4)$

16. Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$ be two quadratic polynomials with real coefficients and satisfy $ac = 2(b + d)$. Then which of the following is (are) correct?

(a) Exactly one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(b) Atleast one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(c) Both $f(x) = 0$ and $g(x) = 0$ must have real roots.

(d) Both $f(x) = 0$ and $g(x) = 0$ must have imaginary roots.

17. The expression $\dfrac{1}{\sqrt{x + 2\sqrt{x-1}}} + \dfrac{1}{\sqrt{x - 2\sqrt{x-1}}}$ simplifies to :

(a) $\dfrac{2}{3-x}$ if $1 < x < 2$ (b) $\dfrac{2}{2-x}$ if $1 < x < 2$

(c) $\dfrac{2\sqrt{x-1}}{(x-2)}$ if $x > 2$ (d) $\dfrac{2\sqrt{x-1}}{x+2}$ if $x > 2$

18. If all values of x which satisfies the inequality $\log_{(1/3)}(x^2 + 2px + p^2 + 1) \ge 0$ also satisfy the inequality $kx^2 + kx - k^2 \le 0$ for all real values of k, then all possible values of p lies in the interval :

(a) $[-1, 1]$ (b) $[0, 1]$ (c) $[0, 2]$ (d) $[-2, 0]$

19. Which of the following statement(s) is/are correct?

(a) The number of quadratic equations having real roots which remain unchanged even after squaring their roots is 3.

(b) The number of solutions of the equation $\tan 2\theta + \tan 3\theta = 0$, in the interval $[0, \pi]$ is equal to 6.

(c) For $x_1, x_2, x_3 > 0$, the minimum value of $\frac{2x_1}{x_2} + \frac{128x_3^2}{x_2^2} + \frac{x_2^3}{4x_1x_3^2}$ equals 24.

(d) The locus of the mid-points of chords of the circle $x^2 + y^2 - 2x - 6y - 1 = 0$, which are passing through origin is $x^2 + y^2 - x - 3y = 0$.

20. If $(a, 0)$ is a point on a diameter inside the circle $x^2 + y^2 = 4$. Then $x^2 - 4x - a^2 = 0$ has :

(a) Exactly one real root in $(-1, 0]$
(b) Exactly one real root in $[2, 5]$
(c) Distinct roots greater than -1
(d) Distinct roots less than 5

21. Let $x^2 - px + q = 0$ where $p \in R$, $q \in R$, $pq \neq 0$ have the roots α, β such that $\alpha + 2\beta = 0$, then:

(a) $2p^2 + q = 0$ (b) $2q^2 + p = 0$ (c) $q < 0$ (d) $q > 0$

22. If a, b, c are rational numbers $(a > b > c > 0)$ and quadratic equation $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ has a root in the interval $(-1, 0)$ then which of the following statement(s) is/are correct ?

(a) $a + c < 2b$
(b) both roots are rational
(c) $ax^2 + 2bx + c = 0$ have both roots negative
(d) $cx^2 + 2bx + a = 0$ have both roots negative

23. For the quadratic polynomial $f(x) = 4x^2 - 8ax + a$, the statements(s) which hold good is/are:

(a) There is only one integral 'a' for which $f(x)$ is non-negative $\forall\ x \in R$
(b) For $a < 0$, the number zero lies between the zeroes of the polynomial
(c) $f(x) = 0$ has two distinct solutions in $(0, 1)$ for a $\in \left(\frac{1}{7}, \frac{4}{7}\right)$
(d) The minimum value of $f(x)$ for minimum value of a for which $f(x)$ is non-negative $\forall\, x \in R$ is 0

24. Given a, b, c are three distinct real numbers satisfying the inequality $a - 2b + 4c > 0$ and the equation $ax^2 + bx + c = 0$ has no real roots. Then the possible value(s) of $\frac{4a + 2b + c}{a + 3b + 9c}$ is/are :

(a) 2 (b) -1 (c) 3 (d) $\sqrt{2}$

25. Let $f(x) = x^2 - 4x + c\ \forall\, x \in R$, where c is a real constant, then which of the following is/are true ?

(a) $f(0) > f(1) > f(2)$
(b) $f(2) > f(3) > f(4)$
(c) $f(1) < f(4) < f(-1)$
(d) $f(0) = f(4) > f(3)$

26. If $0 < a < b < c$ and the roots α, β of the equation $ax^2 + bx + c = 0$ are imaginary, then :

(a) $|\alpha| = |\beta|$ (b) $|\alpha| > 1$ (c) $|\beta| < 1$ (d) $|\alpha| = 1$

27. If x satisfies $|x - 1| + |x - 2| + |x - 3| > 6$, then :

(a) $x \in (-\infty, 1)$ (b) $x \in (-\infty, 0)$ (c) $x \in (4, \infty)$ (d) $(2, \infty)$

28. If both roots of the quadratic equation $ax^2 + x + b - a = 0$ are non real and $b > -1$, then which of the following is/are correct?

(a) $a > 0$ (b) $a < b$ (c) $3a > 2 + 4b$ (d) $3a < 2 + 4b$

29. If a, b are two numbers such that $a^2 + b^2 = 7$ and $a^3 + b^3 = 10$, then :

(a) The greatest value of $|a + b| = 5$
(b) The greatest value of $(a + b)$ is 4
(c) The least value of $(a + b)$ is 1
(d) The least value of $|a + b|$ is 1

30. The number of non-negative integral ordered pair(s) (x, y) for which $(xy - 7)^2 = x^2 + y^2$ holds is greater than or equal to :

(a) 1 (b) 2 (c) 3 (d) 4

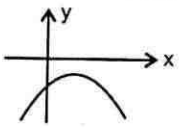
31. If α, β, γ and δ are the roots of the equation $x^4 - bx - 3 = 0$; then an equation whose roots are $\frac{\alpha+\beta+\gamma}{\delta^2}, \frac{\alpha+\beta+\delta}{\gamma^2}, \frac{\alpha+\gamma+\delta}{\beta^2}$ and $\frac{\beta+\gamma+\delta}{\alpha^2}$ is :

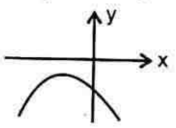
(a) $3x^4 + bx + 1 = 0$
(b) $3x^4 - bx + 1 = 0$
(c) $3x^4 + bx^3 - 1 = 0$
(d) $3x^4 - bx^3 - 1 = 0$

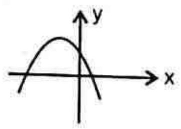
32. The value of k for which both roots of the equation $4x^2 - 2x + k = 0$ are completely in $(-1, 1)$ may be equal to :

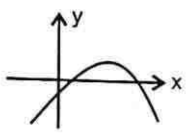
(a) -1 (b) 0 (c) 2 (d) -3

33. If $a, b, c \in R$, then for which of the following graphs of the quadratic polynomial $y = ax^2 - 2bx + c$ $(a \neq 0)$; the product (abc) is negative ?

(a) 

(b) 

(c) 

(d) 

34. If the equation $ax^2 + bx + c = 0$; $a, b, c \in R$ and $a \neq 0$ has no real roots then which of the following is/are always correct ?

(a) $(a + b + c)(a - b + c) > 0$
(b) $(a + b + c)(a - 2b + 4c) > 0$
(c) $(a - b + c)(4a - 2b + c) > 0$
(d) $a(b^2 - 4ac) > 0$

35. If α and β are the roots of the equation $ax^2 + bx + c = 0$; $a, b, c \in R$; $a \neq 0$ then which is (are) correct :

(a) $\alpha^2 + \beta^2 = \frac{b^2 - 2ac}{a^2}$
(b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{b^2 - 2ac}{c^2}$
(c) $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{abc - b^3}{c^3}$
(d) $\alpha\beta(\alpha + \beta) = \frac{-bc}{a^2}$

36. The equation $\cos^2 x - \sin x + \lambda = 0$, $x \in (0, \pi/2)$ has roots then value(s) of λ can be equal to :

(a) 0 (b) -1 (c) $1/2$ (d) 1

37. If the equation $\ln(x^2 + 5x) - \ln(x + a + 3) = 0$ has exactly one solution for x, then possible integral value of a is :

(a) -3 (b) -1 (c) 0 (d) 2

38. The number of non-negative integral ordered pair(s) (x, y) for which $(xy-7)^2 = x^2 + y^2$ holds is greater than or equal to :

(a) 1 (b) 2 (c) 3 (d) 4

39. If $a < 0$, then the value of x satisfying $x^2 - 2a|x-a| - 3a^2 = 0$ is/are

(a) $a(1-\sqrt{2})$ (b) $a(1+\sqrt{2})$ (c) $a(-1-\sqrt{6})$ (d) $a(-1+\sqrt{6})$

40. If $0 < a < b < c$ and the roots α, β of the equation $ax^2 + bx + c = 0$ are imaginary, then

(a) $|\alpha| = |\beta|$ (b) $|\alpha| > 1$ (c) $|\beta| < 1$ (d) $|\alpha| = 1$

41. If x satisfies $|x-1| + |x-2| + |x-3| > 6$, then

(a) $x \in (-\infty, 1)$ (b) $x \in (-\infty, 0)$ (c) $x \in (4, \infty)$ (d) $(2, \infty)$

42. The value of k for which both roots of the equation $4x^2 - 2x + k = 0$ are completely in $(-1, 1)$, may be equal to :

(a) -1 (b) 0 (c) 2 (d) -3

43. Let $\alpha, \beta, \gamma, \delta$ are roots of $x^4 - 12x^3 + \lambda x^2 - 54x + 14 = 0$

If $\alpha + \beta = \gamma + \delta$, then

(a) $\lambda = 45$ (b) $\lambda = -45$

(c) If $\alpha^2 + \beta^2 < \gamma^2 + \delta^2$ then $\dfrac{\alpha\beta}{\gamma\delta} = \dfrac{7}{2}$ (d) If $\alpha^2 + \beta^2 < \gamma^2 + \delta^2 \Rightarrow \dfrac{\alpha\beta}{\gamma\delta} = \dfrac{2}{7}$

44. If $\left(\dfrac{a^3}{a-1}, \dfrac{a^2-3}{a-1}\right); \left(\dfrac{b^3}{b-1}, \dfrac{b^2-3}{b-1}\right); \left(\dfrac{c^3}{c-1}, \dfrac{c^2-3}{c-1}\right)$ lie on $L: lx + my + n = 0$; where a, b, c are real numbers different from 1; then

(a) $a + b + c = -\dfrac{m}{l}$ (b) $abc = \dfrac{m+n}{l}$

(c) $ab + bc + ca = \dfrac{n}{l}$ (d) $abc - (ab + bc + ca) + 3(a + b + c) = 0$

Answers

1.	(a, c, d)	**2.**	(a, b, d)	**3.**	(a, c)	**4.**	(a, c, d)	**5.**	(a, b, c, d)	**6.**	(b, c)
7.	(a, b, c)	**8.**	(a, b, d)	**9.**	(a, c)	**10.**	(a, b, d)	**11.**	(a, b, c, d)	**12.**	(a, c)
13.	(b, d)	**14.**	(c, d)	**15.**	(a, b)	**16.**	(b)	**17.**	(b, c)	**18.**	(a, b, c)
19.	(a, b, d)	**20.**	(a, b, c, d)	**21.**	(a, c)	**22.**	(a, b,c, d)	**23.**	(a, b, d)	**24.**	(a, c, d)
25.	(a, c, d)	**26.**	(a, b)	**27.**	(b, c)	**28.**	(a, b)	**29.**	(a, b, d)	**30.**	(a, b, c, d)
31.	(d)	**32.**	(a, b)	**33.**	(a, c, d)	**34.**	(a, b, d)	**35.**	(a, b, d)	**36.**	(a, c)
37.	(b, c, d)	**38.**	(a, b, c, d)	**39.**	(a, d)	**40.**	(a, b)	**41.**	(b, c)	**42.**	(a, b)
43.	(a, c)	**44.**	(a, c, d)								

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $f(x) = ax^2 + bx + c$, $a \neq 0$, such that $f(-1-x) = f(-1+x)\ \forall x \in R$. Also given that $f(x) = 0$ has no real roots and $4a + b > 0$.

1. Let $\alpha = 4a - 2b + c$, $\beta = 9a + 3b + c$, $\gamma = 9a - 3b + c$, then which of the following is correct ?
   (a) $\beta < \alpha < \gamma$ (b) $\gamma < \alpha < \beta$ (c) $\alpha < \gamma < \beta$ (d) $\alpha < \beta < \gamma$
2. Let $p = b - 4a$, $q = 2a + b$, then pq is :
   (a) negative (b) positive (c) 0 (d) nothing can be said

Paragraph for Question Nos. 3 to 4

If α, β are the roots of equation $(k+1)x^2 - (20k+14)x + 91k + 40 = 0; (\alpha < \beta)\, k > 0$, then answer the following questions.

3. The smaller root (α) lie in the interval :
   (a) (4, 7) (b) (7, 10) (c) (10, 13) (d) None of these
4. The larger root (β) lie in the interval :
   (a) (4, 7) (b) (7, 10) (c) (10, 13) (d) None of these

Paragraph for Question Nos. 5 to 7

Let $f(x) = x^2 + bx + c\ \forall x \in R$, $(b, c \in R)$ attains its least value at $x = -1$ and the graph of $f(x)$ cuts y-axis at $y = 2$.

5. The least value of $f(x)\ \forall x \in R$ is :
   (a) -1 (b) 0 (c) 1 (d) 3/2
6. The value of $f(-2) + f(0) + f(1) =$
   (a) 3 (b) 5 (c) 7 (d) 9
7. If $f(x) = a$ has two distinct real roots, then complete set of values of a is :
   (a) $(1, \infty)$ (b) $(-2, -1)$ (c) $(0, 1)$ (d) $(1, 2)$

Paragraph for Question Nos. 8 to 9

Consider the equation $\log_2^2 x - 4\log_2 x - m^2 - 2m - 13 = 0$, $m \in R$. Let the real roots of the equation be x_1, x_2 such that $x_1 < x_2$.

8. The set of all values of m for which the equation has real roots is :
   (a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) $[1, \infty)$ (d) $(-\infty, \infty)$

9. The sum of maximum value of x_1 and minimum value of x_2 is :

(a) $\dfrac{513}{8}$ (b) $\dfrac{513}{4}$ (c) $\dfrac{1025}{8}$ (d) $\dfrac{257}{4}$

Paragraph for Question Nos. 10 to 11

The equation $x^4 - 2x^3 - 3x^2 + 4x - 1 = 0$ has four distinct real roots x_1, x_2, x_3, x_4 such that $x_1 < x_2 < x_3 < x_4$ and product of two roots is unity, then :

10. $x_1x_2 + x_1x_3 + x_2x_4 + x_3x_4 =$

(a) 0 (b) 1 (c) $\sqrt{5}$ (d) -1

11. $x_2^3 + x_4^3 =$

(a) $\dfrac{2+5\sqrt{5}}{8}$ (b) -4 (c) $\dfrac{27\sqrt{5}+5}{4}$ (d) 18

Paragraph for Question Nos. 12 to 14

Let $f(x)$ be a polynomial of degree 5 with leading coefficient unity, such that $f(1) = 5$, $f(2) = 4$, $f(3) = 3$, $f(4) = 2$ and $f(5) = 1$, then :

12. $f(6)$ is equal to :

(a) 120 (b) -120 (c) 0 (d) 6

13. Sum of the roots of $f(x)$ is equal to :

(a) 15 (b) -15 (c) 21 (d) can't be determine

14. Product of the roots of $f(x)$ is equal to :

(a) 120 (b) -120 (c) 114 (d) -114

Paragraph for Question Nos. 15 to 16

Consider the cubic equation in x, $x^3 - x^2 + (x - x^2)\sin\theta + (x - x^2)\cos\theta + (x-1)\sin\theta\cos\theta = 0$ whose roots are α, β, γ.

15. The value of $\left(\dfrac{\alpha}{2}\right)^2 + \left(\dfrac{\beta}{2}\right)^2 + \left(\dfrac{\gamma}{2}\right)^2 =$

(a) 1 (b) $\dfrac{1}{2}$

(c) $2\cos\theta$ (d) $\dfrac{1}{2}(\sin\theta + \cos\theta + \sin\theta\cos\theta)$

16. Number of values of θ in $[0, 2\pi]$ for which at least two roots are equal, is :

(a) 2 (b) 3 (c) 4 (d) 5

Paragraph for Question Nos. 17 to 18

Let $P(x)$ be a quadratic polynomial with real coefficients such that for all real x the relation $2(1+P(x)) = P(x-1)+P(x+1)$ holds.
If $P(0) = 8$ and $P(2) = 32$ then :

17. The sum of all the coefficient of $P(x)$ is :
(a) 20 (b) 19 (c) 17 (d) 15

18. If the range of $P(x)$ is $[m, \infty)$, then the value of m is :
(a) -12 (b) 15 (c) -17 (d) -5

Paragraph for Question Nos. 19 to 21

Let t be a real number satisfying $2t^3 - 9t^2 + 30 - \lambda = 0$ where $t = x + \dfrac{1}{x}$ and $\lambda \in R$.

19. If the above cubic has three real and distinct solutions for x then exhaustive set of value of λ be :
(a) $3 < \lambda < 10$ (b) $3 < \lambda < 30$ (c) $\lambda = 10$ (d) None of these

20. If the cubic has exactly two real and distinct solutions for x then exhaustive set of values of λ be :
(a) $\lambda \in (-\infty, 3) \cup (30, \infty)$ (b) $\lambda \in (-\infty, -22) \cup (10, \infty) \cup \{3\}$
(c) $\lambda \in \{3, 30\}$ (d) None of these

21. If the cubic has four real and distinct solutions for x then exhaustive set of values of λ be :
(a) $\lambda \in (3, 10)$ (b) $\lambda \in \{3, 10\}$
(c) $\lambda \in (-\infty, -22) \cup (10, \infty)$ (d) None of these

Paragraph for Question Nos. 22 to 23

Consider a quadratic expression $f(x) = tx^2 - (2t-1)x + (5t-1)$

22. If $f(x)$ can take both positive and negative values then t must lie in the interval
(a) $\left(\dfrac{-1}{4}, \dfrac{1}{4}\right)$ (b) $\left(-\infty, \dfrac{-1}{4}\right) \cup \left(\dfrac{1}{4}, \infty\right)$ (c) $\left(\dfrac{-1}{4}, \dfrac{1}{4}\right) - \{0\}$ (d) $(-4, 4)$

23. If $f(x)$ is non-negative $\forall\, x \geq 0$ then t lies in the interval
(a) $\left[\dfrac{1}{5}, \dfrac{1}{4}\right]$ (b) $\left[\dfrac{1}{4}, \infty\right)$ (c) $\left[\dfrac{-1}{4}, \dfrac{1}{4}\right]$ (d) $\left[\dfrac{1}{5}, \infty\right)$

Answers

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(a)	**8.**	(d)	**9.**	(d)	**10.**	(b)
11.	(d)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(b)	**16.**	(d)	**17.**	(b)	**18.**	(c)	**19.**	(c)	**20.**	(b)
21.	(a)	**22.**	(c)	**23.**	(d)														

Exercise-4 : Matching Type Problems

1.

(A)	The least positive integer x, for which $\frac{2x-1}{2x^3+3x^2+x}$ is positive, is equal to	**(P)**	$\frac{4}{3}$
(B)	If the quadratic equation $3x^2+2(a^2+1)x+(a^2-3a+2)=0$ possess roots of opposite sign then a can be equal to	**(Q)**	1
(C)	The roots of the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ can be equal to	**(R)**	6
(D)	If the roots of the equation $x^4-8x^3+bx^2-cx+16=0$ are all real and positive then $2(c-b)$ is equal to	**(S)**	16
		(T)	10

2. Given the inequality $ax+k^2>0$. The complete set of values of 'a' so that

(A)	The inequality is valid for all values of x and k is	**(P)**	R
(B)	There exists a value of x such that the inequality is valid for any value of k is	**(Q)**	ϕ
(C)	There exists a value of k such that the inequality is valid for all values of x is	**(R)**	$\{0\}$
(D)	There exists values of x and k for which inequality is valid is	**(S)**	$R-\{0\}$
		(T)	$\{1\}$

3.

(A)	The real root(s) of the equation $x^4-8x^2-9=0$ are	**(P)**	No real roots
(B)	The real root(s) of the equation $x^{2/3}+x^{1/3}-2=0$ are	**(Q)**	$-3, 3$
(C)	The real root(s) of the equation $\sqrt{3x+1}+1=\sqrt{x}$ are	**(R)**	$-8, 1$

(D)	The real root(s) of the equation $9^x - 10(3^x) + 9 = 0$ are	**(S)**	0, 2

4.

(A)	If a, b are the roots of equation $x^2 + ax + b = 0$ $(a, b \in R)$, then the number of ordered pairs (a, b) is equal to	**(P)**	1
(B)	If $P = \text{cosec}\frac{\pi}{8} + \text{cosec}\frac{2\pi}{8} + \text{cosec}\frac{3\pi}{8} + \text{cosec}\frac{13\pi}{8} + \text{cosec}\frac{14\pi}{8} + \text{cosec}\frac{15\pi}{8}$ and $Q = 8\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$, then $P + Q$ is equal to	**(Q)**	2
(C)	Let $a_1, a_2, a_3 \ldots\ldots$ be positive terms of a G.P. and $a_4, 1, 2, a_{10}$ are the consecutive terms of another G.P. If $\prod_{i=2}^{12} a_i = 4^{\frac{m}{n}}$ where m and n are coprime, then $(m + n)$ equals	**(R)**	3
(D)	For $x, y \in R$, if $x^2 - 2xy + 2y^2 - 6y + 9 = 0$, then the value of $5x - 4y$ is equal to	**(S)**	15

Answers

1.	A → Q ; B → P ; C → R ; D → S
2.	A → Q ; B → S ; C → R ; D → P
3.	A → Q; B → R; C → P; D → S
4.	A → Q; B → P; C → S; D → R

Exercise-5 : Subjective Type Problems

1. Let $f(x) = ax^2 + bx + c$ where a, b, c are integers. If $\sin\frac{\pi}{7}\cdot\sin\frac{3\pi}{7} + \sin\frac{3\pi}{7}\cdot\sin\frac{5\pi}{7} + \sin\frac{5\pi}{7}\cdot\sin\frac{\pi}{7} = f\left(\cos\frac{\pi}{7}\right)$, then find the value of $f(2)$:
2. Let a, b, c, d be distinct integers such that the equation $(x-a)(x-b)(x-c)(x-d)-9=0$ has an integer root 'r', then the value of $a+b+c+d-4r$ is equal to :
3. Consider the equation $(x^2+x+1)^2-(m-3)(x^2+x+1)+m=0$, where m is a real parameter. The number of positive integral values of m for which equation has two distinct real roots, is :
4. The number of positive integral values of m, $m \le 16$ for which the equation given in the above questions has 4 distinct real root is :
5. If the equation $(m^2-12)x^4-8x^2-4=0$ has no real roots, then the largest value of m is $p\sqrt{q}$ where p, q are coprime natural numbers, then $p+q=$
6. The least positive integral value of 'x' satisfying $(e^x-2)\left(\sin\left(x+\frac{\pi}{4}\right)\right)(x-\log_e 2)(\sin x-\cos x)<0$ is :
7. The integral values of x for which $x^2+17x+71$ is perfect square of a rational number are a and b, then $|a-b|=$
8. Let $P(x)=x^6-x^5-x^3-x^2-x$ and $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4-x^3-x^2-1=0$, then $P(\alpha)+P(\beta)+P(\gamma)+P(\delta)=$
9. The number of real values of 'a' for which the largest value of the function $f(x)=x^2+ax+2$ in the interval $[-2, 4]$ is 6 will be :
10. The number of all values of n, (where n is a whole number) for which the equation $\frac{x-8}{n-10}=\frac{n}{x}$ has no solution.
11. The number of negative integral values of m for which the expression $x^2+2(m-1)x+m+5$ is positive $\forall\, x>1$ is :
12. If the expression $ax^4+bx^3-x^2+2x+3$ has the remainder $4x+3$ when divided by x^2+x-2, then $a+4b=$.....
13. Find the smallest value of k for which both the roots of equation $x^2-8kx+16(k^2-k+1)=0$ are real, distinct and have values atleast 4.
14. If x^2-3x+2 is a factor of $x^4-px^2+q=0$, then $p+q=$
15. The sum of all real values of k for which the expression $x^2+2xy+ky^2+2x+k=0$ can be resolved into linear factors is :
16. The curve $y=(a+1)x^2+2$ meets the curve $y=ax+3$, $a \ne -1$ in exactly one point, then $a^2=$

17. Find the number of integral values of 'a' for which the range of function $f(x)=\dfrac{x^2-ax+1}{x^2-3x+2}$ is $(-\infty, \infty)$.

18. When x^{100} is divided by x^2-3x+2, the remainder is $(2^{k+1}-1)x-2(2^k-1)$, then $k=$

19. Let $P(x)$ be a polynomial equation of least possible degree, with rational coefficients, having $\sqrt[3]{7}+\sqrt[3]{49}$ as one of its roots. Then the product of all the roots of $P(x)=0$ is :

20. The range of values k for which the equation $2\cos^4 x-\sin^4 x+k=0$ has atleast one solution is $[\lambda, \mu]$. Find the value of $(9\mu+\delta)$.

21. Let $P(x)$ be a polynomial with real coefficient and $P(x)-P'(x)=x^2+2x+1$. Find $P(1)$.

22. Find the smallest positive integral value of a for which the greater root, of the equation $x^2-(a^2+a+1)x+a(a^2+1)=0$ lies between the roots of the equation $x^2-a^2x-2(a^2-2)=0$

23. If the equation $x^4+kx^2+k=0$ has exactly two distinct real roots, then the smallest integral value of $|k|$ is :

24. Let a, b, c, d be the roots of $x^4-x^3-x^2-1=0$. Also consider $P(x)=x^6-x^5-x^3-x^2-x$, then the value of $P(a)+P(b)+P(c)+P(d)$ is equal to :

25. The number of integral values of a, $a\in[-5, 5]$ for which the equation $x^2+2(a-1)x+a+5=0$ has one root smaller than 1 and the other root greater than 3 is :

26. The number of non-negative integral values of n, $n\le 10$ so that a root of the equation $n^2\sin^2 x-2\sin x-(2n+1)=0$ lies in interval $\left[0, \dfrac{\pi}{2}\right]$ is :

27. Let $f(x)=ax^2+bx+c$, where a, b, c are integers and $a>1$. If $f(x)$ takes the value p, a prime for two distinct integer values of x, then the number of integer values of x for which $f(x)$ takes the value $2p$ is :

28. If x and y are real numbers connected by the equation $9x^2+2xy+y^2-92x-20y+244=0$, then the sum of maximum value of x and the minimum value of y is :

29. Consider two numbers a, b, sum of which is 3 and the sum of their cubes is 7. Then sum of all possible distinct values of a is :

30. If $y^2(y^2-6)+x^2-8x+24=0$ and the minimum value of x^2+y^4 is m and maximum value is M; then find the value of $M-2m$.

31. Consider the equation $x^3-ax^2+bx-c=0$, where a, b, c are rational number, $a\ne 1$. It is given that x_1, x_2 and x_1x_2 are the real roots of the equation. If $(b+c)=2(a+1)$, then $x_1x_2\left(\dfrac{a+1}{b+c}\right)=$

32. Let α satisfy the equation $x^3+3x^2+4x+5=0$ and β satisfy the equation $x^3-3x^2+4x-5=0$, $\alpha, \beta\in R$, then $\alpha+\beta=$

33. Let x, y and z are positive reals and $x^2 + xy + y^2 = 2; y^2 + yz + z^2 = 1$ and $z^2 + zx + x^2 = 3$. If the value of $xy + yz + zx$ can be expressed as $\sqrt{\frac{p}{q}}$ where p and q are relatively prime positive integer find the value of $p - q$:

34. The number of ordered pairs (a, b), where a, b are integers satisfying the inequality $\min(x^2 + (a-b)x + (1-a-b)) > \max(-x^2 + (a+b)x - (1+a+b))\ \forall\ x \in R$, is :

35. The real value of x satisfying $\sqrt[3]{20x + \sqrt[3]{20x + 13}} = 13$ can be expressed as $\frac{a}{b}$ where a and b are relatively prime positive integers. Find the value of b ?

36. If the range of the values of a for which the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$ is (p, q), then find the value of $\left(q - \frac{1}{p}\right)$.

37. Find the number of positive integers satisfying the inequality $x^2 - 10x + 16 < 0$.

38. If $\sin\theta$ and $\cos\theta$ are the roots of the quadratic equation $ax^2 + bx + c = 0\ (ac \neq 0)$. Then find the value of $\frac{b^2 - a^2}{ac}$.

39. Let the inequality $\sin^2 x + a\cos x + a^2 \geq 1 + \cos x$ is satisfied $\forall\ x \in R$, for $a \in (-\infty, k_1] \cup [k_2, \infty)$, then $|k_1| + |k_2| =$

40. α and β are roots of the equation $2x^2 - 35x + 2 = 0$. Find the value of $\sqrt{(2\alpha - 35)^3 (2\beta - 35)^3}$

41. The sum of all integral values of 'a' for which the equation $2x^2 - (1 + 2a)x + 1 + a = 0$ has a integral root.

42. Let $f(x)$ be a polynomial of degree 8 such that $F(r) = \frac{1}{r}, r = 1, 2, 3, \ldots\ldots, 8, 9$, then $\frac{1}{F(10)} =$

43. Let α, β are two real roots of equation $x^2 + px + q = 0, p, q \in R, q \neq 0$. If the quadratic equation $g(x) = 0$ has two roots $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$ such that sum of its roots is equal to product of roots, then then number of integral values q can attain is :

44. If $\cos A$, $\cos B$ and $\cos C$ are the roots of cubic $x^3 + ax^2 + bx + c = 0$, where A, B, C are the angles of a triangle then find the value of $a^2 - 2b - 2c$.

45. Find the number of positive integral values of k for which $kx^2 + (k-3)x + 1 < 0$ for atleast one positive x.

Answers

1.	9	**2.**	0	**3.**	1	**4.**	7	**5.**	5	**6.**	3	**7.**	3	**8.**	6	**9.**	0	**10.**	6
11.	0	**12.**	9	**13.**	2	**14.**	9	**15.**	2	**16.**	4	**17.**	0	**18.**	99	**19.**	56	**20.**	7
21.	2	**22.**	3	**23.**	1	**24.**	6	**25.**	4	**26.**	8	**27.**	0	**28.**	7	**29.**	3	**30.**	4
31.	1	**32.**	0	**33.**	5	**34.**	9	**35.**	5	**36.**	5	**37.**	5	**38.**	2	**39.**	3	**40.**	8
41.	1	**42.**	5	**43.**	3	**44.**	1	**45.**	0										

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Chapter 9 – Sequence and Series

9

SEQUENCE AND SERIES

Exercise-1 : Single Choice Problems

1. If a, b, c are positive numbers and $a+b+c=1$, then the maximum value of $(1-a)(1-b)(1-c)$ is :

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{8}{27}$ (d) $\frac{4}{9}$

2. If $xyz=(1-x)(1-y)(1-z)$ where $0 \le x, y, z \le 1$, then the minimum value of $x(1-z)+y(1-x)+z(1-y)$ is :

(a) $\frac{3}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

3. If $\sec(\alpha-2\beta)$, $\sec\alpha$, $\sec(\alpha+2\beta)$ are in arithmetical progression then $\cos^2\alpha=\lambda\cos^2\beta$ $(\beta \ne n\pi;\ n \in I)$ the value of λ is :

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

4. Let a, b, c, d, e are non-zero and distinct positive real numbers. If a, b, c are in A.P. ; b, c, d are in G.P. and c, d, e are in H.P., then a, c, e are in :

(a) A.P. (b) G.P.

(c) H.P. (d) Nothing can be said

5. If $(m+1)^{\text{th}}$, $(n+1)^{\text{th}}$, and $(r+1)^{\text{th}}$ terms of a non-constant A.P. are in G.P. and m, n, r are in H.P., then the ratio of first term of the A.P to its common difference is :

(a) $-\frac{n}{2}$ (b) $-n$ (c) $-2n$ (d) $+n$

6. If the equation $x^4-4x^3+ax^2+bx+1=0$ has four positive roots, then the value of $(a+b)$ is :

(a) -4 (b) 2

(c) 6 (d) can not be determined

7. If S_1, S_2 and S_3 are the sums of first n natural numbers, their squares and their cubes respectively, then $\dfrac{S_1^4 S_2^2 - S_2^2 S_3^2}{S_1^2 + S_3^2} =$

(a) 4 (b) 2 (c) 1 (d) 0

8. If $S_n = \dfrac{1\cdot 2}{3!} + \dfrac{2\cdot 2^2}{4!} + \dfrac{3\cdot 2^3}{5!} + \ldots\ldots$ upto n terms then the sum of the infinite terms is :

(a) 1 (b) $\dfrac{2}{3}$ (c) e (d) $\dfrac{\pi}{4}$

9. If $\tan\left(\dfrac{\pi}{12} - x\right)$, $\tan\dfrac{\pi}{12}$, $\tan\left(\dfrac{\pi}{12} + x\right)$ in order are three consecutive terms of a G.P. then sum of all the solutions in $[0, 314]$ is $k\pi$. The value of k is :

(a) 4950 (b) 5050 (c) 2525 (d) 5010

10. Let $S_k = 1 + 2 + 3 + \ldots\ldots + k$ and $Q_n = \dfrac{S_2}{S_2 - 1}\cdot\dfrac{S_3}{S_3 - 1}\cdot\dfrac{S_4}{S_4 - 1}\ldots\ldots\dfrac{S_n}{S_n - 1}$, where $k, n \in N$

$\lim_{n\to\infty} Q_n =$

(a) $\dfrac{1}{3}$ (b) 1 (c) 3 (d) 0

11. l, m, n are the p^{th}, q^{th} and r^{th} term of a G.P. all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals :

(a) -1 (b) 2 (c) 1 (d) 0

12. The number of natural numbers < 300 that are divisible by 6 but not by 9 is :

(a) 49 (b) 37 (c) 33 (d) 16

13. If $x, y, z > 0$ and $x + y + z = 1$ then $\dfrac{xyz}{(1-x)(1-y)(1-z)}$ is necessarily.

(a) ≥ 8 (b) $\leq \dfrac{1}{8}$ (c) 1 (d) None of these

14. If the roots of the equation $px^2 + qx + r = 0$, where $2p$, q, $2r$ are in G.P., are of the form α^2, $4\alpha - 4$. Then the value of $2p + 4q + 7r$ is :

(a) 0 (b) 10 (c) 14 (d) 18

15. Let $x_1, x_2, x_3, \ldots\ldots, x_k$ be the divisors of positive integer n (including 1 and n). If $x_1 + x_2 + x_3 + \ldots\ldots + x_k = 75$. Then $\sum_{i=1}^{k}\left(\dfrac{1}{x_i}\right)$ is equal to :

(a) $\dfrac{75}{k}$ (b) $\dfrac{75}{n}$ (c) $\dfrac{1}{n}$ (d) $\dfrac{1}{75}$

16. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in H.P. and $f(k) = \sum_{r=1}^{n} a_r - a_k$ then $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)}, \ldots\ldots, \frac{a_n}{f(n)}$ are in:

(a) A.P. (b) G.P. (c) H.P. (d) None of these

17. If α, β be roots of the equation $375x^2 - 25x - 2 = 0$ and $s_n = \alpha^n + \beta^n$, then $\lim_{n \to \infty} \left(\sum_{r=1}^{n} S_r \right) = \ldots\ldots$

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) 1

18. If $a_i, i = 1, 2, 3, 4$ be four real members of the same sign, then the minimum value of $\sum \frac{a_i}{a_j}, i, j \in \{1, 2, 3, 4\}, i \neq j$ is :

(a) 6 (b) 8 (c) 12 (d) 24

19. Given that x, y, z are positive reals such that $xyz = 32$. The minimum value of $x^2 + 4xy + 4y^2 + 2z^2$ is equal to :

(a) 64 (b) 256 (c) 96 (d) 216

20. In an A.P., five times the fifth term is equal to eight times the eighth term. Then the sum of the first twenty five terms is equal to :

(a) 25 (b) $\frac{25}{2}$ (c) -25 (d) 0

21. Let α, β be two distinct values of x lying in $[0, \pi]$ for which $\sqrt{5}\sin x$, $10\sin x$, $10(4\sin^2 x + 1)$ are 3 consecutive terms of a G.P. Then minimum value of $|\alpha - \beta| =$

(a) $\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{2\pi}{5}$ (d) $\frac{3\pi}{5}$

22. In an infinite G.P., the sum of first three terms is 70. If the extreme terms are multiplied by 4 and the middle term is multiplied by 5, the resulting terms form an A.P. then the sum to infinite terms of G.P. is :

(a) 120 (b) 40 (c) 160 (d) 80

23. The value of the sum $\sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{k}{2^{n+k}}$ is equal to :

(a) 5 (b) 4 (c) 3 (d) 2

24. Let p, q, r are positive real numbers, such that $27pqr \geq (p + q + r)^3$ and $3p + 4q + 5r = 12$, then $p^3 + q^4 + r^5 =$

(a) 3 (b) 6 (c) 2 (d) 4

25. Find the sum of the infinite series $\frac{1}{9} + \frac{1}{18} + \frac{1}{30} + \frac{1}{45} + \frac{1}{63} + \ldots\ldots$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{2}{3}$

26. If S_r denote the sum of first 'r' terms of a non constant A.P. and $\frac{S_a}{a^2} = \frac{S_b}{b^2} = c$, where a, b, c are distinct then S_c =

(a) c^2 (b) c^3 (c) c^4 (d) abc

27. In an infinite G.P. second term is x and its sum is 4, then complete set of values of 'x' is :

(a) $(-8, 0)$ (b) $\left[-\frac{1}{8}, \frac{1}{8}\right) - \{0\}$

(c) $\left[-1, -\frac{1}{8}\right) \cup \left(\frac{1}{8}, 1\right]$ (d) $(-8, 1] - \{0\}$

28. The number of terms of an A.P. is odd. The sum of the odd terms (1^{st}, 3^{rd} etc.,) is 248 and the sum of the even terms is 217. The last term exceeds the first by 56, then :

(a) the number of terms is 17 (b) the first term is 3

(c) the number of terms is 13 (d) the first term is 1

29. Let $A_1, A_2, A_3, \ldots\ldots, A_n$ be squares such that for each $n \geq 1$ the length of a side of A_n equals the length of a diagonal of A_{n+1}. If the side of A_1 be 20 units then the smallest value of 'n' for which area of A_n is less than 1.

(a) 7 (b) 8 (c) 9 (d) 10

30. Let $S_k = \sum_{i=0}^{\infty} \frac{1}{(k+1)^i}$, then $\sum_{k=1}^{n} kS_k$ equal :

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n(n+2)}{2}$ (d) $\frac{n(n+3)}{2}$

31. The sum of the series $\frac{2}{1\cdot 2} + \frac{5}{2\cdot 3}2^1 + \frac{10}{3\cdot 4}2^2 + \frac{17}{4\cdot 5}2^3 + \ldots\ldots$ upto n terms is equal :

(a) $\frac{n2^n}{n+1}$ (b) $\left(\frac{n}{n+1}\right)2^n + 1$ (c) $\frac{n2^n}{n+1} - 1$ (d) $\frac{(n-1)2^n}{n+1}$

32. If $(1\cdot 5)^{30} = k$, then the value of $\sum_{n=2}^{29} (1\cdot 5)^n$, is :

(a) $2k - 3$ (b) $k + 1$ (c) $2k + 7$ (d) $2k - \frac{9}{2}$

33. n arithmetic means are inserted between 7 and 49 and their sum is found to be 364, then n is :

(a) 11 (b) 12 (c) 13 (d) 14

34. The third term of a G.P. is 2. Then the product of the first five terms, is :

(a) 2^3 (b) 2^4 (c) 2^5 (d) none of these

35. The sum of first n terms of an A.P. is $5n^2 + 4n$, its common difference is :

(a) 9 (b) 10 (c) 3 (d) -4

36. If $x + y = a$ and $x^2 + y^2 = b$, then the value of $(x^3 + y^3)$, is :

(a) ab (b) $a^2 + b$ (c) $a + b^2$ (d) $\frac{3ab - a^3}{2}$

37. If $S_1, S_2, S_3, \ldots\ldots, S_n$ are the sum of infinite geometric series whose first terms are 1, 3, 5,……, $(2n-1)$ and whose common ratios are $\frac{2}{3}, \frac{2}{5}, \ldots\ldots, \frac{2}{2n+1}$ respectively, then

$$\left\{\frac{1}{S_1S_2S_3} + \frac{1}{S_2S_3S_4} + \frac{1}{S_3S_4S_5} + \ldots\ldots \text{upto infinite terms}\right\} =$$

(a) $\frac{1}{15}$ (b) $\frac{1}{60}$ (c) $\frac{1}{12}$ (d) $\frac{1}{3}$

38. Sequence $\{t_n\}$ of positive terms is a G.P. If t_6, 2, 5, t_{14} form another G.P. in that order, then the product $t_1t_2t_3\ldots\ldots t_{18}t_{19}$ is equal to :

(a) 10^9 (b) 10^{10} (c) $10^{17/2}$ (d) $10^{19/2}$

39. The minimum value of $\frac{(A^2 + A + 1)(B^2 + B + 1)(C^2 + C + 1)(D^2 + D + 1)}{ABCD}$ where $A, B, C, D > 0$ is :

(a) $\frac{1}{3^4}$ (b) $\frac{1}{2^4}$ (c) 2^4 (d) 3^4

40. If $\sum_1^{20} r^3 = a$, $\sum_1^{20} r^2 = b$ then sum of products of 1, 2, 3, 4...... 20 taking two at a time is :

(a) $\frac{a-b}{2}$ (b) $\frac{a^2 - b^2}{2}$ (c) $a - b$ (d) $a^2 - b^2$

41. The sum of the first $2n$ terms of an A.P. is x and the sum of the next n terms is y, its common difference is :

(a) $\frac{x-2y}{3n^2}$ (b) $\frac{2y-x}{3n^2}$ (c) $\frac{x-2y}{3n}$ (d) $\frac{2y-x}{3n}$

42. The number of non-negative integers 'n' satisfying $n^2 = p + q$ and $n^3 = p^2 + q^2$ where p and q are integers.

(a) 2 (b) 3 (c) 4 (d) Infinite

43. Concentric circles of radii 1, 2, 3 ...... 100 cms are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is equals to:

(a) 1000π (b) 5050π (c) 4950π (d) 5151π

44. If $\log_2 4$, $\log_{\sqrt{2}} 8$ and $\log_3 9^{k-1}$ are consecutive terms of a geometric sequence, then the number of integers that satisfy the system of inequalities $x^2 - x > 6$ and $|x| < k^2$ is :

(a) 193 (b) 194 (c) 195 (d) 196

45. Let T_r be the r^{th} term of an A.P. whose first term is $-\frac{1}{2}$ and common difference is 1, then $\sum_{r=1}^{n}\sqrt{1+T_rT_{r+1}T_{r+2}T_{r+3}} =$

(a) $\frac{n(n+1)(2n+1)}{6}-\frac{5n}{4}$ (b) $\frac{n(n+1)(2n+1)}{6}-\frac{5n}{4}+\frac{1}{4}$

(c) $\frac{n(n+1)(2n+1)}{6}-\frac{5n}{4}+\frac{1}{2}$ (d) $\frac{n(n+1)(2n+1)}{12}-\frac{5n}{8}+1$

46. If $\sum_{r=1}^{n} T_r = \frac{n(n+1)(n+2)}{3}$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{2008}{T_r} =$

(a) 2008 (b) 3012 (c) 4016 (d) 8032

47. The sum of the infinite series, $1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}+\frac{5^2}{5^4}-\frac{6^2}{5^5}+.....$ is :

(a) $\frac{1}{2}$ (b) $\frac{25}{24}$ (c) $\frac{25}{54}$ (d) $\frac{125}{252}$

48. The absolute term in $P(x)=\sum_{r=1}^{n}\left(x-\frac{1}{r}\right)\left(x-\frac{1}{r+1}\right)\left(x-\frac{1}{r+2}\right)$ as n approaches to infinity is :

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$

49. Let a, b, c are positive real numbers such that $p=a^2b+ab^2-a^2c-ac^2$; $q=b^2c+bc^2-a^2b-ab^2$ and $r=ac^2+a^2c-cb^2-bc^2$ and the quadratic equation $px^2+qx+r=0$ has equal roots ; then a, b, c are in :

(a) A.P. (b) G.P. (c) H.P. (d) None of these

50. If T_k denotes the k^{th} term of an H.P. from the begining and $\frac{T_2}{T_6}=9$, then $\frac{T_{10}}{T_4}$ equals :

(a) $\frac{17}{5}$ (b) $\frac{5}{17}$ (c) $\frac{7}{19}$ (d) $\frac{19}{7}$

51. Number of terms common to the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ..., 466 is :

(a) 19 (b) 20 (c) 21 (d) 22

52. The sum of the series $1+\frac{2}{3}+\frac{1}{3^2}+\frac{2}{3^3}+\frac{1}{3^4}+\frac{2}{3^5}+\frac{1}{3^6}+\frac{2}{3^7}+......$ upto infinite terms is equal to :

(a) $\frac{15}{8}$ (b) $\frac{8}{15}$ (c) $\frac{27}{8}$ (d) $\frac{21}{8}$

53. The coefficient of x^8 in the polynomial $(x-1)(x-2)(x-3)......(x-10)$ is :

(a) 2640 (b) 1320 (c) 1370 (d) 2740

54. Let $\alpha = \lim_{n\to\infty} \frac{(1^3 - 1^2) + (2^3 - 2^2) + \ldots + (n^3 - n^2)}{n^4}$, then α is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) non-existent

55. If $16x^4 - 32x^3 + ax^2 + bx + 1 = 0$, $a, b \in R$ has positive real roots only, then $a - b$ is equal to :

(a) -32 (b) 32 (c) 49 (d) -49

56. If ABC is a triangle and $\tan\frac{A}{2}$, $\tan\frac{B}{2}$, $\tan\frac{C}{2}$ are in H.P., then the minimum value of $\cot\frac{B}{2} =$

(a) $\sqrt{3}$ (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

57. If α and β are the roots of the quadratic equation $4x^2 + 2x - 1 = 0$ then the value of $\sum_{r=1}^{\infty} (\alpha^r + \beta^r)$ is :

(a) 2 (b) 3 (c) 6 (d) 0

58. The sum of the series $2^2 + 2\,(4)^2 + 3(6)^2 + \ldots\ldots$ upto 10 terms is equal to :

(a) 11300 (b) 12100 (c) 12300 (d) 11200

59. If a and b are positive real numbers such that $a + b = 6$, then the minimum value of $\left(\frac{4}{a} + \frac{1}{b}\right)$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{3}{2}$

60. The first term of an infinite G.P. is the value of x satisfying the equation $\log_4(4^x - 15) + x - 2 = 0$ and the common ratio is $\cos\left(\frac{2011\pi}{3}\right)$. The sum of G.P. is :

(a) 1 (b) $\frac{4}{3}$ (c) 4 (d) 2

61. Let a, b, c be positive numbers, then the minimum value of $\frac{a^4 + b^4 + c^2}{abc}$ is :

(a) 4 (b) $2^{3/4}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

62. If $xy = 1$; then minimum value of $x^2 + y^2$ is :

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 4

63. Find the value of $\frac{2}{1^3} + \frac{6}{1^3 + 2^3} + \frac{12}{1^3 + 2^3 + 3^3} + \frac{20}{1^3 + 2^3 + 3^3 + 4^3} + \ldots$ upto 60 terms :

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

64. Evaluate : $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)(n+3)....(n+k)}$

(a) $\frac{1}{(k-1)(k-1)!}$ (b) $\frac{1}{k \cdot k!}$ (c) $\frac{1}{(k-1)k!}$ (d) $\frac{1}{k!}$

65. Consider two positive numbers a and b. If arithmetic mean of a and b exceeds their geometric mean by 3/2 and geometric mean of a and b exceeds their harmonic mean by 6/5 then the value of $a^2 + b^2$ will be :

(a) 150 (b) 153 (c) 156 (d) 159

66. Sum of first 10 terms of the series, $S = \frac{7}{2^2 \cdot 5^2} + \frac{13}{5^2 \cdot 8^2} + \frac{19}{8^2 \cdot 11^2} + ...$ is :

(a) $\frac{255}{1024}$ (b) $\frac{88}{1024}$ (c) $\frac{264}{1024}$ (d) $\frac{85}{1024}$

67. $\sum_{r=1}^{10} \frac{r}{1-3r^2+r^4} =$

(a) $-\frac{50}{109}$ (b) $-\frac{54}{109}$ (c) $-\frac{55}{111}$ (d) $-\frac{55}{109}$

68. Let r^{th} term t_r of a series is given by $t_r = \frac{r}{1+r^2+r^4}$. Then $\lim_{n\to\infty} \sum_{r=1}^{n} t_r$ is equal to :

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{1}{4}$

69. The sum of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ...$ to infinite terms, is :

(a) $\frac{31}{12}$ (b) $\frac{41}{16}$ (c) $\frac{45}{16}$ (d) $\frac{35}{16}$

70. The third term of a G.P. is 2. Then the product of the first five terms, is :

(a) 2^3 (b) 2^4 (c) 2^5 (d) none of these

71. If $x_1, x_2, x_3, ... x_{2n}$ are in A.P., then $\sum_{r=1}^{2n} (-1)^{r+1} x_r^2$ is equal to :

(a) $\frac{n}{(2n-1)}(x_1^2 - x_{2n}^2)$ (b) $\frac{2n}{(2n-1)}(x_1^2 - x_{2n}^2)$

(c) $\frac{n}{n-1}(x_1^2 - x_{2n}^2)$ (d) $\frac{n}{2n+1}(x_1^2 - x_{2n}^2)$

72. Let two numbers have arithmatic mean 9 and geometric mean 4. Then these numbers are roots of the equation :

(a) $x^2 + 18x + 16 = 0$ (b) $x^2 - 18x - 16 = 0$

(c) $x^2 + 18x - 16 = 0$ (d) $x^2 - 18x + 16 = 0$

73. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p+q)$ is :

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

74. A person has to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots\ldots = a_{10} = 150$ and $a_{10}, a_{11}, a_{12}, \ldots\ldots$ are in A.P. with common difference -2, then the time taken by him to count all notes is :

(a) 34 minutes (b) 24 minutes (c) 125 minutes (d) 35 minutes

75. A non constant arithmatic progression has common difference d and first term is $(1 - ad)$. If the sum of the first 20 terms is 20, then the value of a is equal to :

(a) $\frac{2}{19}$ (b) $\frac{19}{2}$ (c) $\frac{2}{9}$ (d) $\frac{9}{2}$

76. The value of $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n} =$

(a) $\frac{1}{120}$ (b) $\frac{1}{96}$ (c) $\frac{1}{24}$ (d) $\frac{1}{144}$

77. Find the value of $\frac{2}{1^3} + \frac{6}{1^3+2^3} + \frac{12}{1^3+2^3+3^3} + \frac{20}{1^3+2^3+3^3+4^3} + \ldots\ldots$ up to infinite terms:

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

78. The minimum value of the expression $2^x + 2^{2x+1} + \frac{5}{2^x}, x \in R$ is :

(a) 7 (b) $(7.2)^{1/7}$ (c) 8 (d) $(3.10)^{1/3}$

79. The value of $\sum_{r=1}^{\infty} \frac{(4r+5)5^{-r}}{r(5r+5)}$ is :

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{25}$ (d) $\frac{2}{25}$

Answers

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(a)	**6.**	(b)	**7.**	(d)	**8.**	(a)	**9.**	(a)	**10.**	(c)
11.	(d)	**12.**	(c)	**13.**	(b)	**14.**	(c)	**15.**	(b)	**16.**	(c)	**17.**	(a)	**18.**	(c)	**19.**	(c)	**20.**	(d)
21.	(b)	**22.**	(d)	**23.**	(d)	**24.**	(a)	**25.**	(a)	**26.**	(b)	**27.**	(d)	**28.**	(b)	**29.**	(d)	**30.**	(d)
31.	(a)	**32.**	(d)	**33.**	(c)	**34.**	(c)	**35.**	(b)	**36.**	(d)	**37.**	(b)	**38.**	(d)	**39.**	(d)	**40.**	(a)
41.	(b)	**42.**	(b)	**43.**	(b)	**44.**	(a)	**45.**	(c)	**46.**	(a)	**47.**	(c)	**48.**	(d)	**49.**	(c)	**50.**	(b)
51.	(b)	**52.**	(a)	**53.**	(b)	**54.**	(b)	**55.**	(b)	**56.**	(a)	**57.**	(d)	**58.**	(b)	**59.**	(d)	**60.**	(c)
61.	(d)	**62.**	(b)	**63.**	(c)	**64.**	(c)	**65.**	(d)	**66.**	(d)	**67.**	(d)	**68.**	(a)	**69.**	(d)	**70.**	(c)
71.	(a)	**72.**	(d)	**73.**	(d)	**74.**	(a)	**75.**	(b)	**76.**	(b)	**77.**	(c)	**78.**	(c)	**79.**	(a)		

Exercise-2 : One or More than One Answer is/are Correct

1. If the first and $(2n-1)^{th}$ terms of an A.P., G.P. and H.P. with positive terms are equal and their n^{th} terms are a, b and c respectively, then which of the following options must be correct :

(a) $a+c=2b$ (b) $a \geq b \geq c$

(c) $\dfrac{2ac}{a+c}=b$ (d) $ac=b^2$

2. Let a, b, c are distinct real numbers such that expression ax^2+bx+c, bx^2+cx+a and cx^2+ax+b are always positive then possible value(s) of $\dfrac{a^2+b^2+c^2}{ab+bc+ca}$ may be :

(a) 1 (b) 2 (c) 3 (d) 4

3. If a, b, c are in H.P., where $a > c > 0$, then :

(a) $b > \dfrac{a+c}{2}$ (b) $\dfrac{1}{a-b}-\dfrac{1}{b-c}<0$

(c) $ac > b^2$ (d) $bc(1-a), ac(1-b), ab(1-c)$ are in A.P.

4. In an A.P., let T_r denote r^{th} term from beginning, $T_p=\dfrac{1}{q(p+q)}$, $T_q=\dfrac{1}{p(p+q)}$, then :

(a) $T_1=$ common difference (b) $T_{p+q}=\dfrac{1}{pq}$

(c) $T_{pq}=\dfrac{1}{p+q}$ (d) $T_{p+q}=\dfrac{1}{p^2q^2}$

5. Which of the following statement(s) is(are) correct?

(a) Sum of the reciprocal of all the n harmonic means inserted between a and b is equal to n times the harmonic mean between two given numbers a and b.

(b) Sum of the cubes of first n natural number is equal to square of the sum of the first n natural numbers.

(c) If $a, A_1, A_2, A_3, \ldots\ldots, A_{2n}, b$ are in A.P. then $\sum_{i=1}^{2n} A_i = n(a+b)$.

(d) If the first term of the geometric progression $g_1, g_2, g_3, \ldots\ldots, \infty$ is unity, then the value of the common ratio of the progression such that $(4g_2+5g_3)$ is minimum equals $\dfrac{2}{5}$.

6. If a, b, c are in 3 distinct numbers in H.P., $a, b, c > 0$, then :

(a) $\dfrac{b+c-a}{a}, \dfrac{c+a-b}{b}, \dfrac{a+b-c}{c}$ are in A.P. (b) $\dfrac{b+c}{a}, \dfrac{c+a}{b}, \dfrac{a+b}{c}$ are in A.P.

(c) $a^5+c^5 \geq 2b^5$ (d) $\dfrac{a-b}{b-c}=\dfrac{a}{c}$

7. All roots of equation $x^5 - 40x^4 + \alpha x^3 + \beta x^2 + \gamma x + \delta = 0$ are in G.P. If the sum of their reciprocals is 10, then δ can be equal to :

(a) 32 (b) -32 (c) $\frac{1}{32}$ (d) $-\frac{1}{32}$

8. Let $a_1, a_2, a_3, \ldots\ldots$ be a sequence of non-zero real numbers which are in A.P. for $k \in N$. Let $f_k(x) = a_k x^2 + 2a_{k+1}x + a_{k+2}$

(a) $f_k(x) = 0$ has real roots for each $k \in N$.

(b) Each of $f_k(x) = 0$ has one root in common.

(c) Non-common roots of $f_1(x) = 0, f_2(x) = 0, f_3(x) = 0, \ldots\ldots$ from an A.P.

(d) None of these

9. Given a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. If $a = 2$ and $e = 18$, then the possible value of 'c' can be :

(a) 9 (b) -6 (c) 6 (d) -9

10. The number a, b, c in that order form a three term A.P and $a + b + c = 60$. The number $(a - 2)$, b, $(c + 3)$ in that order form a three term G.P. All possible values of $(a^2 + b^2 + c^2)$ is/are :

(a) 1218 (b) 1208 (c) 1288 (d) 1298

11. If $(x^2 + x + 1) + (x^2 + 2x + 3) + (x^2 + 3x + 5) + \ldots\ldots + (x^2 + 20x + 39) = 4500$, then x is equal to :

(a) 10 (b) -10 (c) 20.5 (d) -20.5

12. For ΔABC, if $81 + 144a^4 + 16b^4 + 9c^4 = 144abc$, (where notations have their usual meaning), then :

(a) $a > b > c$ (b) $A < B < C$

(c) Area of $\Delta ABC = \frac{3\sqrt{3}}{8}$ (d) Triangle ABC is right angled

13. Let $x, y, z \in \left(0, \frac{\pi}{2}\right)$ are first three consecutive terms of an arithmatic progression such that $\cos x + \cos y + \cos z = 1$ and $\sin x + \sin y + \sin z = \frac{1}{\sqrt{2}}$, then which of the following is/are correct ?

(a) $\cot y = \sqrt{2}$ (b) $\cos(x - y) = \frac{\sqrt{3} - \sqrt{2}}{2\sqrt{2}}$

(c) $\tan 2y = \frac{2\sqrt{2}}{3}$ (d) $\sin(x - y) + \sin(y - z) = 0$

14. If the numbers 16, 20, 16, d form a A.G.P., then d can be equal to.

(a) 3 (b) 11 (c) -8 (d) -16

15. Given $\dfrac{\underbrace{1000.....01}_{n\text{zeroes}}}{\underbrace{1000.....01}_{(n+1)\text{zeroes}}} < \dfrac{\underbrace{1000.....01}_{m\text{ zeroes}}}{\underbrace{1000.....01}_{(m+1)\text{zeroes}}}$, then which of the following is true

(a) $m+1<n$ (b) $m<n$ (c) $m<n+1$ (d) $m>n+1$

16. If $S_r = \sqrt{r+\sqrt{r+\sqrt{r+\sqrt{r+\sqrt{....\infty}}}}}$, $r>0$, then which of the following is/are correct.

(a) S_2, S_6, S_{12}, S_{20} are in A.P. (b) S_4, S_9, S_{16} are irrational

(c) $(2S_3-1)^2, (2S_4-1)^2, (2S_5-1)^2$ are in A.P. (d) S_2, S_{12}, S_{56} are in G.P.

17. Consider the A.P. 50, 48, 46, 44, ... If S_n denotes the sum to n terms of this A.P., then

(a) S_n is maximum for $n = 25$ (b) the first negative terms is 26^{th} term

(c) the first negative term is 27^{th} term (d) the maximum value of S_n is 650

18. Let S_n be the sum to n terms of the series $\dfrac{3}{1^2}+\dfrac{5}{1^2+2^2}+\dfrac{7}{1^2+2^2+3^2}+\dfrac{9}{1^2+2^2+3^2+4^2}$ $+......$ then

(a) $S_5 = 5$ (b) $S_{50} = \dfrac{100}{17}$ (c) $S_{1001} = \dfrac{1001}{97}$ (d) $S_\infty = 6$

19. For ΔABC, if $81+144a^4+16b^4+9c^4 = 144abc$, (where notations have their usual meaning), then

(a) $a>b>c$ (b) $A<B<C$

(c) Area of $\Delta ABC = \dfrac{3\sqrt{3}}{8}$ (d) Triangle ABC is right angled

Answers

1.	(b, d)	**2.**	(b, c)	**3.**	(b, c, d)	**4.**	(a, b, c)	**5.**	(b, c)	**6.**	(a, b, c, d)
7.	(a, b)	**8.**	(a, b)	**9.**	(b, c)	**10.**	(b, d)	**11.**	(a, d)	**12.**	(b, c, d)
13.	(a, b)	**14.**	(b)	**15.**	(b, c)	**16.**	(a, b, c, d)	**17.**	(a, c, d)	**18.**	(a, b, d)
19.	(b, c, d)										

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

The first four terms of a sequence are given by $T_1 = 0, T_2 = 1, T_3 = 1, T_4 = 2$.
The general term is given by $T_n = A\alpha^{n-1} + B\beta^{n-1}$ where A, B, α, β are independent of n and A is positive.

1. The value of $(\alpha^2 + \beta^2 + \alpha\beta)$ is equal to :

(a) 1 (b) 2 (c) 5 (d) 4

2. The value of $5(A^2 + B^2)$ is equal to :

(a) 2 (b) 4 (c) 6 (d) 8

Paragraph for Question Nos. 3 to 4

There are two sets A and B each of which consists of three numbers in A.P. whose sum is 15. D and d are their respective common differences such that $D - d = 1$, $D > 0$. If $\frac{p}{q} = \frac{7}{8}$ where p and q are the product of the numbers in those sets A and B respectively.

3. Sum of the product of the numbers in set A taken two at a time is :

(a) 51 (b) 71 (c) 74 (d) 86

4. Sum of the product of the numbers in set B taken two at a time is :

(a) 52 (b) 54 (c) 64 (d) 74

Paragraph for Question Nos. 5 to 7

Let x, y, z are positive reals and $x + y + z = 60$ and $x > 3$.

5. Maximum value of $(x-3)(y+1)(z+5)$ is :

(a) (17) (21) (25) (b) (20) (21) (23) (c) (21) (21) (21) (d) (23) (19) (15)

6. Maximum value of $(x-3)(2y+1)(3z+5)$ is :

(a) $\frac{(355)^3}{3^3 \cdot 6^2}$ (b) $\frac{(355)^3}{3^3 \cdot 6^3}$ (c) $\frac{(355)^3}{3^2 \cdot 6^3}$ (d) None of these

7. Maximum value of xyz is :

(a) 8×10^3 (b) 27×10^3 (c) 64×10^3 (d) 125×10^3

Paragraph for Question Nos. 8 to 10

Two consecutive numbers from n natural numbers $1, 2, 3, \ldots\ldots, n$ are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$.

8. The value of n is :
(a) 48 (b) 50 (c) 52 (d) 49

9. The G.M. of the removed numbers is :
(a) $\sqrt{30}$ (b) $\sqrt{42}$ (c) $\sqrt{56}$ (d) $\sqrt{72}$

10. Let removed numbers are x_1, x_2 then $x_1 + x_2 + n =$
(a) 61 (b) 63 (c) 65 (d) 69

Paragraph for Question Nos. 11 to 13

The sequence $\{a_n\}$ is defined by formula $a_0 = 4$ and $a_{n+1} = a_n^2 - 2a_n + 2$ for $n \geq 0$. Let the sequence $\{b_n\}$ is defined by formula $b_0 = \frac{1}{2}$ and $b_n = \frac{2a_0 a_1 a_2 \ldots\ldots a_{n-1}}{a_n}$ $\forall\ n \geq 1$.

11. The value of a_{10} is equal to :
(a) $1 + 2^{1024}$ (b) 4^{1024} (c) $1 + 3^{1024}$ (d) 6^{1024}

12. The value of n for which $b_n = \frac{3280}{3281}$ is :
(a) 2 (b) 3 (c) 4 (d) 5

13. The sequence $\{b_n\}$ satisfies the recurrence formula :
(a) $b_{n+1} = \frac{2b_n}{1 - b_n^2}$ (b) $b_{n+1} = \frac{2b_n}{1 + b_n^2}$
(c) $\frac{b_n}{1 + 2b_n^2}$ (d) $\frac{b_n}{1 - 2b_n^2}$

Paragraph for Question Nos. 14 to 15

Let $f(n) = \sum_{r=2}^{n} \frac{r}{{}^rC_2\ {}^{r+1}C_2}$, $a = \lim_{n \to \infty} f(n)$ and $x^2 - \left(2a - \frac{1}{2}\right)x + t = 0$ has two positive roots α and β.

14. If value of $f(7) + f(8)$ is $\frac{p}{q}$ where p and q are relatively prime, then $(p - q)$ is :
(a) 53 (b) 55 (c) 57 (d) 59

15. Minimum value of $\frac{4}{\alpha} + \frac{1}{\beta}$ is :
(a) 2 (b) 6 (c) 3 (d) 4

Paragraph for Question Nos. 16 to 17

Given the sequence of number $a_1, a_2, a_3, \ldots\ldots, a_{1005}$

which satisfy $\dfrac{a_1}{a_1+1} = \dfrac{a_2}{a_2+3} = \dfrac{a_3}{a_3+5} = \ldots\ldots = \dfrac{a_{1005}}{a_{1005}+2009}$

Also $a_1 + a_2 + a_3 + \ldots\ldots + a_{1005} = 2010$

16. Nature of the sequence is :

(a) A.P. (b) G.P. (c) A.G.P. (d) H.P.

17. 21^{st} term of the sequence is equal to :

(a) $\dfrac{86}{1005}$ (b) $\dfrac{83}{1005}$ (c) $\dfrac{82}{1005}$ (d) $\dfrac{79}{1005}$

Answers

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(a)	**7.**	(a)	**8.**	(b)	**9.**	(c)	**10.**	(c)
11.	(c)	**12.**	(b)	**13.**	(b)	**14.**	(d)	**15.**	(b)	**16.**	(a)	**17.**	(c)						

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If three unequal numbers a, b, c are in A.P. and $b-a$, $c-b$, a are in G.P., then $\dfrac{a^3+b^3+c^3}{3abc}$ is equal to	**(P)**	1
(B)	Let x be the arithmetic mean and y, z be two geometric means between any two positive numbers, then $\dfrac{y^3+z^3}{2xyz}$ is equal to	**(Q)**	4
(C)	If a, b, c be three positive number which form three successive terms of a G.P. and $c > 4b-3a$, then the common ratio of the G.P. can be equal to	**(R)**	2
(D)	Number of integral values of x satisfying inequality, $-7x^2+8x-9>0$ is	**(S)**	0

2.

	Column-I		Column-II
(A)	The sequence a, b, 10, c, d are in A.P., then $a+b+c+d=$	**(P)**	6
(B)	Six G.M.'s are inserted between 2 and 5, if their product can be expressed as $(10)^n$. Then $n=$	**(Q)**	2
(C)	Let $a_1, a_2, a_3, \ldots, a_{10}$ are in A.P. and $h_1, h_2, h_3, \ldots, h_{10}$ are in H.P. such that $a_1=h_1=1$ and $a_{10}=h_{10}=6$, then $a_4h_7=$	**(R)**	3
(D)	If $\log_3 2$, $\log_3(2^x-5)$ and $\log_3\left(2^x-\dfrac{7}{2}\right)$ are in A.P., then $x=$	**(S)**	20
		(T)	40

3.

	Column-I		Column-II
(A)	The number of real values of x such that three numbers 2^x, 2^{x^2} and 2^{x^3} form a non-constant arithmetic progression in that order, is	**(P)**	0
(B)	Let $S=(a_2-a_3)\left(\dfrac{1}{\sqrt{a_1}+\sqrt{a_2}}+\dfrac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\dfrac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ where $a_1, a_2, a_3, \ldots, a_n$ are n consecutive terms of an A.P. and $a_i>0\ \forall i \in \{1, 2, \ldots, n\}$. If $a_1=225$, $a_n=400$, then the value of $S+7$ is equal to	**(Q)**	1

(C)	Let S_n denote the sum of first n terms of an non constant A.P. and $S_{2n} = 3S_n$, then $\frac{S_{3n}}{2S_n}$ is equal to	**(R)**	2
(D)	If t_1, t_2, t_3, t_4 and t_5 are first 5 terms of an A.P., then $\frac{4(t_1 - t_2 - t_4) + 6t_3 + t_5}{3t_1}$ is equal to	**(S)**	3
		(T)	4

4. **Column-I** contains S and **Column-II** gives last digit of S.

	Column-I		Column-II
(A)	$S = \sum_{n=1}^{11} (2n-1)^2$	**(P)**	0
(B)	$S = \sum_{n=1}^{10} (2n-1)^3$	**(Q)**	1
(C)	$S = \sum_{n=1}^{18} (2n-1)^2(-1)^n$	**(R)**	3
(D)	$S = \sum_{n=1}^{15} (2n-1)^3(-1)^{n-1}$	**(S)**	5
		(T)	8

5.

	Column-I		Column-II
(A)	If $x, y \in R^+$ satisfy $\log_8 x + \log_4 y^2 = 5$ and $\log_8 y + \log_4 x^2 = 7$ then the value of $\frac{x^2 + y^2}{2080} =$	**(P)**	6
(B)	In ΔABC A, B, C are in A.P. and sides a, b and c are in G.P. then $a^2(b-c) + b^2(c-a) + c^2(a-b) =$	**(Q)**	3
(C)	If a, b, c are three positive real numbers then the minimum value of $\frac{b+c}{a} + \frac{a+c}{b} + \frac{a+b}{c}$ is	**(R)**	0
(D)	In ΔABC, $(a+b+c)(b+c-a) = \lambda bc$ where $\lambda \in I$, then greatest value of λ is	**(S)**	2

6. Let $f(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+......+\frac{1}{n}$ such that $P(n)f(n+2)=P(n)f(n)+q(n)$. Where $P(n)$, $Q(n)$ are polynomials of least possible degree and $P(n)$ has leading coefficient unity. Then match the following Column-I with Column-II.

	Column-I		Column-II
(A)	$\sum_{n=1}^{m}\frac{p(n)-2}{n}$	**(P)**	$\frac{m(m+1)}{2}$
(B)	$\sum_{n=1}^{m}\frac{q(n)-3}{2}$	**(Q)**	$\frac{5m(m+7)}{2}$
(C)	$\sum_{n=1}^{m}\frac{p(n)+q^2(n)-11}{n}$	**(R)**	$\frac{3m(m+7)}{2}$
(D)	$\sum_{n=1}^{m}\frac{q^2(n)-p(n)-7}{n}$	**(S)**	$\frac{m(m+7)}{2}$

Answers

1.	A → R ; B → P ; C → Q; D → S
2.	A → R, B → R, C → P, D → R
3.	A → P, B → R, C → S, D → Q
4.	A → Q ; B → P ; C → T ; D → S
5.	A → S; B → R; C → P; D → Q
6.	A → S ; B → P ; C → Q ; D → R

Exercise-5 : Subjective Type Problems

1. Let a, b, c, d are four distinct consecutive numbers in A.P. The complete set of values of x for which $2(a-b)+x(b-c)^2+(c-a)^3 = 2(a-d)+(b-d)^2+(c-d)^3$ is true is $(-\infty, \alpha] \cup [\beta, \infty)$, then $|\alpha|$ is equal to :

2. The sum of all digits of n for which $\sum_{r=1}^{n} r\,2^r = 2 + 2^{n+10}$ is :

3. If $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r+2}{2^{r+1} r(r+1)} = \frac{1}{k}$, then $k =$

4. The value of $\sum_{r=1}^{\infty} \frac{8r}{4r^4+1}$ is equal to :

5. Three distinct non-zero real numbers form an A.P. and the squares of these numbers taken in same order form a G.P. If possible common ratio of G.P. are $3 \pm \sqrt{n}$, $n \in N$ then $n =$

6. If $\sqrt{\underbrace{(1111\ldots\ldots1)}_{2n \text{ times}} - \underbrace{(222\ldots\ldots2)}_{n \text{ times}}} = \underbrace{PPP\ldots\ldots P}_{n \text{ times}}$ then $P =$

7. In an increasing sequence of four positive integers, the first 3 terms are in A.P., the last 3 terms are in G.P. and the fourth term exceed the first term by 30, then the common difference of A.P. lying in interval [1, 9] is :

8. The limit of $\frac{1}{n^4} \sum_{k=1}^{n} k(k+2)(k+4)$ as $n \to \infty$ is equal to $\frac{1}{\lambda}$, then $\lambda =$

9. What is the last digit of $1+2+3+\ldots\ldots+n$ if the last digit of $1^3+2^3+\ldots\ldots+n^3$ is 1?

10. Three distinct positive numbers a, b, c are in G.P., while $\log_c a$, $\log_b c$, $\log_a b$ are in A.P. with non-zero common difference d, then $2d =$

11. The numbers $\frac{1}{3}, \frac{1}{3}\log_x y, \frac{1}{3}\log_y z, \frac{1}{7}\log_z x$ are in H.P. If $y = x^r$ and $z = x^s$, then $4(r+s) =$

12. If $\sum_{k=1}^{\infty} \frac{k^2}{3^k} = \frac{p}{q}$; where p and q are relatively prime positive integers. Find the value of $(p+q)$.

13. The sum of the terms of an infinitely decreasing Geometric Progression (GP) is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ when $x \in [-4, 3]$ and the difference between the first and second term is $f'(0)$. The common ratio $r = \frac{p}{q}$ where p and q are relatively prime positive integers. Find $(p+q)$.

14. A cricketer has to score 4500 runs. Let a_n denotes the number of runs he scores in the n^{th} match. If $a_1 = a_2 = \ldots. a_{10} = 150$ and $a_{10}, a_{11}, a_{12}\ldots.$ are in A.P. with common difference (−2). If N be the total number of matches played by him to score 4500 runs. Find the sum of the digits of N.

15. If $x = 10\sum_{n=3}^{100} \frac{1}{n^2-4}$, then $[x]$ = (where $[\cdot]$ denotes greatest integer function)

16. Let $f(n) = \frac{4n+\sqrt{4n^2-1}}{\sqrt{2n+1}+\sqrt{2n-1}}, n \in N$ then the remainder when $f(1)+f(2)+f(3)+\ldots..+f(60)$ is divided by 9 is.

17. Find the sum of series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{12}+\ldots\ldots\infty$, where the terms are the reciprocals of the positive integers whose only prime factors are two's and three's :

18. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be real numbers in arithmatic progression such that $a_1 = 15$ and a_2 is an integer. Given $\sum_{r=1}^{10}(a_r)^2 = 1185$. If $S_n = \sum_{r=1}^{n} a_r$ and maximum value of n is N for which $S_n \geq S_{(n-1)}$, then find $N-10$.

19. Let the roots of the equation $24x^3 - 14x^2 + kx + 3 = 0$ form a geometric sequence of real numbers. If absolute value of k lies between the roots of the equation $x^2 + \alpha^2 x - 112 = 0$, then the largest integral value of α is :

20. How many ordered pair(s) satisfy $\log\left(x^3 + \frac{1}{3}y^3 + \frac{1}{9}\right) = \log x + \log y$

21. Let a and b be positive integers. The value of xyz is 55 and $\frac{343}{55}$ when a, x, y, z, b are in arithmatic and harmonic progression respectively. Find the value of $(a+b)$

Answers

1.	8	**2.**	9	**3.**	2	**4.**	2	**5.**	8	**6.**	3	**7.**	9
8.	4	**9.**	1	**10.**	3	**11.**	6	**12.**	5	**13.**	5	**14.**	7
15.	5	**16.**	8	**17.**	3	**18.**	6	**19.**	2	**20.**	1	**21.**	8

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Chapter 10 till end (Chapter 26) is in Part 2

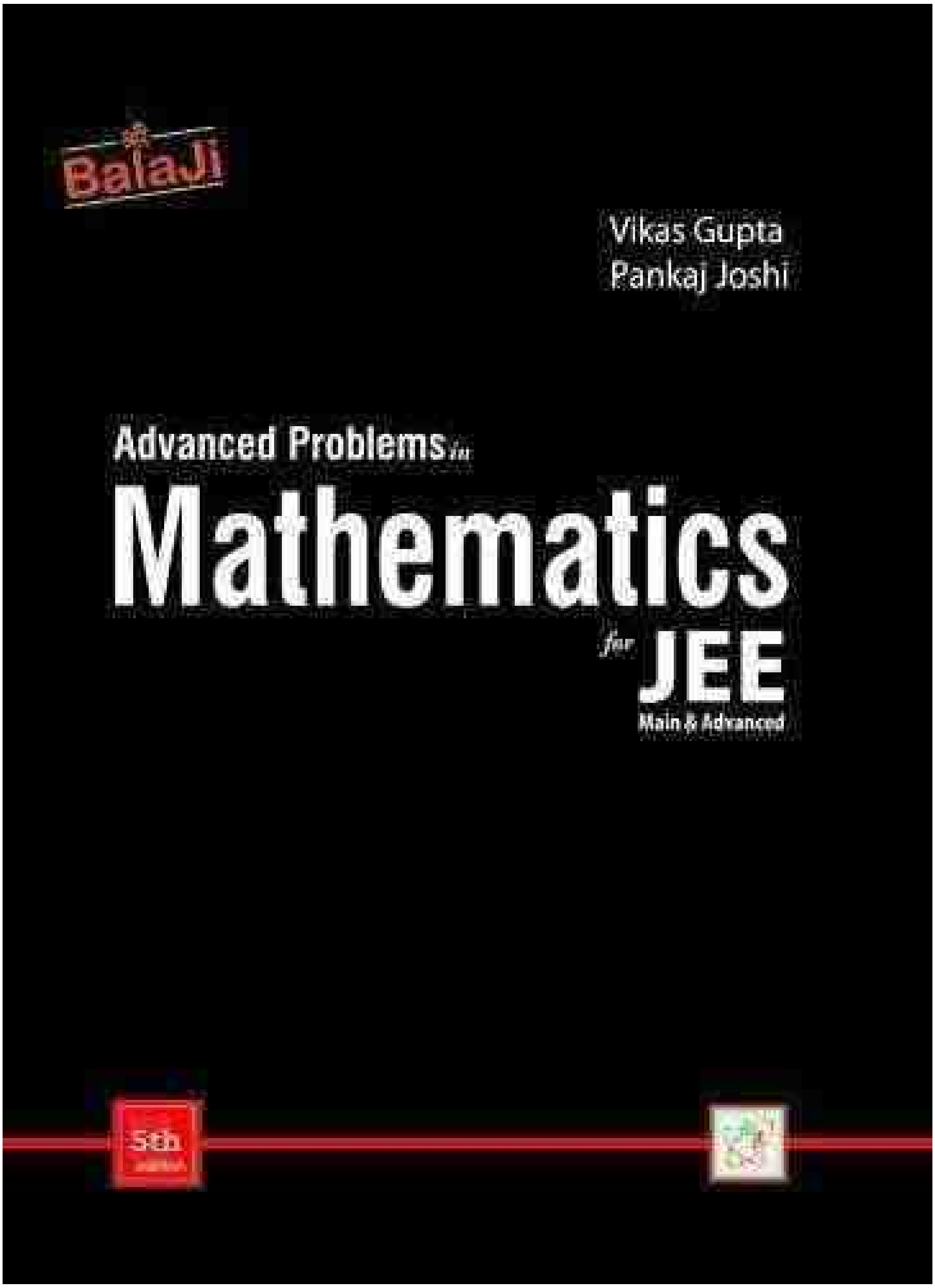
Balaji

Advanced Problems in Mathematics Chapter 10 to 26

for IIT JEE Main and Advanced

by

Vikas Gupta and Pankaj Joshi



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BalaJi

Advanced Problems *in*

MATHEMATICS

for

JEE (MAIN & ADVANCED)

by :

Vikas Gupta
Director
Vibrant Academy India(P) Ltd.
KOTA (Rajasthan)

Pankaj Joshi
Director
Vibrant Academy India(P) Ltd.
KOTA (Rajasthan)

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Chapter 10 – Determinants

10

DETERMINANTS

Exercise-1 : Single Choice Problems

1. If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ then the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ is :

(a) 1 (b) $\frac{3}{2}$ (c) $\frac{3}{8}$ (d) $\frac{9}{4}$

2. Let the following system of equations

$$kx + y + z = 1$$
$$x + ky + z = k$$
$$x + y + kz = k^2$$

has no solution. Find $|k|$.

(a) 0 (b) 1 (c) 2 (d) 3

3. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals :

(a) 2 (b) -1 (c) 1 (d) 0

4. If the system of linear equations

$$x + 2ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 4cy + cz = 0$$

has a non-zero solution, then a, b, c :

(a) are in A.P. (b) are in G.P.

(c) are in H.P. (d) satisfy $a + 2b + 3c = 0$

5. If the number of quadratic polynomials $ax^2 + 2bx + c$ which satisfy the following conditions :

(i) a, b, c are distinct

(ii) $a, b, c \in \{1, 2, 3, \ldots\ldots, 2001, 2002\}$

(iii) $x + 1$ divides $ax^2 + 2bx + c$

is equal to 1000λ, then find the value of λ.

(a) 2002 (b) 2001 (c) 2003 (d) 2004

6. If the system of equations $2x + ay + 6z = 8$, $x + 2y + z = 5$, $2x + ay + 3z = 4$ has a unique solution then 'a' cannot be equal to :

(a) 2 (b) 3 (c) 4 (d) 5

7. If one of the roots of the equation $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix} = 0$ is $x = 2$, then sum of all other five roots is :

(a) -2 (b) 0 (c) $2\sqrt{5}$ (d) $\sqrt{15}$

8. The system of equations

$$kx + (k+1)y + (k-1)z = 0$$
$$(k+1)x + ky + (k+2)z = 0$$
$$(k-1)x + (k+2)y + kz = 0$$

has a nontrivial solution for :

(a) Exactly three real values of k.
(b) Exactly two real values of k.
(c) Exactly one real value of k.
(d) Infinite number of values of k.

9. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in G.P. and $a_i > 0$ for each i, then the determinant

$\Delta = \begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$ is equal to :

(a) 0 (b) $\log\left(\sum_{i=1}^{n^2+n} a_i\right)$ (c) 1 (d) 2

10. If $D_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D_2 = \begin{vmatrix} a_1 + 2a_2 + 3a_3 & 2a_3 & 5a_2 \\ b_1 + 2b_2 + 3b_3 & 2b_3 & 5b_2 \\ c_1 + 2c_2 + 3c_3 & 2c_3 & 5c_2 \end{vmatrix}$ then $\dfrac{D_2}{D_1}$ is equal to :

(a) 10 (b) -10 (c) 20 (d) -20

11. If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ac & b \\ 1 & ab & c \end{vmatrix}$ then :

(a) $\Delta_1 = \Delta_2$ (b) $\Delta_1 = 2\Delta_2$ (c) $\Delta_1 + \Delta_2 = 0$ (d) $\Delta_1 + 2\Delta_2 = 0$

12. The value of the determinant $\begin{vmatrix} 1 & 0 & -1 \\ a & 1 & 1-a \\ b & a & 1+a-b \end{vmatrix}$ depends on :

(a) only a (b) only b (c) neither a nor b (d) both a and b

13. Sum of solutions of the equation $\begin{vmatrix} 1 & 2 & x \\ 2 & 3 & x^2 \\ 3 & 5 & 2 \end{vmatrix} = 10$ is :

(a) 1 (b) -1 (c) 2 (d) 4

14. If $D = \begin{vmatrix} x+d & x+e & x+f \\ x+d+1 & x+e+1 & x+f+1 \\ x+a & x+b & x+c \end{vmatrix}$ then D does not depend on :

(a) a (b) e (c) d (d) x

15. The value of the determinant $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} =$

(a) $xyz(x+y+z)^2$ (b) $(x+y-z)(x+y+z)^2$

(c) $(x+y+z)^3$ (d) $(x+y+z)^2$

16. A rectangle $ABCD$ is inscribed in a circle. Let PQ be the diameter of the circle parallel to the side AB. If $\angle BPC = 30°$, then the ratio of the area of rectangle to the area of circle is :

(a) $\frac{\sqrt{3}}{\pi}$ (b) $\frac{\sqrt{3}}{2\pi}$ (c) $\frac{3}{\pi}$ (d) $\frac{\sqrt{3}}{9\pi}$

17. Let $ab = 1$, $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ then the minimum value of Δ is :

(a) 3 (b) 9 (c) 27 (d) 81

18. The determinant $\begin{vmatrix} 2 & a+b+c+d & ab+cd \\ a+b+c+d & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix} = 0$ for

(a) $a+b+c+d=0$ (b) $ab+cd=0$

(c) $ab(c+d)+cd(a+b)=0$ (d) any a, b, c, d

19. Let $\det A = \begin{vmatrix} l & m & n \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ and

if $(l-m)^2+(p-q)^2 = 9$, $(m-n)^2+(q-r)^2 = 16$, $(n-l)^2+(r-p)^2 = 25$, then the value of $(\det. A)^2$ equals :

(a) 36 (b) 100 (c) 144 (d) 169

20. The number of distinct real values of K such that the system of equations $x+2y+z=1$, $x+3y+4z=K$, $x+5y+10z=K^2$ has infinitely many solutions is :

(a) 0 (b) 4 (c) 2 (d) 3

21. If $\begin{vmatrix} (x+1) & (x+1)^2 & (x+1)^3 \\ (x+2) & (x+2)^2 & (x+2)^3 \\ (x+3) & (x+3)^2 & (x+3)^3 \end{vmatrix}$ is expressed as a polynomial in x, then the term independent of x is :

(a) 0 (b) 2 (c) 12 (d) 16

22. If A, B, C are the angles of triangle ABC, then the minimum value of $\begin{vmatrix} -2 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is equal to :

(a) 0 (b) −1 (c) 1 (d) −2

23. If the system of linear equations

$$x + 2ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 4cy + cz = 0$$

has a non-zero solution then a, b, c are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

24. If a, b and c are the roots of the equation $x^3 + 2x^2 + 1 = 0$, find $\begin{vmatrix} a & b & x \\ b & c & a \\ c & a & b \end{vmatrix}$.

(a) 8 (b) −8 (c) 0 (d) 2

25. The system of homogeneous equation $\lambda x + (\lambda+1)y + (\lambda-1)z = 0$,
$(\lambda+1)x + \lambda y + (\lambda+2)z = 0$, $(\lambda-1)x + (\lambda+2)y + \lambda z = 0$ has non-trivial solution for :

(a) exactly three real values of λ (b) exactly two real values of λ
(c) exactly three real value of λ (d) infinitely many real value of λ

26. If one of the roots of the equation $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix} = 0$ is $x = 2$, then sum of all other five roots is :

(a) −2 (b) 0 (c) $2\sqrt{5}$ (d) $\sqrt{15}$

Answers

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(c)	**7.**	(a)	**8.**	(c)	**9.**	(a)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(b)	**14.**	(d)	**15.**	(c)	**16.**	(a)	**17.**	(c)	**18.**	(d)	**19.**	(c)	**20.**	(c)
21.	(c)	**22.**	(b)	**23.**	(c)	**24.**	(a)	**25.**	(c)	**26.**	(a)								

Exercise-2 : One or More than One Answer is/are Correct

1. Let $f(a, b) = \begin{vmatrix} a & a^2 & 0 \\ 1 & (2a+b) & (a+b)^2 \\ 0 & 1 & (2a+3b) \end{vmatrix}$, then

(a) $(2a+b)$ is a factor of $f(a, b)$
(b) $(a+2b)$ is a factor of $f(a, b)$
(c) $(a+b)$ is a factor of $f(a, b)$
(d) a is a factor of $f(a, b)$

2. If $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 2\sqrt{3}\tan\theta \\ \cos^2\theta & 1+\sin^2\theta & 2\sqrt{3}\tan\theta \\ \cos^2\theta & \sin^2\theta & 1+2\sqrt{3}\tan\theta \end{vmatrix} = 0$ then θ may be :

(a) $\frac{\pi}{6}$
(b) $\frac{5\pi}{6}$
(c) $\frac{7\pi}{6}$
(d) $\frac{11\pi}{6}$

3. Let $\Delta = \begin{vmatrix} a & a+d & a+3d \\ a+d & a+2d & a \\ a+2d & a & a+d \end{vmatrix}$ then :

(a) Δ depends on a
(b) Δ depends on d
(c) Δ is independent of a, d
(d) $\Delta = 0$

4. The value(s) of λ for which the system of equations

$$(1-\lambda)x + 3y - 4z = 0$$
$$x - (3+\lambda)y + 5z = 0$$
$$3x + y - \lambda z = 0$$

possesses non-trivial solutions.

(a) -1
(b) 0
(c) 1
(d) 2

5. Let $D(x) = \begin{vmatrix} x^2+4x-3 & 2x+4 & 13 \\ 2x^2+5x-9 & 4x+5 & 26 \\ 8x^2-6x+1 & 16x-6 & 104 \end{vmatrix} = \alpha x^3 + \beta x^2 + \gamma x + \delta$ then :

(a) $\alpha+\beta = 0$
(b) $\beta+\gamma = 0$
(c) $\alpha+\beta+\gamma+\delta = 0$
(d) $\alpha+\beta+\gamma = 0$

6. Let $D(x) = \begin{vmatrix} x^2+4x-3 & 2x+4 & 13 \\ 2x^2+5x-9 & 4x+5 & 26 \\ 8x^2-6x+1 & 16x-6 & 104 \end{vmatrix} = \alpha x^3 + \beta x^2 + \gamma x + \delta$ then :

(a) $\alpha+\beta = 0$
(b) $\beta+\gamma = 0$
(c) $\alpha+\beta+\gamma+\delta = 0$
(d) $\alpha+\beta+\gamma = 0$

7. If the system of equations

$$ax + y + 2z = 0$$
$$x + 2y + z = b$$
$$2x + y + az = 0$$

has no solution then $(a+b)$ can be equals to :

(a) -1
(b) 2
(c) 3
(d) 4

8. If the system of equations

$$ax + y + 2z = 0$$
$$x + 2y + z = b$$
$$2x + y + az = 0$$

has no solution then $(a + b)$ can be equal to

(a) -1 (b) 2 (c) 3 (d) 4

Answers

1.	(b, c, d)	**2.**	(b, d)	**3.**	(a, b)	**4.**	(a, b)	**5.**	(a, b, d)	**6.**	(a, b, d)
7.	(b, c, d)	**8.**	(b)								

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Consider the system of equations

$$2x + \lambda y + 6z = 8$$
$$x + 2y + \mu z = 5$$
$$x + y + 3z = 4$$

The system of equations has :

1. No solution if :

(a) $\lambda = 2, \mu = 3$ (b) $\lambda \neq 2, \mu = 3$ (c) $\lambda \neq 2, \mu \neq 3$ (d) $\lambda = 2, \mu \in R$

2. Exactly one solution if :

(a) $\lambda \neq 2, \mu \neq 3$ (b) $\lambda = 2, \mu = 3$ (c) $\lambda \neq 2, \mu = 3$ (d) $\lambda = 2, \mu \in R$

3. Infinitely many solutions if :

(a) $\lambda \neq 2, \mu \neq 3$ (b) $\lambda = 2, \mu \neq 3$ (c) $\lambda \neq 2, \mu = 3$ (d) $\lambda = 2, \mu \in R$

Answers

1.	(b)	**2.**	(a)	**3.**	(d)

Exercise-4 : Subjective Type Problems

1. If 3^n is a factor of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^nC_1 & {}^{n+3}C_1 & {}^{n+6}C_1 \\ {}^nC_2 & {}^{n+3}C_2 & {}^{n+6}C_2 \end{vmatrix}$ then the maximum value of n is ....

2. Find the value of λ for which $\begin{vmatrix} 2a_1+b_1 & 2a_2+b_2 & 2a_3+b_3 \\ 2b_1+c_1 & 2b_2+c_2 & 2b_3+c_3 \\ 2c_1+a_1 & 2c_2+a_2 & 2c_3+a_3 \end{vmatrix} = \lambda \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

3. Find the co-efficient of x in the expansion of the determinant $\begin{vmatrix} (1+x)^2 & (1+x)^4 & (1+x)^6 \\ (1+x)^3 & (1+x)^6 & (1+x)^9 \\ (1+x)^4 & (1+x)^8 & (1+x)^{12} \end{vmatrix}$.

4. If $x, y, z \in R$ and $\begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix} = 2$ then find the value of

$$\begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4y^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2z^3(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix}.$$

5. If the system of equations :

$$2x+3y-z=0$$
$$3x+2y+kz=0$$
$$4x+y+z=0$$

have a set of non-zero integral solutions then, find the smallest positive value of z.

6. Find $a \in R$ for which the system of equations $2ax-2y+3z=0$; $x+ay+2z=0$ and $2x+az=0$ also have a non-trivial solution.

7. If three non-zero distinct real numbers form an arithmatic progression and the squares of these numbers taken in the same order constitute a geometric progression. Find the sum of all possible common ratios of the geometric progression.

8. Let $\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 6a_1 & 2a_2 & 2a_3 \\ 3b_1 & b_2 & b_3 \\ 12c_1 & 4c_2 & 4c_3 \end{vmatrix}$ and $\Delta_3 = \begin{vmatrix} 3a_1+b_1 & 3a_2+b_2 & 3a_3+b_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix}$

then $\Delta_3 - \Delta_2 = k\Delta_1$, find k.

9. The minimum value of determinant $\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 2 \end{vmatrix}$ $\forall\ \theta \in R$ is :

10. For a unique value of μ & λ, the system of equations given by

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$2x + 5y + \lambda z = \mu$$

has infinitely many solutions, then $\dfrac{\mu - \lambda}{4}$ is equal to

11. Let $\lim\limits_{n \to \infty} n \sin (2\pi e \underline{\lfloor n}) = k\pi$, where $n \in N$. Find k :

12. If the system of linear equations

$$(\cos\theta)x + (\sin\theta)y + \cos\theta = 0$$
$$(\sin\theta)x + (\cos\theta)y + \sin\theta = 0$$
$$(\cos\theta)x + (\sin\theta)y - \cos\theta = 0$$

is consistent, then the number of possible values of θ, $\theta \in [0, 2\pi]$ is :

Answers

1.	3	**2.**	9	**3.**	0	**4.**	4	**5.**	5	**6.**	2	**7.**	6
8.	3	**9.**	3	**10.**	7	**11.**	2	**12.**	2				

❑❑❑

Chapter 11 – Complex Numbers

11 COMPLEX NUMBERS

Exercise-1 : Single Choice Problems

1. Let t_1, t_2, t_3 be three distinct points on circle $|t|=1$. If θ_1, θ_2 and θ_3 be the arguments of t_1, t_2, t_3 respectively then $\cos(\theta_1-\theta_2)+\cos(\theta_2-\theta_3)+\cos(\theta_3-\theta_1)$

(a) $\geq -\frac{3}{2}$ (b) $\leq -\frac{3}{2}$

(c) $\geq \frac{3}{2}$ (d) ≤ 2

2. The number of points of intersection of the curves represented by

$$\arg(z-2-7i)=\cot^{-1}(2) \text{ and } \arg\left(\frac{z-5i}{z+2-i}\right)=\pm\frac{\pi}{2}$$

(a) 0 (b) 1 (c) 2 (d) None of these

3. All three roots of $az^3+bz^2+cz+d=0$, have negative real part, $(a, b, c \in R)$ then :

(a) All a, b, c, d have the same sign (b) a, b, c have same sign

(c) a, b, d have same sign (d) b, c, d have same sign

4. Let z_1 and z_2 be two roots of the equation $z^2+az+b=0$, z being complex number. Further, assume that the origin, z_1 and z_2 form an equilateral triangle, then :

(a) $a^2=b$ (b) $a^2=2b$ (c) $a^2=3b$ (d) $a^2=4b$

5. If z and ω are two non-zero complex numbers such that $|z\omega|=1$, and $\arg(z)-\arg(\omega)=\frac{\pi}{2}$, then $\bar{z}\omega$ is equal to :

(a) 1 (b) -1 (c) i (d) $-i$

6. If ω be an imaginary n^{th} root of unity, then $\sum_{r=1}^{n}(ar+b)\,\omega^{r-1}$ is equal to :

(a) $\frac{n(n+1)a}{2\omega}$ (b) $\frac{nb}{1-n}$ (c) $\frac{na}{\omega-1}$ (d) None of these

7. If α, β are complex numbers then the maximum value of $\dfrac{\alpha\bar{\beta}+\bar{\alpha}\beta}{|\alpha\beta|}$ is equal to :

(a) 1 (b) 2 (c) greater than 2 (d) less than 1

8. Let z_1, z_2, z_3 and z_4 be the roots of the equation $z^4+z^3+2=0$, then the value of $\prod_{r=1}^{4}(2z_r+1)$ is equal to :

(a) 28 (b) 29 (c) 30 (d) 31

9. If $\arg\left(\dfrac{z-6-3i}{z-3-6i}\right)=\dfrac{\pi}{4}$, then :

(a) minimum value of $|z|$ is $6\sqrt{2}-3$ (b) Maximum value of $|z|$ is $6\sqrt{2}+3$

(c) minimum value of $|z|$ is $15\sqrt{2}-6$ (d) Maximum value of $|z|$ is $15\sqrt{2}+6$

10. If $z_1 \neq -z_2$ and $|z_1+z_2|=\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}\right|$ then :

(a) at least one of z_1, z_2 is unimodular (b) both z_1, z_2 are unimodular

(c) $z_1 \cdot z_2$ is unimodular (d) z_1-z_2 is unimodular

11. If $|z-i|\le 2$ and $z_1=5+3i$, then the maximum value of $|iz+z_1|$ is :

(a) $5+\sqrt{13}$ (b) $5+\sqrt{2}$ (c) 7 (d) 8

12. If z_1, z_2, z_3 are vertices of a triangle such that $|z_1-z_2|=|z_1-z_3|$ then $\arg\left(\dfrac{2z_1-z_2-z_3}{z_3-z_2}\right)$ is :

(a) $\pm\dfrac{\pi}{3}$ (b) 0 (c) $\pm\dfrac{\pi}{2}$ (d) $\pm\dfrac{\pi}{6}$

13. It is given that complex numbers z_1 and z_2 satisfy $|z_1|=2$ and $|z_2|=3$. If the included angle of their corresponding vectors is 60°, then $\left|\dfrac{z_1+z_2}{z_1-z_2}\right|$ can be expressed as $\dfrac{\sqrt{n}}{7}$, where 'n' is a natural number then $n=$

(a) 126 (b) 119 (c) 133 (d) 19

14. If all the roots of $z^3+az^2+bz+c=0$ are of unit modulus, then :

(a) $|a|\le 3$ (b) $|b|\le 3$ (c) $|c|=1$ (d) All of the above

15. Let z be a complex number satisfying $\dfrac{1}{2}\le|z|\le 4$, then sum of greatest and least values of $\left|z+\dfrac{1}{z}\right|$ is :

(a) $\dfrac{65}{4}$ (b) $\dfrac{65}{16}$ (c) $\dfrac{17}{4}$ (d) 17

16. If $|z-2i|\le\sqrt{2}$, then the maximum value of $|3+i(z-1)|$ is :

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $2+\sqrt{2}$ (d) $3+2\sqrt{2}$

17. Let $x - \frac{1}{x} = (\sqrt{2})i$ where $i = \sqrt{-1}$. Then the value of $x^{2187} - \frac{1}{x^{2187}}$ is :

(a) $i\sqrt{2}$ (b) $-i\sqrt{2}$ (c) -2 (d) $\frac{i}{\sqrt{2}}$

18. If $z = re^{i\theta}(r > 0 \ \& \ 0 \le \theta < 2\pi)$ is a root of the equation $z^8 - z^7 + z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 = 0$ then number of values of 'θ' is :

(a) 6 (b) 7 (c) 8 (d) 9

19. Let P and Q be two points on the circle $|w| = r$ represented by w_1 and w_2 respectively, then the complex number representing the point of intersection of the tangents at P and Q is :

(a) $\frac{w_1 w_2}{2(w_1 + w_2)}$ (b) $\frac{2w_1 \overline{w}_2}{w_1 + w_2}$ (c) $\frac{2w_1 w_2}{w_1 + w_2}$ (d) $\frac{2\overline{w}_1 w_2}{w_1 + w_2}$

20. If z_1, z_2, z_3 are complex number, such that $|z_1| = 2, |z_2| = 3, |z_3| = 4$, then maximum value of $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$ is :

(a) 58 (b) 29 (c) 87 (d) None of these

21. If $Z = \frac{7+i}{3+4i}$, then find Z^{14}:

(a) 2^7 (b) $(-2)^7$ (c) $(2^7)i$ (d) $(-2^7)i$

22. If $|Z - 4| + |Z + 4| = 10$, then the difference between the maximum and the minimum values of $|Z|$ is :

(a) 2 (b) 3 (c) $\sqrt{41} - 5$ (d) 0

Answers

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(d)	**6.**	(c)	**7.**	(b)	**8.**	(d)	**9.**	(b)	**10.**	(c)
11.	(c)	**12.**	(c)	**13.**	(c)	**14.**	(d)	**15.**	(c)	**16.**	(b)	**17.**	(a)	**18.**	(c)	**19.**	(c)	**20.**	(c)
21.	(c)	**22.**	(a)																

Exercise-2 : One or More than One Answer is/are Correct

1. Let Z_1 and Z_2 are two non-zero complex number such that $|Z_1 + Z_2| = |Z_1| = |Z_2|$, then $\frac{Z_1}{Z_2}$ may be :

(a) $1+\omega$
(b) $1+\omega^2$
(c) ω
(d) ω^2

2. Let z_1, z_2 and z_3 be three distinct complex numbers, satisfying $|z_1| = |z_2| = |z_3| = 1$. Which of the following is/are true :

(a) If $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then $\arg\left(\frac{z - z_1}{z - z_2}\right) > \frac{\pi}{4}$ where $|z| > 1$

(b) $|z_1 z_2 + z_2 z_3 + z_3 z_1| = |z_1 + z_2 + z_3|$

(c) $\text{Im}\left(\frac{(z_1+z_2)(z_2+z_3)(z_3+z_1)}{z_1 \cdot z_2 \cdot z_3}\right) = 0$

(d) If $|z_1 - z_2| = \sqrt{2}|z_1 - z_3| = \sqrt{2}|z_2 - z_3|$, then $\text{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = 0$

3. The triangle formed by the complex numbers z, iz, $i^2 z$ is :

(a) equilateral
(b) isosceles
(c) right angled
(d) isosceles but not right angled

4. If $A(z_1)$, $B(z_2)$, $C(z_3)$, $D(z_4)$ lies on $|z| = 4$ (taken in order), where $z_1 + z_2 + z_3 + z_4 = 0$ then:

(a) Max. area of quadrilateral $ABCD = 32$
(b) Max. area of quadrilateral $ABCD = 16$
(c) The triangle ΔABC is right angled
(d) The quadrilateral $ABCD$ is rectangle

5. Let z_1, z_2 and z_3 be three distinct complex numbers satisfying $|z_1| = |z_2| = |z_3| = 1$. Which of the following is/are true ?

(a) If $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then $\arg\left(\frac{z - z_1}{z - z_2}\right) > \frac{\pi}{4}$ where $|z| > 1$

(b) $|z_1 z_2 + z_2 z_3 + z_3 z_1| = |z_1 + z_2 + z_3|$

(c) $\text{Im}\left(\frac{(z_1+z_2)(z_2+z_3)(z_3+z_1)}{z_1 \cdot z_2 \cdot z_3}\right) = 0$

(d) If $|z_1 - z_2| = \sqrt{2}|z_1 - z_3| = \sqrt{2}|z_2 - z_3|$, then $\text{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = 0$

6. If $z_1 = a+ib$ and $z_2 = c+id$ are two complex numbers where $a, b, c, d \in R$ and $|z_1| = |z_2| = 1$ and $\text{Im}(z_1\bar{z}_2) = 0$. If $w_1 = a+ic$ and $w_2 = b+id$, then :

(a) $\text{Im}(w_1\overline{w}_2) = 0$ (b) $\text{Im}(\overline{w}_1 w_2) = 0$

(c) $\text{Im}\left(\dfrac{w_1}{w_2}\right) = 0$ (d) $\text{Re}\left(\dfrac{w_1}{\overline{w}_2}\right) = 0$

7. The solutions of the equation $z^4 + 4iz^3 - 6z^2 - 4iz - i = 0$ represent vertices of a convex polygon in the complex plane. The area of the polygon is :

(a) $2^{1/2}$ (b) $2^{3/2}$ (c) $2^{5/2}$ (d) $2^{5/4}$

8. Least positive argument of the 4^{th} root of the complex number $2 - i\sqrt{12}$ is :

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{12}$ (c) $\dfrac{5\pi}{12}$ (d) $\dfrac{7\pi}{12}$

9. Let ω be the imaginary cube root of unity and $(a + b\omega + c\omega^2)^{2015} = (a + b\omega^2 + c\omega)$ where a, b, c are unequal real numbers. Then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ equals :

(a) 0 (b) 1 (c) 2 (d) 3

10. Let n be a positive integer and a complex number with unit modulus is a solution of the equation $z^n + z + 1 = 0$ then the value of n can be :

(a) 62 (b) 155 (c) 221 (d) 196

Answers

1.	(c, d)	**2.**	(b, c, d)	**3.**	(b, c)	**4.**	(a, c, d)	**5.**	(b, c, d)	**6.**	(a, b, c)
7.	(d)	**8.**	(c)	**9.**	(b)	**10.**	(a, b, c)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $f(z)$ is of the form $\alpha z+\beta$, where α, β are constants and α, β, z are complex numbers such that $|\alpha| \neq |\beta|$. $f(z)$ satisfies following properties :

(i) If imaginary part of z is non zero, then $f(z)+\overline{f(z)} = f(\bar{z})+\overline{f(\bar{z})}$

(ii) If real part of z is zero, then $f(z)+\overline{f(z)}=0$

(iii) If z is real, then $\overline{f(z)}\,f(z) > (z+1)^2 \;\forall\; z \in R$.

1. $\dfrac{4x^2}{(f(1)-f(-1))^2}+\dfrac{y^2}{(f(0))^2}=1$, $x, y \in R$, in (x, y) plane will represent :

(a) hyperbola (b) circle (c) ellipse (d) pair of line

2. Consider ellipse $S : \dfrac{x^2}{(\text{Re}(\alpha))^2}+\dfrac{y^2}{(\text{Im}(\beta))^2}=1$, $x, y \in R$ in (x, y) plane, then point $(1, 1)$ will lie :

(a) outside the ellipse S (b) inside the ellipse S

(c) on the ellipse S (d) none of these

Paragraph for Question Nos. 3 to 5

Let z_1 and z_2 be complex numbers, such what $z_1^2-4z_2=16+20i$. Also suppose that roots α and β of $t^2+z_1t+z_2+m=0$ for some complex number m satisfy $|\alpha-\beta|=2\sqrt{7}$, then :

3. The complex number 'm' lies on :

(a) a square with side 7 and centre (4, 5) (b) a circle with radius 7 and centre (4, 5)

(c) a circle with radius 7 and centre (–4, 5) (d) a square with side 7 and centre (–4, 5)

4. The greatest value of $|m|$ is :

(a) $5\sqrt{21}$ (b) $5+\sqrt{23}$ (c) $7+\sqrt{43}$ (d) $7+\sqrt{41}$

5. The least value of $|m|$ is :

(a) $7-\sqrt{41}$ (b) $7-\sqrt{43}$ (c) $5-\sqrt{23}$ (d) $5+\sqrt{21}$

Paragraph for Question Nos. 6 to 7

Let $z_1=3$ and $z_2=7$ represent two points A and B respectively on complex plane. Let the curve C_1 be the locus of point $P(z)$ satisfying $|z-z_1|^2+|z-z_2|^2=10$ and the curve C_2 be the locus of point $P(z)$ satisfying $|z-z_1|^2+|z-z_2|^2=16$.

6. Least distance between curves C_1 and C_2 is :

(a) 4 (b) 3 (c) 2 (d) 1

7. The locus of point from which tangents drawn to C_1 and C_2 are perpendicular, is :

(a) $|z-5|=4$ (b) $|z-3|=2$ (c) $|z-5|=3$ (d) $|z-5|=\sqrt{5}$

Paragraph for Question Nos. 8 to 9

In the Argand plane Z_1, Z_2 and Z_3 are respectively the vertices of an isosceles triangle ABC with $AC=BC$ and $\angle CAB=\theta$. If $I(Z_4)$ is the incentre of triangle, then :

8. The value of $\left(\frac{AB}{IA}\right)^2\left(\frac{AC}{AB}\right)$ is equal to :

(a) $\left|\frac{(Z_2-Z_1)(Z_1-Z_3)}{(Z_4-Z_1)}\right|$ (b) $\left|\frac{(Z_2-Z_1)(Z_3-Z_1)}{(Z_4-Z_1)}\right|$

(c) $\left|\frac{(Z_2-Z_1)(Z_3-Z_1)}{(Z_4-Z_1)^2}\right|$ (d) $\left|\frac{(Z_2+Z_1)(Z_3+Z_1)}{(Z_4+Z_1)}\right|$

9. The value of $(Z_4-Z_1)^2(1+\cos\theta)\sec\theta$ is :

(a) $(Z_2-Z_1)(Z_3-Z_1)$ (b) $\frac{(Z_2-Z_1)(Z_3-Z_1)}{Z_4-Z_1}$

(c) $\frac{(Z_2-Z_1)(Z_3-Z_1)}{(Z_4-Z_1)^2}$ (d) $(Z_2-Z_1)(Z_3-Z_1)^2$

Answers

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(a)	**6.**	(d)	**7.**	(d)	**8.**	(c)	**9.**	(a)

Exercise-4 : Matching Type Problems

1. In a ΔABC, the side lengths BC, CA and AB are consecutive positive integers in increasing order.

	Column-I		Column-II
(A)	If z_1, z_2 and z_3 be the affixes of vertices A, B and C respectively in argand plane, such that $\left\lvert \arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right)\right\rvert = \left\lvert 2\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)\right\rvert$, then biggest side of the triangle is	**(P)**	2
(B)	Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of vertices A, B and C respectively. If $(\vec{c}-\vec{a})\cdot(\vec{b}-\vec{c}) = 0$ then the value of $\lvert \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\rvert$ equals to	**(Q)**	3
(C)	Let the equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ represent the lines AB and AC respectively and $\left\lvert\frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}\right\rvert = \frac{4}{3}$ then the value of $s - c$ (where s is the semiperimeter) $a = BC$, $b = CA$, $c = AB$	**(R)**	4
(D)	If the altitudes of ΔABC are in harmonic progression then the side length 'b' can be	**(S)**	6
		(T)	12

2. Let $ABCDEF$ is a regular hexagon $A(z_1)$, $B(z_2)$, $C(z_3)$, $D(z_4)$, $E(z_5)$, $F(z_6)$ in argand plane where A, B, C, D, E and F are taken in anticlockwise manner. If $z_1 = -2$, $z_3 = 1 - \sqrt{3}\,i$.

	Column-I		Column-II
(A)	If $z_2 = a + ib$, then $2a^2 + b^2$ is equal to	**(P)**	8
(B)	The square of the inradius of hexagon is	**(Q)**	7
(C)	The area of region formed by point $P(z)$ lying inside the incircle of hexagon and satisfying $\frac{\pi}{3} \le \arg(z) \le \frac{5\pi}{6}$ is $\frac{m}{n}\pi$, where m, n are relatively prime natural numbers, then $m + n$ is equal to	**(R)**	5
(D)	The value of $z_4^2 - z_1^2 - z_2^2 - z_3^2 - z_5^2 - z_6^2$ is equal to	**(S)**	3
		(T)	2

3.

	Column-I		Column-II
(A)	Let ω be a non real cube root of unity then the number of distinct elements in the set $\{(1+\omega+\omega^2+\ldots+\omega^n)^m;$ $n, m \in N\}$ is :	**(P)**	3
(B)	Let ω and ω^2 be non real cube root of unity. The least possible degree of a polynomial with real co-efficients having roots $2\omega, (2+3\omega), (2+3\omega)^2, (2-\omega-\omega^2)$ is	**(Q)**	4
(C)	Let $\alpha = 6+4i$ and $\beta = 2+4i$ are two complex numbers on Argand plane. A complex number z satisfying amp $\left(\dfrac{z-\alpha}{z-\beta}\right)=\dfrac{\pi}{6}$ moves on a major segment of a circle whose radius is	**(R)**	5
(D)	Let z_1, z_2, z_3 are complex numbers denoting the vertices of an equilateral triangle ABC having circumradius equals to unity. If P denotes any arbitrary point on its circumcircle then the value of $\frac{1}{2}((PA)^2+(PB)^2+(PC)^2)$ equals to	**(S)**	7

Answers

1.	A $\to$ S ; B $\to$ T ; C $\to$ S ; D $\to$ Q, R, S, T
2.	A $\to$ R ; B $\to$ S ; C $\to$ Q ; D $\to$ P
3.	A $\to$ S ; B $\to$ R ; C $\to$ Q ; D $\to$ P

Exercise-5 : Subjective Type Problems

1. Let complex number 'z' satisfy the inequality $2 \le |z| \le 4$. A point P is selected in this region at random. The probability that argument of P lies in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is $\frac{1}{K}$, then $K =$

2. Let z be a complex number satisfying $|z-3| \le |z-1|$, $|z-3| \le |z-5|$, $|z-i| \le |z+i|$ and $|z-i| \le |z-5i|$. Then the area of region in which z lies is A square units, where $A =$

3. Complex number z_1 and z_2 satisfy $z + \bar{z} = 2|z-1|$ and $\arg(z_1 - z_2) = \frac{\pi}{4}$. Then the value of $\text{Im}(z_1 + z_2)$ is :

4. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 36$, then $|z_1 + z_2 + z_3|$ is equal to :

5. If $|z_1|$ and $|z_2|$ are the distances of points on the curve $5z\bar{z} - 2i(z^2 - \bar{z}^2) - 9 = 0$ which are at maximum and minimum distance from the origin, then the value of $|z_1| + |z_2|$ is equal to :

6. Let $\frac{1}{a_1+\omega} + \frac{1}{a_2+\omega} + \frac{1}{a_3+\omega} + \ldots\ldots + \frac{1}{a_n+\omega} = i$

where $a_1, a_2, a_3, \ldots\ldots\ldots a_n \in R$ and ω is imaginary cube root of unity, then evaluate $\sum_{r=1}^{n} \frac{2a_r - 1}{a_r^2 - a_r + 1}$.

7. If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 9$, then value of $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|^{1/3}$ is :

8. The sum of maximum and minimum modulus of a complex number z satisfying $|z - 25i| \le 15$, $i = \sqrt{-1}$ is S, then $\frac{S}{10}$ is :

Answers

1.	4	**2.**	6	**3.**	2	**4.**	6	**5.**	4	**6.**	0	**7.**	6
8.	5												

□□□

Chapter 12 – Matrices

12 MATRICES

Exercise-1 : Single Choice Problems

1. Let $A = BB^T + CC^T$, where $B = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$, $C = \begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix}$; $\theta \in R$. Then A is :

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

2. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is :

(a) A is a zero matrix (b) $A^2 = I$, where I is a unit matrix

(c) A^{-1} does not exist (d) $A = (-1)I$, where I is a unit matrix

3. Let $A = [a_{ij}]_{3\times 3}$ be such that $a_{ij} = \begin{bmatrix} 3; \text{ when } \hat{i} = \hat{j} \\ 0; \quad \hat{i} \neq \hat{j} \end{bmatrix}$, then $\left\{ \dfrac{\det(\operatorname{adj}(\operatorname{adj} A))}{5} \right\}$ equals :

(where $\{\cdot\}$ denotes fractional part function)

(a) $\dfrac{2}{5}$ (b) $\dfrac{1}{5}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{1}{3}$

4. If $A^{-1} = \begin{bmatrix} \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\beta & 0 \\ 0 & 0 & \sin^2\gamma \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} \cos^2\alpha & 0 & 0 \\ 0 & \cos^2\beta & 0 \\ 0 & 0 & \cos^2\gamma \end{bmatrix}$ where α, β, γ are any real numbers and $C = (A^{-5} + B^{-5}) + 5A^{-1}B^{-1}(A^{-3} + B^{-3}) + 10A^{-2}B^{-2}(A^{-1} + B^{-1})$ then find $|C|$.

(a) 0 (b) 1 (c) 2 (d) 3

5. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$; then $A^{-1} =$

(a) A (b) A^2 (c) A^3 (d) A^4

6. Let $M = [a_{ij}]_{3\times 3}$ where $a_{ij} \in \{-1, 1\}$. Find the maximum possible value of $\det(M)$.

(a) 3 (b) 4 (c) 5 (d) 6

7. Let matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$; if $xyz = 2\lambda$ and $8x + 4y + 3z = \lambda + 28$, then (adj A) A equals :

(a) $\begin{bmatrix} \lambda+1 & 0 & 0 \\ 0 & \lambda+1 & 0 \\ 0 & 0 & \lambda+1 \end{bmatrix}$ (b) $\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

(c) $\begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda^2 \end{bmatrix}$ (d) $\begin{bmatrix} \lambda+2 & 0 & 0 \\ 0 & \lambda+2 & 0 \\ 0 & 0 & \lambda+2 \end{bmatrix}$

8. If the trace of matrix $A = \begin{pmatrix} x-2 & e^x & -\sin x \\ \cos x^2 & x^2 - x + 3 & \ln|x| \\ 0 & \tan^{-1} x & x-7 \end{pmatrix}$ is zero, then x is equal to :

(a) -2 or 3 (b) -3 or -2 (c) -3 or 2 (d) 2 or 3

9. If $A = [a_{ij}]_{2\times 2}$ where $a_{ij} = \begin{cases} i+j, & i \neq j \\ i^2 - 2j, & i = j \end{cases}$ then A^{-1} is equal to :

(a) $\frac{1}{9}\begin{bmatrix} 0 & 3 \\ 3 & 1 \end{bmatrix}$ (b) $\frac{1}{9}\begin{bmatrix} 0 & -3 \\ 3 & -1 \end{bmatrix}$ (c) $\frac{1}{9}\begin{bmatrix} 0 & -3 \\ -3 & -1 \end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix} 0 & 3 \\ 3 & 1 \end{bmatrix}$

10. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then :

(a) $a = b = 1$ (b) $a = \cos 2\theta$, $b = \sin 2\theta$

(c) $a = \sin 2\theta$, $b = \cos 2\theta$ (d) $a = 1$, $b = \sin 2\theta$

11. A square matrix P satisfies $P^2 = I - P$, where I is identity matrix. If $P^n = 5I - 8P$, then n is :

(a) 4 (b) 5 (c) 6 (d) 7

12. Let matrix $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ where $x, y, z \in N$. If det. (adj. (adj. A)) $= 2^8 \cdot 3^4$ then the number of such matrices A is :

[Note : adj. A denotes adjoint of square matrix A.]

(a) 220 (b) 45 (c) 55 (d) 110

13. If A is a 2×2 non singular matrix, then adj (adj A) is equal to :

(a) A^2 (b) A (c) A^{-1} (d) $(A^{-1})^2$

14. $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and $MA = A^{2m}$, $m \in N, a, b \in R$, for some matrix M, then which one of the following is correct :

(a) $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$ (b) $M = (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $M = (a^m + b^m)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $M = (a^2 + b^2)^{m-1}\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

15. Let A be a square matrix satisfying $A^2 + 5A + 5I = 0$. The inverse of $A + 2I$ is equal to :

(a) $A - 2I$ (b) $A + 3I$ (c) $A - 3I$ (d) non-existent

16. Let $A = \begin{bmatrix} 3 & -5 \\ 7 & -12 \end{bmatrix}$ and $B = \begin{bmatrix} 12 & -5 \\ 7 & -3 \end{bmatrix}$ be two given matrices, then $(AB)^{-1}$ is :

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

17. If matrix $A = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$ then the value of $|\text{adj. } A|$ equals to :

(a) 2 (b) 3 (c) 4 (d) 6

18. If for the matrix $A = \begin{bmatrix} \cos\theta & 2\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $A^{-1} = A^T$ then number of possible value(s) of θ in $[0, 2\pi]$ is:

(a) 2 (b) 3 (c) 1 (d) 4

19. Let M be a column vector (not null vector) and $A = \dfrac{MM^T}{M^TM}$ the matrix A is :

(where M^T is transpose matrix of M)

(a) idempotant (b) nilpotent (c) involutary (d) none of these

20. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $P = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, $Q = P^TAP$, find $PQ^{2014}P^T$:

(a) $\begin{pmatrix} 1 & 2^{2014} \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 4028 \\ 0 & 1 \end{pmatrix}$

(c) $(P^T)^{2013}A^{2014}P^{2013}$ (d) $P^TA^{2014}P$

21. If M be a square matrix of order 3 such that $|M| = 2$, then $\left|adj\left(\dfrac{M}{2}\right)\right|$ equals to :

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{8}$ (d) $\dfrac{1}{16}$

22. If A is matrix of order 3 such that $|A| = 5$ and $B = adj A$, then the value of $\left||A^{-1}|(AB)^T\right|$ is equal to (where $|A|$ denotes determinant of matrix A. A^T denotes transpose of matrix A, A^{-1} denotes inverse of matrix A. $adj A$ denotes adjoint of matrix A)

(a) 5 (b) 1 (c) 25 (d) $\dfrac{1}{25}$

Answers

1.	(c)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(c)	**6.**	(b)	**7.**	(b)	**8.**	(c)	**9.**	(a)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(b)	**14.**	(d)	**15.**	(b)	**16.**	(b)	**17.**	(a)	**18.**	(b)	**19.**	(a)	**20.**	(b)
21.	(d)	**22.**	(b)																

Exercise-2 : One or More than One Answer is/are Correct

1. If A and B are two orthogonal matrices of order n and $\det(A) + \det(B) = 0$, then which of the following must be correct ?

(a) $\det(A + B) = \det(A) + \det(B)$ (b) $\det(A + B) = 0$

(c) A and B both are singular matrices (d) $A + B = 0$

2. Let M be a 3×3 matrix satisfying $M^3 = 0$. Then which of the following statement(s) are true:

(a) $\left|\frac{1}{2}M^2 + M + I\right| \neq 0$ (b) $\left|\frac{1}{2}M^2 - M + I\right| = 0$

(c) $\left|\frac{1}{2}M^2 + M + I\right| = 0$ (d) $\left|\frac{1}{2}M^2 - M + I\right| \neq 0$

3. Let $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then :

(a) $A_{\alpha+\beta} = A_\alpha A_\beta$ (b) $A_\alpha^{-1} = A_{-\alpha}$

(c) $A_\alpha^{-1} = -A_\alpha$ (d) $A_\alpha^2 = -I$

4. $A^3 - 2A^2 - A + 2I = 0$ if $A =$

(a) I (b) $2I$ (c) $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

5. Let A be a 3×3 symmetric invertible matrix with real positive elements. Then the number of zero elements in A^{-1} are less than or equal to :

(a) 0 (b) 1 (c) 2 (d) 3

Answers

1.	(a, b)	**2.**	(a, d)	**3.**	(a, b)	**4.**	(a, b, c, d)	**5.**	(d)		

Exercise-3 : Matching Type Problems

1. Consider a square matrix A of order 2 which has its elements as 0, 1, 2 and 4. Let N denotes the number of such matrices.

	Column-I		Column-II
(A)	Possible non-negative value of $\det(A)$ is	**(P)**	2
(B)	Sum of values of determinants corresponding to N matrices is	**(Q)**	4
(C)	If absolute value of $(\det(A))$ is least, then possible value of $\lvert \operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))\rvert$	**(R)**	-2
(D)	If $\det(A)$ is least, then possible value of $\det(4A^{-1})$ is	**(S)**	0
		(T)	8

2.

	Column-I		Column-II
(A)	If A is an idempotent matrix and I is an identify matrix of the same order, then the value of n, such that $(A+I)^n = I + 127A$ is	**(P)**	9
(B)	If $(I-A)^{-1} = I + A + A^2 + \ldots\ldots A^7$, then $A^n = O$ where n is	**(Q)**	10
(C)	If A is matrix such that $a_{ij} = (i+j)(i-j)$, then A is singular if order of matrix is	**(R)**	7
(D)	If a non-singular matrix A is symmetic, such that A^{-1} is also symmetric, then order of A can be	**(S)**	8

3.

	Column-I		Column-II
(A)	Number of ordered pairs (x, y) of real numbers satisfying $\sin x + \cos y = 0$, $\sin^2 x + \cos^2 y = \frac{1}{2}$, $0 < x < \pi$ and $0 < y < \pi$, is equal to	**(P)**	0
(B)	Given $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are three vectors such that $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are unit like vectors and $\lvert\vec{\mathbf{a}}\rvert = 4$. If $\vec{\mathbf{a}} + \lambda\vec{\mathbf{c}} = 2\vec{\mathbf{b}}$ then the sum of all possible values of λ is equal to	**(Q)**	2

(C)	If $P = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10Q = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & t \\ 1 & -2 & 3 \end{bmatrix}$ and $Q = P^{-1}$, then the value of t is equal to	**(R)**	4
(D)	If $y = \tan u$ where $u = v - \frac{1}{v}$ and $v = \ln x$, then the value of $\frac{dy}{dx}$ at $x = e$ is equal to λ then $[\lambda]$ is equal to (where [·] denotes greatest integer function)	**(S)**	5

4.

	Column-I		Column-II
(A)	If P and Q are variable points on $C_1 : x^2 + y^2 = 4$ and $C_2 : x^2 + y^2 - 8x - 6y + 24 = 0$ respectively then the maximum value of PQ, is equal to	**(P)**	1
(B)	Let P, Q, R be invertible matrices of second order such that $A = PQ^{-1}, B = QR^{-1}, C = RP^{-1}$, then the value of det. $(ABC + BCA + CAB)$ is equal to	**(Q)**	2
(C)	The perpendicular distance of the point whose position vector is (1, 3, 5) from the line $\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ is equal to	**(R)**	9
(D)	Let $f(x)$ be a continuous function in $[-1, 1]$ such that $f(x) = \begin{cases} \dfrac{\ln(px^2 + qx + r)}{x^2} & ; \ -1 \le x < 0 \\ 1 & ; \ x = 0 \ ; \\ \dfrac{\sin(e^{x^2} - 1)}{x^2} & ; \ 0 < x \le 1 \end{cases}$ then the value of $(p + q + r)$, is equal to	**(S)**	8

5.

	Column-I		Column-II
(A)	$\lim\limits_{n \to \infty} \frac{1}{\sqrt{n}} \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}}\right)$ has the value equal to	**(P)**	1

(B)	Let $A=[a_{ij}]$ be a 3×3 matrix where $a_{ij}=\begin{cases}2\cos t\,; & \text{if } i=j\\ 1\,; & \text{if } \lvert i-j\rvert=1\\ 0\,; & \text{otherwise}\end{cases}$ then maximum value of $\det(A)$ is	**(Q)**	2
(C)	Let $f(x)=x^3+px^2+qx+6$; where $p, q\in R$ and $f'(x)<0$ in largest possible interval $\left(-\frac{5}{3},-1\right)$ then value of $q-p$ is	**(R)**	3
(D)	If $4^x-2^{x+2}+5+\lvert\lvert b-1\rvert-3\rvert=\lvert\sin y\rvert$; $x, y, b\in R$ then the sum of the possible values of b is λ then $(\lambda+1)$ equals	**(S)**	4

Answers

1.	A → P, Q, T; B → S ; C → P, R ; D → R
2.	A → R; B → P, Q, S ; C → P, R; D → P, Q, R, S
3.	A → Q ; B → R ; C → S ; D → P
4.	A → S ; B → R ; C → P ; D → Q
5.	A → Q ; B → S ; C → P ; D → R

Exercise-4 : Subjective Type Problems

1. A and B are two square matrices. Such that $A^2B = BA$ and if $(AB)^{10} = A^k \cdot B^{10}$. Find the value of $k - 1020$.
2. Let A_n and B_n be square matrices of order 3, which are defined as :
   $A_n = [a_{ij}]$ and $B_n = [b_{ij}]$ where $a_{ij} = \dfrac{2i + j}{3^{2n}}$ and $b_{ij} = \dfrac{3i - j}{2^{2n}}$ for all i and j, $1 \le i, j \le 3$.
   If $l = \lim\limits_{n\to\infty} \text{Tr.}\ (3A_1 + 3^2A_2 + 3^3A_3 + \ldots\ldots + 3^nA_n)$ and
   $m = \lim\limits_{n\to\infty} \text{Tr.}\ (2B_1 + 2^2B_2 + 2^3B_3 + \ldots\ldots + 2^nB_n)$, then find the value of $\dfrac{(l+m)}{3}$
   [Note : Tr. (P) denotes the trace of matrix P.]
3. Let A be a 2×3 matrix whereas B be a 3×2 matrix. If det. $(AB) = 4$, then the value of det. (BA), is :
4. Find the maximum value of the determinant of an arbitrary 3×3 matrix A, each of whose entries $a_{ij} \in \{-1, 1\}$.
5. The set of natural numbers is divided into array of rows and columns in the form of matrices as
   $A_1 = [1], A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ and so on. Let the trace of A_{10} be λ. Find unit digit of λ?

Answers

1.	3	2.	7	3.	0	4.	4	5.	5				

❑❑❑

Chapter 13 – Permutation and Combination

13 PERMUTATION AND COMBINATIONS

Exercise-1 : Single Choice Problems

1. The number of 3-digit numbers containing the digit 7 exactly once :
   (a) 225 (b) 220 (c) 200 (d) 180
2. Let $A = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, $B = \{y_1, y_2, y_3, y_4\}$. The total number of function $f : A \to B$ that are onto and there are exactly three elements x in A such that $f(x) = y_1$ is :
   (a) 11088 (b) 10920 (c) 13608 (d) None of these
3. The number of arrangements of the word "IDIOTS" such that vowels are at the places which form three consecutive terms of an A.P. is :
   (a) 36 (b) 72 (c) 24 (d) 108
4. Consider all the 5 digit numbers where each of the digits is chosen from the set $\{1, 2, 3, 4\}$. Then the number of numbers, which contain all the four digits is :
   (a) 240 (b) 244 (c) 586 (d) 781
5. How many ways are there to arrange the letters of the word "GARDEN" with the vowels in alphabetical order ?
   (a) 120 (b) 480 (c) 360 (d) 240
6. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is :
   (a) $3x^2 - 19x + 3 = 0$ (b) $3x^2 + 19x - 3 = 0$
   (c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 5x + 3 = 0$
7. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is :
   (a) 140 (b) 196 (c) 280 (d) 346
8. Let set $A = \{1, 2, 3, \ldots, 22\}$. Set B is a subset of A and B has exactly 11 elements, find the sum of elements of all possible subsets B.
   (a) $252\,{}^{21}C_{11}$ (b) $230\,{}^{21}C_{10}$
   (c) $253\,{}^{21}C_9$ (d) $253\,{}^{21}C_{10}$

9. The value of $\left[\dfrac{2009!+2006!}{2008!+2007!}\right]=$

([·] denotes greatest integer function.)

(a) 2009 (b) 2008 (c) 2007 (d) 1

10. If $p_1, p_2, p_3, \ldots\ldots, p_{m+1}$ are distinct prime numbers. Then the number of factors of $p_1^n p_2 p_3 \ldots\ldots p_{m+1}$ is :

(a) $m(n+1)$ (b) $(n+1)2^m$ (c) $n \cdot 2^m$ (d) 2^{nm}

11. A basket ball team consists of 12 pairs of twin brothers. On the first day of training, all 24 players stand in a circle in such a way that all pairs of twin brothers are neighbours. Number of ways this can be done is :

(a) $(12)!\,2^{11}$ (b) $(11)!\,2^{12}$ (c) $(12)!\,2^{12}$ (d) $(11)!\,2^{11}$

12. Let 'm' denotes the number of four digit numbers such that the left most digit is odd, the second digit is even and all four digits are different and 'n' denotes the number of four digit numbers such that left most digit is even, second digit is odd and all four digits are different. If $m = nk$, then k equals :

(a) $\dfrac{4}{5}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{5}{4}$ (d) $\dfrac{4}{3}$

13. The number of three digit numbers of the form xyz such that $x < y$ and $z \le y$ is :

(a) 156 (b) 204 (c) 240 (d) 276

14. A and B are two sets and their intersection has 3 elements. If A has 1920 more subsets than B has, then the number of elements of A union B is :

(a) 12 (b) 14 (c) 15 (d) 16

15. All possible 120 permutations of WDSMC are arranged in dictionary order, as if each were an ordinary five-letter word. The last letter of the 86th word in the list, is :

(a) W (b) D (c) M (d) C

16. The number of permutation of all the letters $AAAABBBC$ in which the A's appear together in a block of 4 letters or the B's appear together in a block of 3 letters is :

(a) 44 (b) 50 (c) 60 (d) 89

17. Number of zero's at the ends of $\prod_{n=5}^{30}(n)^{n+1}$ is :

(a) 111 (b) 147 (c) 137 (d) None of these

18. The number of positive integral pairs (x, y) satisfying the equation $x^2 - y^2 = 3370$ is :

(a) 0 (b) 1 (c) 2 (d) 4

19. The number of ways of selecting 'n' things out of '$3n$' things of which 'n' are of one kind and alike and 'n' are of second kind and alike and the rest unlike is :

(a) $n\,2^{n-1}$ (b) $(n-1)2^{n-1}$ (c) $(n+1)2^{n-1}$ (d) $(n+2)2^{n-1}$

20. If x, y, z are three natural numbers in A.P. such that $x + y + z = 30$, then the possible number of ordered triplet (x, y, z) is :

(a) 18 (b) 19 (c) 20 (d) 21

21. A dice is rolled 4 times, the numbers appearing are listed. The number of different throws, such that the largest number appearing in the list is not 4, is : :

(a) 175 (b) 625 (c) 1040 (d) 1121

22. Let m denotes the number of ways in which 5 boys and 5 girls can be arranged in a line alternately and n denotes the number of ways in which 5 boys and 5 girls an be arranged in a cirlce so that no two boys are together. if $m = kn$ then the value of k is :

(a) 2 (b) 5 (c) 6 (d) 10

23. Number of ways in which 4 students can sit in 7 chair in a row, if there is no empty chair between any two students is :

(a) 24 (b) 28 (c) 72 (d) 96

24. Number of zero's at the ends of $\prod_{n=5}^{30}(n)^{n+1}$ is :

(a) 111 (b) 147 (c) 137 (d) None

25. The number of words of four letters consisting of equal number of vowels and consonants (of english language) with repetition permitted is :

(a) 51030 (b) 50030 (c) 63050 (d) 66150

26. Ten different letters of an alphabet are given. Words with five letters are formed with these given letters. Then the number of words which have atleast one letter repeated is :

(a) 30240 (b) 69760 (c) 69780 (d) 99784

27. Number of four digit numbers in which at least one digit occurs more than once, is:

(a) 4464 (b) 4644 (c) 4446 (d) 6444

28. In a game of minesweeper, a number on a square denotes the number of mines that share at least one vertex with that square. A square with a number may not have a mine, and the blank squares are undetermined. In how many ways can the mines be placed in the given configuration on blank squares:

	2		1		2

(a) 120 (b) 105 (c) 95 (d) 100

29. Let the product of all the divisors of 1440 be P. If P is divisible by 24^x, then the maximum value of x is :

(a) 28 (b) 30 (c) 32 (d) 36

30. Let N be the number of 4-digit numbers which contain not more than 2 different digits. The sum of the digits of N is :

(a) 18 (b) 19 (c) 20 (d) 21

31. The number of different permutations of all the letters of the word PERMUTATION such that any two consecutive letters in the arrangement are neither both vowels nor both identical is :

(a) $63 \times \underline{|6} \times \underline{|5}$ (b) $8 \times \underline{|6} \times \underline{|5}$ (c) $57 \times \underline{|5} \times \underline{|5}$ (d) $7 \times \underline{|7} \times \underline{|5}$

32. A batsman can score 0, 1, 2, 3, 4 or 6 runs from a ball. The number of different sequences in which he can score exactly 30 runs in an over of six balls :

(a) 4 (b) 72 (c) 56 (d) 71

33. A batsman can score 0, 2, 3, or 4 runs for each ball he receives. If N is the number of ways of scoring a total of 20 runs in one over of six balls, then N is divisible by:

(a) 5 (b) 7 (c) 14 (d) 16

34. The number of non-negative integral solutions of the equation $x + y + z = 5$ is :

(a) 20 (b) 19 (c) 21 (d) 25

35. The number of solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 101$, where $x_i's$ are odd natural numbers is :

(a) ${}^{105}C_4$ (b) ${}^{52}C_5$ (c) ${}^{52}C_4$ (d) ${}^{50}C_4$

36. An ordinary dice is rolled 4 times, numbers appearing on them are listed. The number of different throws, such that the largest number appearing on them is NOT 4, is :

(a) 175 (b) 625 (c) 1121 (d) 1040

37. Number of four letter words can be formed using the letters of word VIBRANT if letter V is must included, are :

(a) 840 (b) 480 (c) 120 (d) 240

38. The number of rectangles that can be obtained by joining four of the twelve vertices of a 12-sided regular polygon is :

(a) 66 (b) 30 (c) 24 (d) 15

39. Number of five digit integers, with sum of the digits equal to 43 are :

(a) 5 (b) 10 (c) 15 (d) 35

Answers

1.	(a)	**2.**	(d)	**3.**	(d)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(b)	**8.**	(d)	**9.**	(b)	**10.**	(b)
11.	(b)	**12.**	(c)	**13.**	(d)	**14.**	(c)	**15.**	(b)	**16.**	(a)	**17.**	(c)	**18.**	(a)	**19.**	(d)	**20.**	(b)
21.	(d)	**22.**	(d)	**23.**	(d)	**24.**	(c)	**25.**	(d)	**26.**	(b)	**27.**	(a)	**28.**	(c)	**29.**	(b)	**30.**	(a)
31.	(c)	**32.**	(d)	**33.**	(d)	**34.**	(c)	**35.**	(c)	**36.**	(c)	**37.**	(b)	**38.**	(d)	**39.**	(c)		

Exercise-2 : One or More than One Answer is/are Correct

1. The number of 5 letter words formed with the letters of the word CALCULUS is divisible by :

(a) 2 (b) 3 (c) 5 (d) 7

2. The coefficient of x^{50} in the expansion of $\sum_{k=0}^{100} {}^{100}C_k (x-2)^{100-k} 3^k$ is also equal to :

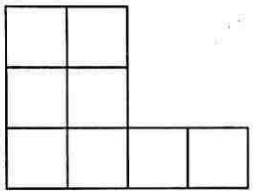
(a) Number of ways in which 50 identical books can be distributed in 100 students, if each student can get atmost one book.

(b) Number of ways in which 100 different white balls and 50 identical red balls can be arranged in a circle, if no two red balls are together.

(c) Number of dissimilar terms in $(x_1 + x_2 + x_3 + \ldots + x_{50})^{51}$.

(d) $\dfrac{2\cdot 6\cdot 10\cdot 14 \ldots\ldots 198}{50!}$

3. Number of ways in which the letters of the word "NATION" can be filled in the given figure such that no row remains empty and each box contains not more than one letter, are :



(a) $11\underline{|6}$ (b) $12\underline{|6}$ (c) $13\underline{|6}$ (d) $14\underline{|6}$

4. Let a, b, c, d be non zero distinct digits. The number of 4 digit numbers $abcd$ such that $ab + cd$ is even is divisible by :

(a) 3 (b) 4 (c) 7 (d) 11

Answers

1.	(a, b, c)	**2.**	(a, d)	**3.**	(c)	**4.**	(a, b, d)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Consider all the six digit numbers that can be formed using the digits 1, 2, 3, 4, 5 and 6, each digit being used exactly once. Each of such six digit numbers have the property that for each digit, not more than two digits smaller than that digit appear to the right of that digit.

1. A six digit number which does not satisfy the property mentioned above, is :

(a) 315426 (b) 135462 (c) 234651 (d) None of these

2. Number of such six digit numbers having the desired property is :

(a) 120 (b) 144 (c) 162 (d) 210

Answers

1.	(d)	**2.**	(c)

Exercise-4 : Matching Type Problems

1. All letters of the word BREAKAGE are to be jumbled. The number of ways of arranging them so that :

	Column-I		Column-II
(A)	The two A's are not together	**(P)**	720
(B)	The two E's are together but not two A's	**(Q)**	1800
(C)	Neither two A's nor two E's are together	**(R)**	5760
(D)	No two vowels are together	**(S)**	6000
		(T)	7560

2. Consider the letters of the word MATHEMATICS. Set of repeating letters = { M, A, T}, set of non repeating letters = { H, E, I, C, S }:

	Column-I		Column-II
(A)	The number of words taking all letters of the given word such that atleast one repeating letter is at odd position is	**(P)**	$28 \cdot (7!)$
(B)	The number of words formed taking all letters of the given word in which no two vowels are together is	**(Q)**	$\frac{(11)!}{(2!)^3}$
(C)	The number of words formed taking all letters of the given word such that in each word both M's are together and both T's are together but both A's are not together is	**(R)**	$210(7!)$
(D)	The number of words formed taking all letters of the given word such that relative order of vowels and consonants does not change is	**(S)**	$840\,(7!)$
		(T)	$\frac{4!7!}{(2!)^3}$

Answers

1.	$A \to T$; $B \to Q$; $C \to R$; $D \to P$
2.	$A \to Q$; $B \to R$; $C \to P$; $D \to T$

Exercise-5 : Subjective Type Problems

1. The number of ways in which eight digit number can be formed using the digits from 1 to 9 without repetition if first four places of the numbers are in increasing order and last four places are in decreasing order is N, then find the value of $\frac{N}{70}$.

2. Number of ways in which the letters of the word DECISIONS be arranged so that letter N be somewhere to the right of the letter "D" is $\frac{\underline{|9}}{\lambda}$. Find λ.

4. There are 10 stations enroute. A train has to be stopped at 3 of them. Let N be the ways in which the train can be stopped if atleast two of the stopping stations are consecutive. Find the value of $\sqrt{N}$.

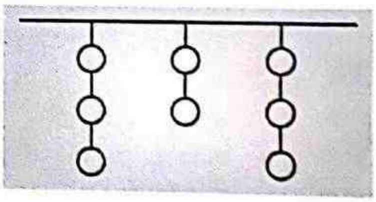
5. There are 10 girls and 8 boys in a class room including Mr. Ravi, Ms. Rani and Ms. Radha. A list of speakers consisting of 8 girls and 6 boys has to be prepared. Mr. Ravi refuses to speak if Ms. Rani is a speaker. Ms. Rani refuses to speak if Ms. Radha is a speaker. The number of ways the list can be prepared is a 3 digit number $n_1 n_2 n_3$, then $|n_3 + n_2 - n_1| =$

6. Nine people sit around a round table. The number of ways of selecting four of them such that they are not from adjacent seats, is

7. Let the number of arrangements of all the digits of the numbers 12345 such that atleast 3 digits will not come in it's original position is N. Then the unit digit of N is

8. The number of triangles with each side having integral length and the longest side is of 11 units is equal to k^2, then the value of 'k' is equal to

9. 8 clay targets are arranged as shown. If N be the number of different ways they can be shot (one at a time) if no target can be shot until the target(s) below it have been shot. Find the ten's digit of N.



10. There are n persons sitting around a circular table. They start singing a 2 minute song in pairs such that no two persons sitting together will sing together. This process is continued for 28 minutes. Find n .

11. The number of ways to choose 7 distinct natural numbers from the first 100 natural numbers such that any two chosen numbers differ atleast by 7 can be expressed as ${}^nC_7$. Find the number of divisors of n.

12. Four couples (husband and wife) decide to form a committee of four members. The number of different committees that can be formed in which no couple finds a place is λ, then the sum of digits of λ is :

13. The number of ways in which $2n$ objects of one type, $2n$ of another type and $2n$ of a third type can be divided between 2 persons so that each may have $3n$ objects is $\alpha n^2 + \beta n + \gamma$. Find the value of $(\alpha + \beta + \gamma)$.

14. Let N be the number of integral solution of the equation $x + y + z + w = 15$ where $x \geq 0, y > 5$, $z \geq 2$ and $w \geq 1$. Find the unit digit of N.

Answers

1.	9	**2.**	8	**3.**	8	**4.**	8	**5.**	5	**6.**	9	**7.**	9
8.	6	**9.**	6	**10.**	7	**11.**	7	**12.**	7	**13.**	7	**14.**	4

□□□

Chapter 14 – Binomial Theorem

14

BIONMIAL THEOREM

Exercise-1 : Single Choice Problems

1. Let $N = 2^{1224} - 1$, $\alpha = 2^{153} + 2^{77} + 1$ and $\beta = 2^{408} - 2^{204} + 1$. Then which of the following statement is correct ?

(a) α divides N but β does not
(b) β divides N but α does not
(c) α and β both divide N
(d) neither α nor β divides N

2. If $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r - {}^nC_1 \cdot a_{r-1} + {}^nC_2 a_{r-2} - {}^nC_3 a_{r-3} + \ldots\ldots + (-1)^r\, {}^nC_r a_0$ is equal to : (r is not multiple of 3)

(a) 0
(b) ${}^nC_r$
(c) a_r
(d) 1

3. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals :

(a) $-\dfrac{5}{3}$
(b) $\dfrac{3}{5}$
(c) $\dfrac{-3}{10}$
(d) $\dfrac{10}{3}$

4. If $(1+x)^{2010} = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_{2010} x^{2010}$ then the sum of series $C_2 + C_5 + C_8 + \ldots\ldots + C_{2009}$ equals to :

(a) $\dfrac{1}{2}(2^{2010} - 1)$
(b) $\dfrac{1}{3}(2^{2010} - 1)$
(c) $\dfrac{1}{2}(2^{2009} - 1)$
(d) $\dfrac{1}{3}(2^{2009} - 1)$

5. Let $\alpha_n = (2+\sqrt{3})^n$. Find $\lim_{n\to\infty} (\alpha_n - [\alpha_n])$ ([.] denotes greatest integer function)

(a) 1
(b) $\dfrac{1}{2}$
(c) $\dfrac{1}{3}$
(d) $\dfrac{2}{3}$

6. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \ldots\ldots - {}^{20}C_{20}$ is not divisible by :

(a) 3
(b) 7
(c) 11
(d) 19

7. The value of the expression $\log_2\left(1+\frac{1}{2}\sum_{k=1}^{11} {}^{12}C_k\right)$:

(a) 11 (b) 12 (c) 13 (d) 14

8. The constant term in the expansion of $\left(x+\frac{1}{x^3}\right)^{12}$ is :

(a) 26 (b) 169 (c) 260 (d) 220

9. If $\frac{3}{4!}+\frac{4}{5!}+\frac{5}{6!}+......+50 \text{ term} = \frac{1}{3!}-\frac{1}{(k+3)!}$, then sum of coefficients in the expansion $(1+2x_1+3x_2+......+100x_{100})^k$ is :

(where $x_1, x_2, x_3, ..., x_{100}$ are independent variables)

(a) $(5050)^{49}$ (b) $(5050)^{51}$

(c) $(5050)^{52}$ (d) $(5050)^{50}$

10. Statement-1: The remainder when $(128)^{(128)^{128}}$ is divided by 7 is 3.

because

Statement-2: $(128)^{128}$ when divided by 3 leaves the remainder 1.

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

11. If $n > 3$, then $xyz\,{}^nC_0-(x-1)(y-1)(z-1)\,{}^nC_1+(x-2)(y-2)(z-2)\,{}^nC_2-(x-3)(y-3)(z-3)\,{}^nC_3+.....+(-1)^n(x-n)(y-n)(z-n)\,{}^nC_n$ equals :

(a) xyz (b) $x+y+z$

(c) $xy+yz+zx$ (d) 0

12. If $\alpha_1, \alpha_2, ..., \alpha_n$ are the n; n^{th} roots of unity, $\alpha_r = e^{\frac{i2(r-1)\pi}{n}}$, $r = 1, 2, \ldots n$ then ${}^nC_1\alpha_1+{}^nC_2\alpha_2+.....+{}^nC_n\alpha_n$ is equal to:

(a) $\left(1+\frac{\alpha_2}{\alpha_1}\right)^n-1$ (b) $\frac{\alpha_1}{2}[(1+\alpha_1)^n-1]$ (c) $\frac{\alpha_1+\alpha_{n-1}-1}{2}$ (d) $(\alpha_1+\alpha_{n-1})^n-1$

13. The remainder when $2^{30}\cdot 3^{20}$ is divided by 7 is :

(a) 1 (b) 2 (c) 4 (d) 6

14. ${}^{26}C_0+{}^{26}C_1+{}^{26}C_2+.....+{}^{26}C_{13}$ is equal to :

(a) $2^{25}-\frac{1}{2}\cdot{}^{26}C_{13}$ (b) $2^{25}+\frac{1}{2}\cdot{}^{26}C_{13}$ (c) 2^{13} (d) $2^{26}+\frac{1}{2}\cdot{}^{26}C_{13}$

15. If a_r is the coefficient of x^r in the expansion of $(1+x+x^2)^n\ (n \in N)$. Then the value of $(a_1 + 4a_4 + 7a_7 + 10a_{10} + \ldots\ldots)$ is equal to :

(a) 3^{n-1} (b) 2^n (c) $\frac{1}{3}\cdot 2^n$ (d) $n \cdot 3^{n-1}$

16. Let $\binom{n}{k}$ represents the combination of 'n' things taken 'k' at a time, then the value of the sum

$$\binom{99}{97}+\binom{98}{96}+\binom{97}{95}+\ldots\ldots+\binom{3}{1}+\binom{2}{0} \text{ equals:}$$

(a) $\binom{99}{97}$ (b) $\binom{100}{98}$ (c) $\binom{99}{98}$ (d) $\binom{100}{97}$

17. The last digit of $9! + 3^{9966}$ is :

(a) 1 (b) 3 (c) 7 (d) 9

18. Let x be the 7^{th} term from the beginning and y be the 7^{th} term from the end in the expansion of $\left(3^{1/3} + \frac{1}{4^{1/3}}\right)^n$. If $y = 12x$ then the value of n is :

(a) 9 (b) 8 (c) 10 (d) 11

19. The expression $({}^{10}C_0)^2 - ({}^{10}C_1)^2 + ({}^{10}C_2)^2 - \ldots\ldots + ({}^{10}C_8)^2 - ({}^{10}C_9)^2 + ({}^{10}C_{10})^2$ equals :

(a) $10\,!$ (b) $({}^{10}C_5)^2$ (c) $-{}^{10}C_5$ (d) ${}^{10}C_5$

20. The ratio of the co-efficients to x^{15} to the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$ is :

(a) $1:4$ (b) $1:32$ (c) $7:64$ (d) $7:16$

21. In the expansion of $(1+x)^2(1+y)^3(1+z)^4(1+w)^5$, the sum of the coefficient of the terms of degree 12 is :

(a) 61 (b) 71 (c) 81 (d) 91

22. If $\sum_{r=0}^{n}\left(\frac{r^3+2r^2+3r+2}{(r+1)^2}\right){}^nC_r = \frac{2^4+2^3+2^2-2}{3}$

then the value of n is :

(a) 2 (b) 2^2 (c) 2^3 (d) 2^4

Answers

1.	(c)	**2.**	(a)	**3.**	(c)	**4.**	(b)	**5.**	(a)	**6.**	(c)	**7.**	(a)	**8.**	(d)	**9.**	(d)	**10.**	(d)
11.	(d)	**12.**	(a)	**13.**	(b)	**14.**	(b)	**15.**	(d)	**16.**	(d)	**17.**	(d)	**18.**	(a)	**19.**	(c)	**20.**	(b)
21.	(d)	**22.**	(a)																

Exercise-2 : One or More than One Answer is/are Correct

1. The number $N = {}^{20}C_7 - {}^{20}C_8 + {}^{20}C_9 - {}^{20}C_{10} + \ldots\ldots - {}^{20}C_{20}$ is divisible by :

(a) 3 (b) 4 (c) 7 (d) 19

2. If $(1+x+x^2+x^3)^{100} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{300}x^{300}$ then which of the following statement(s) is/are correct ?

(a) $a_1 = 100$

(b) $a_0 + a_1 + a_2 + \ldots\ldots + a_{300}$ is divisible by 1024

(c) coefficients equidistant from beginning and end are equal

(d) $a_0 + a_2 + a_4 + \ldots\ldots + a_{300} = a_1 + a_3 + a_5 + \ldots\ldots + a_{299}$

3. $\sum_{r=0}^{4}(-1)^r \, {}^{16}C_r$ is divisible by :

(a) 5 (b) 7 (c) 11 (d) 13

4. The expansion of $\left(\sqrt{x} + \frac{1}{2\sqrt[4]{x}}\right)^n$ is arranged in decreasing powders of x. If coefficient of first three terms form an A.P. then in expansion, the integral powers of x are :

(a) 0 (b) 2 (c) 4 (d) 8

5. Let $(1+x^2)^2(1+x)^n = \sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2, a_3 are in AP, then n is (given that ${}^nC_r = 0$, if $n < r$):

(a) 6 (b) 4 (c) 3 (d) 2

6. $\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n}\binom{n}{i}\binom{n}{j}\binom{n}{k}$, $\binom{n}{r} = {}^nC_r$:

(a) is less than 500 if $n = 3$

(b) is greater than 600 if $n = 3$

(c) is less than 5000 if $n = 4$

(d) is greater than 4000 if $n = 4$

7. If ${}^{100}C_6 + 4 . {}^{100}C_7 + 6 . {}^{100}C_8 + 4 . {}^{100}C_9 + {}^{100}C_{10}$ has the value equal to ${}^xC_y$; then the possible value(s) of $x + y$ can be :

(a) 112 (b) 114 (c) 196 (d) 198

8. If the co-efficient of x^{2r} is greater than half of the co-efficient of x^{2r+1} in the expansion of $(1+x)^{15}$; then the possible value of 'r' equal to :

(a) 5 (b) 6 (c) 7 (d) 8

9. Let $f(x) = 1 + x^{111} + x^{222} + x^{333} \ldots\ldots\ldots + x^{999}$ then $f(x)$ is divisible by

(a) $x+1$

(b) x

(c) $x-1$

(d) $1 + x^{222} + x^{444} + x^{666} + x^{888}$

Answers

1.	(a, b, c, d)	**2.**	(a, b, c, d)	**3.**	(a, b, d)	**4.**	(a, c, d)	**5.**	(b, c, d)	**6.**	(c, d)
7.	(b, d)	**8.**	(a, b, c)	**9.**	(a, d)						

Exercise-3 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If ${}^{n-1}C_r = (k^2-3)\,{}^nC_{r+1}$ and $k \in R^+$, then least value of $5[k]$ is (where [·] represents greatest integer function)	**(P)**	10
(B)	$\sum_{i=0}^{m} {}^{20}C_i\ {}^{40}C_{m-i}$, where ${}^nC_r = 0$ if $r > n$, is maximum when $\frac{m}{5}$ is	**(Q)**	5
(C)	Number of non-negative integral solutions of inequation $x+y+z \le 4$ is	**(R)**	35
(D)	Let $A = \{1, 2, 3, 4, 5\}$, $f : A \to A$, The number of onto functions such that $f(x) = x$ for atleast 3 distinct $x \in A$, is not a multiple of	**(S)**	6
		(T)	12

2.

	Column-I		Column-II
(A)	Number of real solution of $(x^2+6x+7)^2 + 6(x^2+6x+7) + 7 = x$ is/are	**(P)**	15
(B)	If $P = \sum_{r=0}^{n} {}^nC_r$; $q = \sum_{r=0}^{m} {}^mC_r\,(15)^r$ $(m, n \in N)$ and if $P = q$ and m, n are least then $m + n =$	**(Q)**	5
(C)	Remainder when $1! + 3! + 5! + \ldots\ldots + 2011!$ is divided by 56 is	**(R)**	3
(D)	Inequality $\left\lvert 1 - \frac{\lvert x\rvert}{1+\lvert x\rvert} \right\rvert \ge \frac{1}{2}$ holds for x, then number of integral values of 'x' is/are	**(S)**	0

3. Match the following

	Column-I		Column-II
(A)	If the sum of first 84 terms of the series $\frac{4+\sqrt{3}}{1+\sqrt{3}} + \frac{8+\sqrt{15}}{\sqrt{3}+\sqrt{5}} + \frac{12+\sqrt{35}}{\sqrt{5}+\sqrt{7}} + \ldots..$ is 549k, then k is equal to	**(P)**	3

(B)	If $x, y \in R$, $x^2 + y^2 - 6x + 8y + 24 = 0$, the greatest value of $\frac{16}{5}\cos^2\left(\sqrt{x^2+y^2}\right) - \frac{24}{5}\sin\left(\sqrt{x^2+y^2}\right)$ is	**(Q)**	2
(C)	If $(\sqrt{3}+1)^6 + (\sqrt{3}-1)^6 = 416$, if $xyz = [(\sqrt{3}+1)^6]$, $x, y, z \in N$, (where $[\cdot]$ denotes the greatest integer function), then the number of ordered triplets (x, y, z) is	**(R)**	5
(D)	If $(1+x)(1+x^2)(1+x^4)\ldots\ldots(1+x^{128}) = \sum_{r=0}^{n} x^r$, then $\frac{n}{85}$ is equal to	**(S)**	9

Answers

1.	$A \to Q$; $B \to S$; $C \to R$; $D \to P, Q, R, S, T$
2.	$A \to S$; $B \to Q$; $C \to P$; $D \to R$
3.	$A \to Q$; $B \to R$; $C \to S$; $D \to P$

Exercise-4 : Subjective Type Problems

1. The sum of the series $3\cdot {}^{2007}C_0 - 8\cdot {}^{2007}C_1 + 13\cdot {}^{2007}C_2 - 18\cdot {}^{2007}C_3 + \ldots\ldots$ upto 2008 terms is K, then K is :

2. In the polynomial function $f(x) = (x-1)(x^2-2)(x^3-3)\ldots\ldots(x^{11}-11)$ the coefficient of x^{60} is :

3. If $\sum_{r=0}^{3n} a_r(x-4)^r = \sum_{r=0}^{3n} A_r(x-5)^r$ and $a_k = 1 \;\forall\; K \geq 2n$ and $\sum_{r=0}^{3n} d_r(x-8)^r = \sum_{r=0}^{3n} B_r(x-9)^r$ and $\sum_{r=0}^{3n} d_r(x-12)^r = \sum_{r=0}^{3n} D_r(x-13)^r$ and $d_K = 1 \;\forall\; K \geq 2n$. The find the value of $\dfrac{A_{2n}+D_{2n}}{B_{2n}}$.

4. If $3^{101} - 2^{100}$ is divided by 11, the remainder is

5. Find the hundred's digit in the co-efficient of x^{17} in the expansion of $(1+x^5+x^7)^{20}$.

6. Let $x = (3\sqrt{6}+7)^{89}$. If $\{x\}$ denotes the fractional part of 'x' then find the remainder when $x\{x\} + (x\{x\})^2 + (x\{x\})^3$ is divided by 31.

7. Let $n \in N; S_n = \sum_{r=0}^{3n} ({}^{3n}C_r)$ and $T_n = \sum_{r=0}^{n} ({}^{3n}C_{3r})$. Find $|S_n - 3T_n|$.

8. Find the sum of possible real values of x for which the sixth term of $\left(3^{\log_3\sqrt{9^{|x-2)|}}} + 7^{\frac{1}{5}\log_7(3^{|x-2|-9})}\right)^7$ equal 567 :

9. Let q be a positive integer with $q \leq 50$.
If the sum ${}^{98}C_{30} + 2.\,{}^{97}C_{30} + 3.\,{}^{96}C_{30} + \ldots\ldots + 68.\,{}^{31}C_{30} + 69.\,{}^{30}C_{30} = {}^{100}C_q$
Find the sum of the digits of q.

10. The remainder when $\left(\sum_{k=1}^{5} {}^{20}C_{2k-1}\right)^6$ is divided by 11, is :

11. Let $a = 3^{\frac{1}{223}} + 1$ and for all $n \geq 3$, let
$f(n) = {}^nC_0.a^{n-1} - {}^nC_1.a^{n-2} + {}^nC_2.a^{n-3} - \ldots\ldots + (-1)^{n-1}\cdot{}^nC_{n-1}\cdot a^0$.
If the value of $f(2007) + f(2008) = 3^7 k$ where $k \in N$ then find k

12. In the polynomial $(x-1)(x^2-2)(x^3-3)\ldots(x^{11}-11)$, the coefficient of x^{60} is :

13. Let the sum of all divisiors of the form $2^p\cdot 3^q$ (with p, q positive integers) of the number $19^{88}-1$ be λ. Find the unit digit of λ.

14. Find the sum of possible real values of x for which the sixth term of $\left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{\left(\frac{1}{5}\right)\log_7 (3^{|x-2|-9}}\right)^7$ equals 567.

15. Let $1+\sum_{r=1}^{10}(3^r \cdot {}^{10}C_r + r \cdot {}^{10}C_r) = 2^{10}(\alpha \cdot 4^5 + \beta)$ where $\alpha, \beta \in N$ and $f(x) = x^2 - 2x - k^2 + 1$. If α, β lies between the roots of $f(x) = 0$. Then find the smallest positive integral value of k.

16. Let $S_n = {}^nC_0\,{}^nC_1 + {}^nC_1\,{}^nC_2 + \ldots\ldots + {}^nC_{n-1}\,{}^nC_n$ if $\frac{S_{n+1}}{S_n} = \frac{15}{4}$; find the sum of all possible values of $n\ (n \in N)$

Answers

1.	0	**2.**	1	**3.**	2	**4.**	2	**5.**	4	**6.**	0	**7.**	2
8.	4	**9.**	5	**10.**	3	**11.**	9	**12.**	(1)	**13.**	(4)	**14.**	(4)
15.	5	**16.**	6										

□□□

Chapter 15 – Probability

15

PROBABILITY

Exercise-1 : Single Choice Problems

1. The boy comes from a family of two children; What is the probability that the other child is his sister ? :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

2. If A be any event in sample space then the maximum value of $3\sqrt{P(A)} + 4\sqrt{P(\bar{A})}$ is :

(a) 4 (b) 2
(c) 5 (d) Can not be determined

3. Let A and B be two events, such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are :

(a) equally likely and mutually exclusive (b) equally likely but not independent
(c) independent but not equally likely (d) mutually exclusive and independent

4. Let n ordinary fair dice are rolled once. The probability that at least one of the dice shows an odd number is $\left(\frac{31}{32}\right)$ than 'n' is equal to :

(a) 3 (b) 4 (c) 5 (d) 6

5. Three a's, three b's and three c's are placed randomly in a 3×3 matrix. The probability that no row or column contain two identical letters can be expressed as $\frac{p}{q}$, where p and q are coprime then $(p + q)$ equals to :

(a) 151 (b) 161 (c) 141 (d) 131

6. A set contains $3n$ members. Let P_n be the probability that S is partitioned into 3 disjoint subsets with n members in each subset such that the three largest members of S are in different subsets. Then $\lim_{n \to \infty} P_n =$

(a) 2/7 (b) 1/7 (c) 1/9 (d) 2/9

7. Three different numbers are selected at random from the set $A = \{1, 2, 3, \ldots\ldots, 10\}$. Then the probability that the product of two numbers equal to the third number is $\frac{p}{q}$, where p and q are relatively prime positive integers then the value of $(p+q)$ is :

(a) 39 (b) 40 (c) 41 (d) 42

8. Mr. *A*'s T.V. has only 4 channels ; all of them quite boring so he naturally desires to switch (change) channel after every one minute. The probability that he is back to his original channel for the first time after 4 minutes can be expressed as $\frac{m}{n}$; where m and n are relatively prime numbers. Then $(m+n)$ equals :

(a) 27 (b) 31 (c) 23 (d) 33

9. Letters of the word TITANIC are arranged to form all the possible words. What is the probability that a word formed starts either with a T or a vowel ?

(a) $\frac{2}{7}$ (b) $\frac{4}{7}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

10. A mapping is selected at random from all mappings $f : A \to A$ where set $A = \{1, 2, 3, \ldots\ldots, n\}$

If the probability that mapping is injective is $\frac{3}{32}$, then the value of n is :

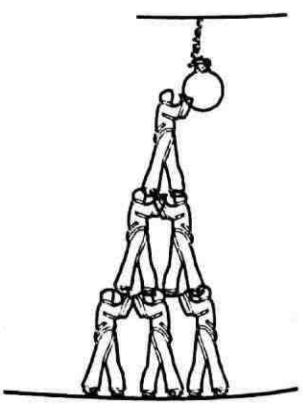
(a) 3 (b) 4 (c) 8 (d) 16

11. A 4 digit number is randomly picked from all the 4 digit numbers, then the probability that the product of its digit is divisible by 3 is :

(a) $\frac{107}{125}$ (b) $\frac{109}{125}$

(c) $\frac{111}{125}$ (d) None of these

12. To obtain a gold coin; 6 men, all of different weight, are trying to build a human pyramid as shown in the figure. Human pyramid is called "stable" if some one not in the bottom row is "supported by" each of the two closest people beneath him and no body can be supported by anybody of lower weight. Formation of 'stable' pyramid is the only condition to get a gold coin. What is the probability that they will get gold coin ?



(a) $\frac{1}{45}$ (b) $\frac{2}{45}$

(c) $\frac{4}{45}$ (d) $\frac{1}{\lfloor 5}$

13. From a pack of 52 playing cards; half of the cards are randomly removed without looking at them. From the remaining cards, 3 cards are drawn randomly. The probability that all are king.

(a) $\frac{1}{(25)(17)(13)}$ (b) $\frac{1}{(25)(15)(13)}$

(c) $\frac{1}{(52)(17)(13)}$ (d) $\frac{1}{(13)(51)(17)}$

14. A bag contains 10 white and 3 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. The probability that the procedure of drawing balls will come to an end at the seventh draw is :

(a) $\frac{15}{286}$ (b) $\frac{105}{286}$ (c) $\frac{35}{286}$ (d) $\frac{7}{286}$

15. Let S be the set of all function from the set $\{1, 2, ..., 10\}$ to itself. One function is selected from S, the probability that the selected function is one-one onto is :

(a) $\frac{9!}{10^9}$ (b) $\frac{1}{10}$ (c) $\frac{100}{10!}$ (d) $\frac{9!}{10^{10}}$

16. Two friends visit a restaurant randomly during 5 pm to 6 pm. Among the two, whoever comes first waits for 15 min and then leaves. The probability that they meet is :

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) $\frac{7}{16}$ (d) $\frac{9}{16}$

17. Three numbers are randomly selected from the set $\{10, 11, 12, \ldots\ldots, 100\}$. Probability that they form a Geometric progression with integral common ratio greater than 1 is :

(a) $\frac{15}{{}^{91}C_3}$ (b) $\frac{16}{{}^{91}C_3}$ (c) $\frac{17}{{}^{91}C_3}$ (d) $\frac{18}{{}^{91}C_3}$

Answers

1.	(a)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(c)	**8.**	(b)	**9.**	(d)	**10.**	(b)
11.	(a)	**12.**	(a)	**13.**	(a)	**14.**	(a)	**15.**	(a)	**16.**	(c)	**17.**	(d)						

Exercise-2 : One or More than One Answer is/are Correct

1. A consignment of 15 record players contain 4 defectives. The record players are selected at random, one by one and examined. The one examined is not put back. Then :

(a) Probability of getting exactly 3 defectives in the examination of 8 record players is $\frac{{}^4C_3\ {}^{11}C_5}{{}^{15}C_8}$.

(b) Probability that 9^{th} one examined is the last defective is $\frac{8}{197}$.

(c) Probability that 9^{th} examined record player is defective, given that there are 3 defectives in first 8 players examined is $\frac{1}{7}$.

(d) Probability that 9^{th} one examined is the last defective is $\frac{8}{195}$.

2. If $A_1, A_2, A_3, \ldots\ldots, A_{1006}$ be independent events such that $P(A_i) = \frac{1}{2i}\ (i = 1, 2, 3, \ldots\ldots, 1006)$ and probability that none of the events occurs be $\frac{\alpha!}{2^{\alpha}(\beta!)^2}$, then :

(a) β is of form $4k + 2$, $k \in I$

(b) $\alpha = 2\beta$

(c) β is a composite number

(d) α is of form $4k$, $k \in I$

3. A bag contains four tickets marked with 112, 121, 211, 222 one ticket is drawn at random from the bag. let E_i $(i = 1,2,3)$ denote the event that i^{th} digit on the ticket is 2. Then :

(a) E_1 and E_2 are independent

(b) E_2 and E_3 are independent

(c) E_3 and E_1 are independent

(d) E_1, E_2, E_3 are independent

4. For two events A and B let, $P(A) = \frac{3}{5}, P(B) = \frac{2}{3}$, then which of the following is/are correct ?

(a) $P(A \cap \overline{B}) \leq \frac{1}{3}$

(b) $P(A \cup B) \geq \frac{2}{3}$

(c) $\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$

(d) $\frac{1}{10} \leq P(\overline{A}/B) \leq \frac{3}{5}$

Answers

1.	(a, c, d)	**2.**	(a, b, c, d)	**3.**	(a, b, c)	**4.**	(a, b, c, d)		

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

There are four boxes B_1, B_2, B_3 and B_4. Box B_i has i cards and on each card a number is printed, the numbers are from 1 to i. A box is selected randomly, the probability of selecting box B_i is $\frac{i}{10}$ and then a card is drawn.

Let E_i represent the event that a card with number 'i' is drawn. Then :

1. $P(E_1)$ is equal to :

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

2. $P(B_3|E_2)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Paragraph for Question Nos. 3 to 5

Mr. A randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a three digit number. Mr. B randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3 digit number.

3. The probability that Mr. A's 3 digit number is always greater than Mr. B's 3 digit number is :

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

4. The probability that A and B has the same 3 digit number is :

(a) $\frac{7}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{84}$ (d) $\frac{1}{72}$

5. The probability that Mr. A's number is larger than Mr. B's number, is :

(a) $\frac{37}{56}$ (b) $\frac{39}{56}$ (c) $\frac{31}{56}$ (d) none of these

Paragraph for Question Nos. 6 to 7

In an experiment a coin is tossed 10 times.

6. Probability that no two heads are consecutive is :

(a) $\frac{143}{2^{10}}$ (b) $\frac{9}{2^6}$ (c) $\frac{2^7-1}{2^{10}}$ (d) $\frac{2^6-1}{2^6}$

7. The probability of the event that "exactly four heads occur and occur alternately" is :

(a) $1-\frac{4}{2^{10}}$ (b) $1-\frac{7}{2^{10}}$ (c) $\frac{4}{2^{10}}$ (d) $\frac{5}{2^{10}}$

Paragraph for Question Nos. 8 to 10

The rule of an "obstacle course" specifies that at the n^{th} obstacle a person has to toss a fair 6 sided die n times. If the sum of points in these n tosses is bigger than 2^n, the person is said to have crossed the obstacle.

8. The maximum obstacles a person can cross :

(a) 4 (b) 5 (c) 6 (d) 7

9. The probability that a person crosses the first three obstacles :

(a) $\frac{143}{216}$ (b) $\frac{100}{243}$ (c) $\frac{216}{243}$ (d) $\frac{100}{216}$

10. The probability that a person crosses the first two obstacles but fails to cross the third obstacle.

(a) $\frac{36}{243}$ (b) $\frac{116}{216}$ (c) $\frac{35}{243}$ (d) $\frac{143}{243}$

Paragraph for Question Nos. 11 to 12

In an objective paper, there are two sections of 10 questions each. For 'section 1', each question has 5 options and only one option is correct and 'section 2' has 4 options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in 'section 1' is 1 and in 'section 2' is 3. (There is no negative marking).

11. If a candidate attempts only two questions by gussing, one from 'section 1' and one from 'section 2', the probability that he scores in both questions is :

(a) $\frac{74}{75}$ (b) $\frac{1}{25}$ (c) $\frac{1}{15}$ (d) $\frac{1}{75}$

12. If a candidate in total attempts 4 questions all by gussing, then the probability of scoring 10 marks is :

(a) $\frac{1}{15}\left(\frac{1}{15}\right)^2$ (b) $\frac{4}{5}\left(\frac{1}{15}\right)^3$ (c) $\frac{1}{5}\left(\frac{14}{15}\right)^3$ (d) None of these

Answers

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(c)	**8.**	(a)	**9.**	(b)	**10.**	(c)
11.	(d)	**12.**	(d)																

Exercise-4 : Matching Type Problems

1. A is a set containing n elements, A subset P (may be void also) is selected at random from set A and the set A is then reconstructed by replacing the elements of P. A subset Q (may be void also) of A is again chosen at random. The probability that

	Column-I		Column-II
(A)	Number of elements in P is equal to the number of elements in Q is	(P)	$\frac{{}^{2n}C_n}{4^n}$
(B)	The number of elements in P is more than that in Q is	(Q)	$\frac{(2^{2n} - {}^{2n}C_n)}{2^{2n+1}}$
(C)	$P \cap Q = \phi$ is	(R)	$\frac{{}^{2n}C_{n+1}}{4^n}$
(D)	Q is a subset of P is	(S)	$\left(\frac{3}{4}\right)^n$
		(T)	$\frac{{}^{2n}C_n}{4^{n-1}}$

Answers

1.	$A \to P$; $B \to Q$; $C \to S$; $D \to S$

Exercise-5 : Subjective Type Problems

1. Mr. A writes an article. The article originally is error free. Each day Mr. B introduces one new error into the article. At the end of the day, Mr. A checks the article and has $\frac{2}{3}$ chance of catching each individual error still in the article. After 3 days, the probability that the article is error free can be expressed as $\frac{p}{q}$ where p and q are relatively prime positive integers. Let $\lambda = q - p$, then find the sum of the digits of λ.
2. India and Australia play a series of 7 one-day matches. Each team has equal probability of winning a match. No match ends in a draw. If the probability that India wins atleast three consecutive matches can be expressed as $\frac{p}{q}$ where p and q are relatively prime positive integers. Find the unit digit of p.
3. Two hunters A and B set out to hunt ducks. Each of them hits as often as he misses when shooting at ducks. Hunter A shoots at 50 ducks and hunter B shoots at 51 ducks. The probability that B bags more ducks than A can be expressed as $\frac{p}{q}$ in its lowest form. Find the value of $(p+q)$.
4. If $a, b, c \in N$, the probability that $a^2 + b^2 + c^2$ is divisible by 7 is $\frac{m}{n}$ where m, n are relatively prime natural numbers, then $m + n$ is equal to :
5. A fair coin is tossed 10 times. If the probability that heads never occur on consecutive tosses be $\frac{m}{n}$ (where m, n are coprime and $m, n \in N$), then the value of $(n - 7m)$ equals to :
6. A bag contains 2 red, 3 green and 4 black balls. 3 balls are drawn randomly and exactly 2 of them are found to be red. If p denotes the chance that one of the three balls drawn is green ; find the value of $7p$.
7. There are 3 different pairs (*i.e.*, 6 units say a, a, b, b, c, c) of shoes in a lot. Now three person come and pick the shoes randomly (each gets 2 units). Let p be the probability that no one is able to wear shoes (*i.e.*, no one gets a correct pair), then the value of $\frac{13p}{4-p}$, is :
8. A fair coin is tossed 12 times. If the probability that two heads do not occur consecutively is p, then the value of $\frac{[\sqrt{4096p} - 1]}{2}$ is, where [ ] denotes greatest integer function :
9. X and Y are two weak students in mathematics and their chances of solving a problem correctly are 1/8 and 1/12 respectively. They are given a question and they obtain the same answer. If the probability of common mistake is $\frac{1}{1001}$, then probability that the answer was correct is a / b (a and b are coprimes). Then $| a - b | =$

10. Seven digit numbers are formed using digits 1, 2, 3, 4, 5, 6, 7, 8, 9 without repetition. The probability of selecting a number such that product of any 5 consecutive digits is divisible by either 5 or 7 is P. Then $12P$ is equal to

11. Assume that for every person the probability that he has exactly one child, excactly 2 children and exactly 3 children are $\frac{1}{4}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. The probability that a person will have 4 grand children can be expressed as $\frac{p}{q}$ where p and q are relatively prime positive integers. Find the value of $5p - q$.

12. Mr. B has two fair 6-sided dice, one whose faces are numbered 1 to 6 and the second whose faces are numbered 3 to 8. Twice, he randomly picks one of dice (each dice equally likely) and rolls it. Given the sum of the resulting two rolls is 9. The probability he rolled same dice twice is $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $(m+n)$.

Answers

1.	7	**2.**	7	**3.**	3	**4.**	8	**5.**	1	**6.**	3	**7.**	2
8.	9	**9.**	1	**10.**	7	**11.**	7	**12.**	7				

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Chapter 16 – Logarithms

16

LOGARITHMS

Exercise-1 : Single Choice Problems

1. Solution set of the in equality $\log_{10^2} x - 3(\log_{10} x)(\log_{10}(x-2)) + 2\log_{10^2}(x-2) < 0$, is :
(a) $(0, 4)$ (b) $(-\infty, 1)$ (c) $(4, \infty)$ (d) $(2, 4)$

2. The number of real solution/s of the equation $9^{\log_3(\log_e x)} = \log_e x - (\log_e x)^2 + 1$ is :
(a) 0 (b) 1 (c) 2 (d) 3

3. If a, b, c are positive numbers such that $a^{\log_3 7} = 27$, $b^{\log_7 11} = 49$, $c^{\log_{11} 25} = \sqrt{11}$, then the sum of digits of $S = a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}$ is :
(a) 15 (b) 17 (c) 19 (d) 21

4. Least positive integral value of 'a' for which $\log_{\left(x+\frac{1}{x}\right)}(a^2 - 3a + 3) > 0$; $(x > 0)$:
(a) 1 (b) 2 (c) 3 (d) 4

5. Let $P = \dfrac{5}{\dfrac{1}{\log_2 x} + \dfrac{1}{\log_3 x} + \dfrac{1}{\log_4 x} + \dfrac{1}{\log_5 x}}$ and $(120)^P = 32$, then the value of x be :
(a) 1 (b) 2 (c) 3 (d) 4

6. If x, y, z be positive real numbers such that $\log_{2x}(z) = 3$, $\log_{5y}(z) = 6$ and $\log_{xy}(z) = \dfrac{2}{3}$ then the value of z is :
(a) $\dfrac{1}{5}$ (b) $\dfrac{1}{10}$ (c) $\dfrac{3}{5}$ (d) $\dfrac{4}{9}$

7. Sum of values of x and y satisfying $\log_x(\log_3(\log_x y)) = 0$ and $\log_y 27 = 1$ is :
(a) 27 (b) 30 (c) 33 (d) 36

8. $\log_{0.01} 1000 + \log_{0.1} 0.0001$ is equal to :
(a) -2 (b) 3 (c) $-5/2$ (d) $5/2$

9. If $\log_{12} 27 = a$, then $\log_6 16 =$

(a) $2\left(\frac{3-a}{3+a}\right)$ (b) $3\left(\frac{3-a}{3+a}\right)$ (c) $4\left(\frac{3-a}{3+a}\right)$ (d) None of these

10. If $\log_2(\log_2(\log_3 x)) = \log_2(\log_3(\log_2 y)) = 0$ then the value of $(x+y)$ is :

(a) 17 (b) 9 (c) 21 (d) 19

11. Suppose that a and b are positive real numbers such that $\log_{27} a + \log_9 b = \frac{7}{2}$ and $\log_{27} b + \log_9 a = \frac{2}{3}$. Then the value of $a \cdot b$ is :

(a) 81 (b) 243 (c) 27 (d) 729

12. If $2^a = 5$, $5^b = 8$, $8^c = 11$ and $11^d = 14$, then the value of 2^{abcd} is :

(a) 1 (b) 2 (c) 7 (d) 14

13. Which of the following conditions necessarily imply that the real number x is rational ?

(I) x^2 is rational (II) x^3 and x^5 are rational (III) x^2 and x^3 are rational

(a) I and II only (b) I and III only (c) II and III only (d) III only

14. The value of $\frac{\log_8 17}{\log_9 23} - \frac{\log_{2\sqrt{2}} 17}{\log_3 23}$ is equal to :

(a) -1 (b) 0 (c) $\frac{\log_2 17}{\log_3 23}$ (d) $\frac{4(\log_2 17)}{3(\log_3 23)}$

15. The true solution set of inequality $\log_{(2x-3)}(3x-4) > 0$ is equal to :

(a) $\left(\frac{4}{3}, \frac{5}{3}\right) \cup (2, \infty)$ (b) $\left(\frac{3}{2}, \frac{5}{3}\right) \cup (2, \infty)$ (c) $\left(\frac{4}{3}, \frac{3}{2}\right) \cup (2, \infty)$ (d) $\left(\frac{2}{3}, \frac{4}{3}\right) \cup (2, \infty)$

16. If P is the number of natural numbers whose logarithm to the base 10 have the characteristic p and Q is the number of natural numbers logarithm of whose reciprocals to the base 10 have the characteristic $-q$ then $\log_{10} P - \log_{10} Q$ has the value equal to :

(a) $p-q+1$ (b) $p-q$ (c) $p+q-1$ (d) $p-q-1$

17. If $2^{2010} = a_n 10^n + a_{n-1} 10^{n-1} + \ldots\ldots\ldots + a_2 10^2 + a_1 \cdot 10 + a_0$, where $a_i \in \{0, 1, 2, \ldots\ldots\ldots, 9\}$ for all $i = 0, 1, 2, 3, \ldots\ldots\ldots, n$, then $n =$

(a) 603 (b) 604 (c) 605 (d) 606

18. The number of zeros after decimal before the start of any significant digit in the number $N = (0.15)^{20}$ are :

(a) 15 (b) 16 (c) 17 (d) 18

19. $\log_2[\log_4(\log_{10} 16^4 + \log_{10} 25^8)]$ simplifies to :

(a) an irrational (b) an odd prime

(c) a composite (d) unity

20. The sum of all the solutions to the equation $2\log x - \log(2x-75) = 2$:

(a) 30 (b) 350 (c) 75 (d) 200

21. $x^{\log_x a \cdot \log_a y \cdot \log_y z}$ is equal to :

(a) x (b) y (c) z (d) x^z

22. Number of solution(s) of the equation $x^{x\sqrt{x}} = (x\sqrt{x})^x$ is/are :

(a) 0 (b) 1 (c) 2 (d) 3

23. The difference of roots of the equation $(\log_{27} x^3)^2 = \log_{27} x^6$ is :

(a) $\frac{2}{3}$ (b) 1 (c) 9 (d) 8

24. If $\log_{10} x + \log_{10} y = 2$, $x - y = 15$ then :

(a) (x, y) lies on the line $y = 4x + 3$ (b) (x, y) lies on $y^2 = 4x$

(c) (x, y) lies on $x = 4y$ (d) (x, y) lies on $4x = y$

25. Product of all values of x satisfying the equation

$$\sqrt{2^x \sqrt[3]{4^x (0.125)^{1/x}}} = 4(\sqrt[3]{2}) \text{ is :}$$

(a) $\frac{14}{5}$ (b) 3 (c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

26. Sum of all values of x satisfying the equation

$$25^{(2x-x^2+1)} + 9^{(2x-x^2+1)} = 34(15^{(2x-x^2)}) \text{ is :}$$

(a) 1 (b) 2 (c) 3 (d) 4

27. If $a^x = b^y = c^z = d^w$, then $\log_a(bcd) =$

(a) $z\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{w}\right)$ (b) $y\left(\frac{1}{x}+\frac{1}{z}+\frac{1}{w}\right)$ (c) $x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$ (d) $\frac{xyz}{w}$

28. If $x = \frac{4}{(\sqrt{5}+1)(\sqrt[4]{5}+1)(\sqrt[8]{5}+1)(\sqrt[16]{5}+1)}$. Then the value of $(1+x)^{48}$ is :

(a) 5 (b) 25 (c) 125 (d) 625

29. If $\log_x \log_{18}(\sqrt{2}+\sqrt{8}) = \frac{1}{3}$, then the value of $32x =$

(a) 2 (b) 4 (c) 6 (d) 8

30. Let $n \in N$, $f(n) = \begin{cases} \log_8 n & \text{if } \log_8 n \text{ is integer} \\ 0 & \text{otherwise} \end{cases}$, then the value of $\sum_{n=1}^{2011} f(n)$ is :

(a) 2011 (b) 2011×1006 (c) 6 (d) 2^{2011}

31. If the equation $\frac{\log_{12}(\log_8(\log_4 x))}{\log_5(\log_4(\log_y(\log_2 x)))} = 0$ has a solution for 'x' when $c < y < b$, $y \neq a$, where 'b' is as large as possible and 'c' is as small as possible, then the value of $(a+b+c)$ is equals to :

(a) 18 (b) 19 (c) 20 (d) 21

32. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval :

(a) $(2, \infty)$ (b) $(1, 2)$ (c) $(-2, -1)$ (d) $\left(1, \frac{3}{2}\right)$

33. The absolute integral value of the solution of the equation $\sqrt{7^{2x^2-5x-6}} = (\sqrt{2})^{3\log_2 49}$

(a) 2 (b) 1 (c) 4 (d) 5

34. Let $1 \le x \le 256$ and M be the maximum value of $(\log_2 x)^4 + 16(\log_2 x)^2 \log_2\left(\frac{16}{x}\right)$. The sum of the digits of M is :

(a) 9 (b) 11 (c) 13 (d) 15

35. Let $1 \le x \le 256$ and M be the maximum value of $(\log_2 x)^4 + 16(\log_2 x)^2 \log_2\left(\frac{16}{x}\right)$. The sum of the digits of M is :

(a) 9 (b) 11 (c) 13 (d) 15

36. Number of real solution(s) of the equation $9\log_3(\log nx) = ln\, x - (ln^2 x) + 1$ is :

(a) 0 (b) 1 (c) 2 (d) 3

37. The number of real values of the parameter λ for which $(\log_{16} x)^2 - \log_{16} x + \log_{16} \lambda = 0$ with real coefficients will have exactly one solution is :

(a) 1 (b) 2 (c) 3 (d) 4

38. A rational number which is 50 times its own logarithm to the base 10 is :

(a) 1 (b) 10 (c) 100 (d) 1000

39. If $x = \log_5(1000)$ and $y = \log_7(2058)$, then

(a) $x > y$ (b) $x < y$ (c) $x = y$ (d) none of these

40. $7\log\left(\frac{16}{15}\right) + 5\log\left(\frac{25}{24}\right) + 3\log\left(\frac{81}{80}\right)$ is equal to :

(a) 0 (b) 1 (c) $\log 2$ (d) $\log 3$

41. $\log_{10} \tan 1° + \log_{10} \tan 2° + \ldots\ldots + \log_{10} \tan 89°$ is equal to :

(a) 0 (b) 1 (c) 27 (d) 81

42. $\log_7 \log_7 \sqrt{7\sqrt{(7\sqrt{7})}}$ is equal to :

(a) $3\log_2 7$ (b) $3\log_7 2$ (c) $1 - 3\log_7 2$ (d) $1 - 3\log_2 7$

43. If $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$, then x is equal to :

(a) 2 (b) 3 (c) 10 (d) 30

44. $x^{\log_{10}\left(\frac{y}{z}\right)} \cdot y^{\log_{10}\left(\frac{z}{x}\right)} \cdot z^{\log_{10}\left(\frac{x}{y}\right)}$ is equal to :

(a) 0 (b) 1 (c) -1 (d) 2

45. The solution set of the equation : $\log_x 2 \log_{2x} 2 = \log_{4x} 2$ is :

(a) $\{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}$ (b) $\{1/2, 2\}$ (c) $\{1/4, 2^2\}$ (d) none of these

46. The least value of the expression $2\log_{10} x - \log_x 0.01$ is $(x > 1)$

(a) 2 (b) 4 (c) 6 (d) 8

47. If $\sqrt{\log_2 x} - 0.5 = \log_2 \sqrt{x}$, then x equals to :

(a) odd integer (b) prime number

(c) composite number (d) irrational

48. If x_1 and x_2 are the roots of the equation $e^2 x^{\ln x} = x^3$ with $x_1 > x_2$, then

(a) $x_1 = 2x_2$ (b) $x_1 = x_2^2$ (c) $2x_1 = x_2^2$ (d) $x_1^2 = x_2^2$

49. Let M denote antilog$_{32}$0.6 and N denote the value of $49^{(1-\log_7 2)} + 5^{-\log_5 4}$. Then M.N is :

(a) 100 (b) 400 (c) 50 (d) 200

50. If $\log_2(\log_2(\log_3 x)) = \log_3(\log_3(\log_2 y)) = 0$, then $x - y$ is equal to :

(a) 0 (b) 1 (c) 8 (d) 9

51. $\left|\log_{\frac{1}{2}} 10 + \left|\log_4 625 - \left|\log_{\frac{1}{2}} 5\right|\right|\right| =$

(a) $\log_{1/2} 2$ (b) $\log_2 5$ (c) $\log_2 2$ (d) $\log_2 25$

52. If $\log_4 5 = a$ and $\log_5 6 = b$, then $\log_3 2$ is equal to :

(a) $\frac{1}{2a+1}$ (b) $\frac{1}{2b+1}$ (c) $2ab+1$ (d) $\frac{1}{2ab-1}$

53. If $x = \log_a bc$; $y = \log_b ac$ and $z = \log_c ab$ then which of the following is equal to unity ?

(a) $x+y+z$ (b) xyz

(c) $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ (d) $(1+x)+(1+y)+(1+z)$

54. $x^{\log_x a \cdot \log_a y \cdot \log_y z}$ is equal to :

(a) x (b) y (c) z (d) a

55. Number of value(s) of 'x' satisfying the equation $x^{\log_{\sqrt{x}}(x-3)} = 9$ is/are

(a) 0 (b) 1 (c) 2 (d) 6

56. $\log_{0.01} 1000 + \log_{01} 0.0001$ is equal to :

(a) -2 (b) 3 (c) $-\frac{5}{2}$ (d) $\frac{5}{2}$

57. If $7\log_a \frac{16}{15} + 5\log_a \frac{25}{24} + 3\log_a \frac{81}{80} = 8$, then $a =$

(a) $2^{1/8}$ (b) $(10)^{1/8}$ (c) $(30)^{1/8}$ (d) 1

58. $\log_8(128) - \log_9 \cot\left(\frac{\pi}{3}\right) =$

(a) $\frac{31}{12}$ (b) $\frac{19}{12}$ (c) $\frac{13}{12}$ (d) $\frac{11}{12}$

59. The value of $\left(\frac{1}{\sqrt{27}}\right)^{2-\left(\frac{\log_5 16}{2\log_5 9}\right)}$ equals to :

(a) $\frac{5\sqrt{2}}{27}$ (b) $\frac{\sqrt{2}}{27}$ (c) $\frac{4\sqrt{2}}{27}$ (d) $\frac{2\sqrt{2}}{27}$

60. The sum of all the roots of the equation $\log_2(x-1) + \log_2(x+2) - \log_2(3x-1) = \log_2 4$

(a) 12 (b) 2 (c) 10 (d) 11

61. $\frac{(\log_{100} 10)(\log_2(\log_4 2))(\log_4 \log_2^2(256)^2)}{\log_4 8 + \log_8 4} =$

(a) $-\frac{6}{13}$ (b) $-\frac{1}{2}$ (c) $-\frac{8}{13}$ (d) $-\frac{12}{13}$

62. Let $\lambda = \log_5 \log_5(3)$. If $3^{k+5^{-\lambda}} = 405$, then the value of k is :

(a) 3 (b) 5 (c) 4 (d) 6

63. A circle has a radius $\log_{10}(a^2)$ and a circumference of $\log_{10}(b^4)$. Then the value of $\log_a b$ is equal to :

(a) $\frac{1}{4\pi}$ (b) $\frac{1}{\pi}$ (c) 2π (d) π

64. If $2^x = 3^y = 6^{-z}$, the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 3

65. The value of $\log_{(\sqrt{2}-1)}(5\sqrt{2}-7)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

66. The value of $\log_{ab}\left(\frac{\sqrt[3]{a}}{\sqrt{b}}\right)$, if $\log_{ab} a = 4$ is equal to :

(a) 2 (b) $\frac{13}{6}$ (c) $\frac{15}{6}$ (d) $\frac{17}{6}$

67. Identify the correct option

(a) $\log_2 3 < \log_{1/4} 5$ (b) $\log_5 7 < \log_8 3$

(c) $\log_{\sqrt[3]{2}} \sqrt{3} > \log_{\sqrt[3]{2}} \sqrt{5}$ (d) $2^{\frac{1}{4}} > \left(\frac{3}{2}\right)^{1/3}$

68. Sum of all values of x satisfying the system of equations $5(\log_y x + \log_x y) = 26$, $xy = 64$ is :

(a) 42 (b) 34 (c) 32 (d) 2

69. The product of all values of x satisfying the equations $\log_3 a - \log_x a = \log_{x/3} a$ is :

(a) 3 (b) $\frac{3}{2}$ (c) 18 (d) 27

70. The value of $x + y + z$ satisfying the system of equations

$$\log_2 x + \log_4 y + \log_4 z = 2 \quad \text{is}$$
$$\log_3 y + \log_9 z + \log_9 x = 2$$
$$\log_4 z + \log_{16} x + \log_{16} y = 2$$

(a) $\frac{175}{12}$ (b) $\frac{349}{24}$ (c) $\frac{353}{24}$ (d) $\frac{112}{3}$

71. $\left(\frac{1}{49}\right)^{1+\log_7 2} + 5^{-\log_{\frac{1}{5}} 7} =$

(a) $7\frac{1}{196}$ (b) $7\frac{3}{196}$ (c) $7\frac{5}{196}$ (d) $7\frac{1}{98}$

72. The number of real values of x satisfying the equation $\log_2(3-x) - \log_2\left(\frac{\sin\left(\frac{3\pi}{4}\right)}{(5-x)}\right)$

$= \frac{1}{2} + \log_2(x+7)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

73. If $\log_k x \log_5 k = \log_x 5, k \neq 1, k > 0$, then sum of all values of x is :

(a) 5 (b) $\frac{24}{5}$ (c) $\frac{26}{5}$ (d) $\frac{37}{5}$

74. The product of all values of x satisfying the equation $|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$, is :

(a) 162 (b) $\frac{162}{\sqrt{3}}$ (c) $\frac{81}{\sqrt{3}}$ (d) 81

75. The number of values of x satisfying the equation $\log_2(9^{x-1} + 7) = 2 + \log_2(3^{x-1} + 1)$ is :

(a) 1 (b) 2 (c) 3 (d) 0

76. Which is the correct order for a given number $\alpha, \alpha > 1$

(a) $\log_2 \alpha < \log_3 \alpha < \log_e \alpha < \log_{10} \alpha$ (b) $\log_{10} \alpha < \log_3 \alpha < \log_e \alpha < \log_2 \alpha$

(c) $\log_{10} \alpha < \log_e \alpha < \log_2 \alpha < \log_3 \alpha$ (d) $\log_3 \alpha < \log_e \alpha < \log_2 \alpha < \log_{10} \alpha$

77. Let $1 \le x \le 256$ and M be the maximum value of $(\log_2 x)^4 + 16(\log_2 x)^2 \log_2\left(\frac{16}{x}\right)$. The sum of the digits of M is :

(a) 9 (b) 11 (c) 13 (d) 15

78. If $T_r = \dfrac{1}{\log_{2^r} 4}$ (where $r \in N$), then the value of $\sum_{r=1}^{4} T_r$ is :

(a) 3 (b) 4 (c) 5 (d) 10

79. In which of the following intervals does $\dfrac{1}{\log_{1/2}(1/3)} + \dfrac{1}{\log_{1/5}(1/3)}$ lies

(a) $(1, 2)$ (b) $(2, 3)$ (c) $(3, 4)$ (d) $(4, 5)$

80. If $\sin\theta = \dfrac{1}{2}\left(a + \dfrac{1}{a}\right)$ and $\sin 3\theta = \dfrac{k}{2}\left(a^3 + \dfrac{1}{a^3}\right)$, then $k + 6$ is equal to :

(a) 3 (b) 4 (c) 5 (d) -4

81. Complete set of real values of x for which $\log_{(2x-3)}(x^2 - 5x - 6)$ is defined is :

(a) $\left(\dfrac{3}{2}, \infty\right)$ (b) $(6, \infty)$ (c) $\left(\dfrac{3}{2}, 6\right)$ (d) $\left(\dfrac{3}{2}, 2\right) \cup (2, \infty)$

Answers

1.	(c)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(b)	**6.**	(b)	**7.**	(b)	**8.**	(d)	**9.**	(c)	**10.**	(a)
11.	(b)	**12.**	(d)	**13.**	(c)	**14.**	(b)	**15.**	(b)	**16.**	(a)	**17.**	(c)	**18.**	(b)	**19.**	(d)	**20.**	(d)
21.	(c)	**22.**	(c)	**23.**	(d)	**24.**	(c)	**25.**	(d)	**26.**	(d)	**27.**	(c)	**28.**	(c)	**29.**	(b)	**30.**	(c)
31.	(b)	**32.**	(a)	**33.**	(c)	**34.**	(c)	**35.**	(c)	**36.**	(b)	**37.**	(a)	**38.**	(c)	**39.**	(a)	**40.**	(c)
41.	(a)	**42.**	(c)	**43.**	(c)	**44.**	(b)	**45.**	(a)	**46.**	(b)	**47.**	(b)	**48.**	(b)	**49.**	(a)	**50.**	(b)
51.	(c)	**52.**	(d)	**53.**	(c)	**54.**	(c)	**55.**	(b)	**56.**	(d)	**57.**	(a)	**58.**	(a)	**59.**	(d)	**60.**	(d)
61.	(d)	**62.**	(c)	**63.**	(d)	**64.**	(a)	**65.**	(d)	**66.**	(d)	**67.**	(d)	**68.**	(b)	**69.**	(d)	**70.**	(c)
71.	(a)	**72.**	(b)	**73.**	(c)	**74.**	(a)	**75.**	(b)	**76.**	(b)	**77.**	(c)	**78.**	(c)	**79.**	(b)	**80.**	(c)
81.	(b)																		

Exercise-2 : One or More than One Answer is/are Correct

1. The values of 'x' satisfies the equation $\dfrac{1-2(\log x^2)^2}{\log x - 2(\log x)^2} = 1$ (is/are) :

(where log is logarithm to the base 10)

(a) $\dfrac{1}{\sqrt{10}}$ (b) $\dfrac{1}{\sqrt{20}}$ (c) $\sqrt[3]{10}$ (d) $\sqrt{10}$

2. If $\log_a x = b$ for permissible values of a and x then identify the statement(s) which can be correct?

(a) If a and b are two irrational numbers then x can be rational.

(b) If a rational and b irrational then x can be rational.

(c) If a irrational and b rational then x can be rational.

(d) If a rational and b rational then x can be rational.

3. Consider the quadratic equation, $(\log_{10} 8)x^2 - (\log_{10} 5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational ?

(a) Sum of the roots (b) Product of the roots

(c) Sum of the coefficients (d) Discriminant

4. Let $A =$ Minimum $(x^2 - 2x + 7)$, $x \in R$ and $B =$ Minimum $(x^2 - 2x + 7), x \in [2, \infty)$, then :

(a) $\log_{(B-A)}(A+B)$ is not defined (b) $A + B = 13$

(c) $\log_{(2B-A)} A < 1$ (d) $\log_{(2A-B)} A > 1$

Answers

1.	(a, c)	**2.**	(a, b, c, d)	**3.**	(c, d)	**4.**	(a, b, c, d)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Let $\log_3 N = \alpha_1 + \beta_1$

$\log_5 N = \alpha_2 + \beta_2$

$\log_7 N = \alpha_3 + \beta_3$

where α_1, α_2 and α_3 are integers and $\beta_1, \beta_2, \beta_3 \in [0, 1)$.

1. Number of integral values of N if $\alpha_1 = 4$ and $\alpha_2 = 2$:

(a) 46 (b) 45 (c) 44 (d) 47

2. Largest integral value of N if $\alpha_1 = 5$, $\alpha_2 = 3$ and $\alpha_3 = 2$.

(a) 342 (b) 343 (c) 243 (d) 242

3. Difference of largest and smallest integral values of N if $\alpha_1 = 5$, $\alpha_2 = 3$ and $\alpha_3 = 2$.

(a) 97 (b) 100 (c) 98 (d) 99

Paragraph for Question Nos. 4 to 5

If $\log_{10}|x^3 + y^3| - \log_{10}|x^2 - xy + y^2| + \log_{10}|x^3 - y^3| - \log_{10}|x^2 + xy + y^2| = \log_{10} 221.$ Where x, y are integers, then

4. If $x = 111$, then y can be :

(a) ± 111 (b) ± 2 (c) ± 110 (d) ± 109

5. If $y = 2$, then value of x can be :

(a) ± 111 (b) ± 15 (c) ± 2 (d) ± 110

Paragraph for Question Nos. 6 to 7

Given a right triangle ABC right angled at C and whose legs are given $1 + 4\log_{p^2}(2p)$, $1 + 2^{\log_2(\log_2 p)}$ and hypotenuse is given to be $1 + \log_2(4p)$. The area of ΔABC and circle circumscribing it are Δ_1 and Δ_2 respectively, then

6. $\Delta_1 + \dfrac{4\Delta_2}{\pi}$ is equal to :

(a) 31 (b) 28 (c) $3 + \dfrac{1}{\sqrt{2}}$ (d) 199

7. The value of $\sin\left(\dfrac{\pi(25p^2\Delta_1 + 2)}{6}\right) =$

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\sqrt{2}}$ (c) $\dfrac{\sqrt{3}}{2}$ (d) 1

Answers

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(b)	**6.**	(a)	**7.**	(c)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If $a = 3(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}})$, $b = \sqrt{(42)(30)+36}$, then the value of $\log_a b$ is equal to	**(P)**	-1
(B)	If $a = (\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}})$, $b = \sqrt{11+6\sqrt{2}} - \sqrt{11-6\sqrt{2}}$ then the value of $\log_a b$ is equal to	**(Q)**	1
(C)	If $a = \sqrt{3+2\sqrt{2}}$, $b = \sqrt{3-2\sqrt{2}}$, then the value of $\log_a b$ is equal to	**(R)**	2
(D)	If $a = \sqrt{7+\sqrt{7^2-1}}$, $b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to	**(S)**	$\frac{3}{2}$
		(T)	None of these

Answers

1.	$A \rightarrow R$; $B \rightarrow S$; $C \rightarrow P$; $D \rightarrow P$

Exercise-5 : Subjective Type Problems

1. The number $N = 6^{\log_{10} 40} \cdot 5^{\log_{10} 36}$ is a natural number. Then sum of digits of N is :
2. The minimum value of 'c' such that $\log_b(a^{\log_2 b}) = \log_a(b^{\log_2 b})$ and $\log_a(c-(b-a)^2) = 3$, where $a, b \in N$ is :
3. How many positive integers b have the property that $\log_b 729$ is a positive integer ?
4. The number of negative integral values of x satisfying the inequality $\log_{\left(x+\frac{5}{2}\right)}\left(\frac{x-5}{2x-3}\right)^2 < 0$ is :
5. $\frac{6}{5}a^{(\log_a x)(\log_{10} a)(\log_a 5)} - 3^{\log_{10}\left(\frac{x}{10}\right)} = 9^{\log_{100} x + \log_4 2}$ (where $a > 0$, $a \neq 1$), then $\log_3 x = \alpha + \beta$, α is integer, $\beta \in [0, 1)$, then $\alpha =$
6. If $\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 a + \log_5 b}{2}$, then $\frac{a^4+b^4}{a^2b^2} =$
7. Let a, b, c, d are positive integers such that $\log_a b = \frac{3}{2}$ and $\log_c d = \frac{5}{4}$. If $(a-c) = 9$. Find the value of $(b-d)$.
8. The number of real values of x satisfying the equation
$$\log_{10}\sqrt{1+x} + 3\log_{10}\sqrt{1-x} = 2 + \log_{10}\sqrt{1-x^2} \text{ is :}$$
9. The ordered pair (x, y) satisfying the equation
$$x^2 = 1 + 6\log_4 y \text{ and } y^2 = 2^x y + 2^{2x+1}$$
are (x_1, y_1) and (x_2, y_2), then find the value of $\log_2|x_1 x_2 y_1 y_2|$.
10. If $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = 1 - a\log_7 2$ and $\log_{15}\log_{15}\sqrt{15\sqrt{15\sqrt{15\sqrt{15}}}} = 1 - b\log_{15} 2$, then $a + b =$
11. The number of ordered pair(s) of (x, y) satisfying the equations
$\log_{(1+x)}(1-2y+y^2) + \log_{(1-y)}(1+2x+x^2) = 4$ and $\log_{(1+x)}(1+2y) + \log_{(1-y)}(1+2x) = 2$
12. If $\log_b n = 2$ and $\log_n(2b) = 2$, then $nb =$
13. If $\log_y x + \log_x y = 2$, and $x^2 + y = 12$, then the value of xy is :
14. If x, y satisfy the equation, $y^x = x^y$ and $x = 2y$, then $x^2 + y^2 =$
15. Find the number of real values of x satisfying the equation.
$$\log_2(4^{x+1}+4)\cdot\log_2(4^x+1) = \log_{1/\sqrt{2}}\sqrt{\frac{1}{8}}$$
16. If $x_1, x_2 (x_1 > x_2)$ are the two solutions of the equation
$3^{\log_2 x} - 12(x^{\log_{16} 9}) = \log_3\left(\frac{1}{3}\right)^{3^3}$, then the value of $x_1 - 2x_2$ is :

17. Find the number of real values of x satisfying the equation $9^{2\log_9 x} + 4x + 3 = 0$.

18. If $\log_{16}(\log_{\sqrt[4]{3}}(\log_{\sqrt[3]{5}}(x))) = \frac{1}{2}$; find x.

19. The value $\left[\frac{1}{6}\left(\frac{2\log_{10}(1728)}{1+\frac{1}{2}\log_{10}(0.36)+\frac{1}{3}\log_{10}8}\right)^{1/2}\right]^{-1}$ is :

Answers

1.	9	**2.**	8	**3.**	4	**4.**	0	**5.**	4	**6.**	47	**7.**	93
8.	0	**9.**	7	**10.**	7	**11.**	1	**12.**	2	**13.**	9	**14.**	20
15.	1	**16.**	8	**17.**	0	**18.**	5	**19.**	2				

□□□

Co-ordinate Geometry

Chapter 17 – Straight Lines

17

STRAIGHT LINES

Exercise-1 : Single Choice Problems

1. The ratio in which the line segment joining (2, – 3) and (5, 6) is divided by the x-axis is :

(a) 3 : 1
(b) 1 : 2
(c) $\sqrt{3} : 2$
(d) $\sqrt{2} : 3$

2. If L is the line whose equation is $ax + by = c$. Let M be the reflection of L through the y-axis, and let N be the reflection of L through the x-axis. Which of the following must be true about M and N for all choices of a, b and c ?

(a) The x-intercepts of M and N are equal
(b) The y-intercepts of M and N are equal
(c) The slopes of M and N are equal
(d) The slopes of M and N are reciprocal

3. The complete set of real values of 'a' such that the point $P(a, \sin a)$ lies inside the triangle formed by the lines $x - 2y + 2 = 0$; $x + y = 0$ and $x - y - \pi = 0$, is :

(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$
(b) $\left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{2\pi}{2}, 2\pi\right)$
(c) $(0, \pi)$
(d) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

4. Let m be a positive integer and let the lines $13x + 11y = 700$ and $y = mx - 1$ intersect in a point whose coordinates are integer. Then m equals to :

(a) 4
(b) 5
(c) 6
(d) 7

5. If $P \equiv \left(\frac{1}{x_p}, p\right); Q = \left(\frac{1}{x_q}, q\right); R = \left(\frac{1}{x_r}, r\right)$

where $x_k \neq 0$, denotes the k^{th} terms of a H.P. for $k \in N$, then:

(a) ar. $(\Delta PQR) = \frac{p^2q^2r^2}{2}\sqrt{(p-q)^2+(q-r)^2+(r-p)^2}$

(b) ΔPQR is a right angled triangle

(c) the points P, Q, R are collinear

(d) None of these

6. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value :

(a) 1 (b) −1 (c) 2 (d) −2

7. A piece of cheese is located at (12, 10) in a coordinate plane. A mouse is at (4,−2) and is running up the line $y = -5x + 18$. At the point (a, b), the mouse starts getting farther from the cheese rather than closer to it. The value of $(a + b)$ is:

(a) 6 (b) 10

(c) 18 (d) 14

8. The vertex of right angle of a right angled triangle lies on the straight line $2x + y - 10 = 0$ and the two other vertices, at points (2, −3) and (4, 1) then the area of triangle in sq. units is:

(a) $\sqrt{10}$ (b) 3 (c) $\frac{33}{5}$ (d) 11

9. Given the family of lines, $a(2x + y + 4) + b(x - 2y - 3) = 0$. Among the lines of the family, the number of lines situated at a distance of $\sqrt{10}$ from the point $M(2,-3))$ is:

(a) 0 (b) 1

(c) 2 (d) ∞

10. Point $(0, \beta)$ lies on or inside the triangle formed by the lines $y = 0$, $x + y = 8$ and $3x - 4y + 12 = 0$. Then β can be :

(a) 2 (b) 4 (c) 8 (d) 12

11. If the lines $x + y + 1 = 0$; $4x + 3y + 4 = 0$ and $x + \alpha y + \beta = 0$, where $\alpha^2 + \beta^2 = 2$, are concurrent then:

(a) $\alpha = 1, \beta = -1$ (b) $\alpha = 1, \beta = \pm 1$

(c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

12. A straight line through the origin 'O' meets the parallel lines $4x + 2y = 9$ and $2x + y = -6$ at points P and Q respectively. Then the point 'O' divides the segment PQ in the ratio :

(a) 1 : 2 (b) 4 : 3 (c) 2 : 1 (d) 3 : 4

13. If the points $(2a, a)$, $(a, 2a)$ and (a, a) enclose a triangle of area 72 units, then co-ordinates of the centroid of the triangle may be :

(a) (4, 4) (b) (−4, 4) (c) (12, 12) (d) (16, 16)

14. Let $g(x) = ax + b$, where $a < 0$ and g is defined from [1, 3] onto [0, 2] then the value of $\cot(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|))$ is equal to :

(a) $g(1)$ (b) $g(2)$ (c) $g(3)$ (d) $g(1) + g(3)$

15. If the distances of any point P from the points $A(a+b,\ a-b)$ and $B(a-b,\ a+b)$ are equal, then locus of P is :

(a) $ax+by=0$ (b) $ax-by=0$ (c) $bx+ay=0$ (d) $x-y=0$

16. If the equation $4y^3-8a^2yx^2-3ay^2x+8x^3=0$ represent three straight lines, two of them are perpendicular then sum of all possible values of a is equal to :

(a) $\frac{3}{8}$ (b) $\frac{-3}{4}$ (c) $\frac{1}{4}$ (d) -2

17. The orthocentre of the triangle formed by the lines $x-7y+6=0$, $2x-5y-6=0$ and $7x+y-8=0$ is :

(a) $(8, 2)$ (b) $(0, 0)$ (c) $(1, 1)$ (d) $(2, 8)$

18. All the chords of the curve $2x^2+3y^2-5x=0$ which subtend a right angle at the origin are concurrent at :

(a) $(0, 1)$ (b) $(1, 0)$ (c) $(-1, 1)$ (d) $(1, -1)$

19. From a point $P\equiv(3, 4)$ perpendiculars PQ and PR are drawn to line $3x+4y-7=0$ and a variable line $y-1=m(x-7)$ respectively, then maximum area of ΔPQR is :

(a) 10 (b) 12 (c) 6 (d) 9

20. The equation of two adjacent sides of rhombus are given by $y=x$ and $y=7x$. The diagonals of the rhombus intersect each other at the point $(1, 2)$. Then the area of the rhombus is :

(a) $\frac{10}{3}$ (b) $\frac{20}{3}$ (c) $\frac{40}{3}$ (d) $\frac{50}{3}$

21. The point $P(3, 3)$ is reflected across the line $y=-x$. Then it is translated horizontally 3 units to the left and vertically 3 units up. Finally, it is reflected across the line $y=x$. What are the coordinates of the point after these transformations ?

(a) $(0, -6)$ (b) $(0, 0)$

(c) $(-6, 6)$ (d) $(-6, 0)$

22. The equations $x=t^3+9$ and $y=\frac{3t^3}{4}+6$ represents a straight line where t is a parameter. Then y-intercept of the line is :

(a) $-\frac{3}{4}$ (b) 9 (c) 6 (d) 1

23. The combined equation of two adjacent sides of a rhombus formed in first quadrant is $7x^2-8xy+y^2=0$; then slope of its longer diagonal is :

(a) $-\frac{1}{2}$ (b) -2 (c) 2 (d) $\frac{1}{2}$

24. The number of integral points inside the triangle made by the line $3x+4y-12=0$ with the coordinate axes which are equidistant from at least two sides is/are :

(an integral point is a point both of whose coordinates are integers.)

(a) 1 (b) 2 (c) 3 (d) 4

25. The area of triangle formed by the straight lines whose equations are $y = 4x + 2$, $2y = x + 3$ and $x = 0$ is :

(a) $\frac{25}{7\sqrt{2}}$ (b) $\frac{\sqrt{2}}{28}$ (c) $\frac{1}{28}$ (d) $\frac{15}{7}$

26. In a triangle ABC, if A is (1, 2) and the equations of the medians through B and C are $x + y = 5$ and $x = 4$ respectively then B must be :

(a) (1, 4) (b) (7, −2) (c) (4, 1) (d) (−2, 7)

27. The equation of image of pair of lines $y = |x - 1|$ with respect to y-axis is :

(a) $x^2 - y^2 - 2x + 1 = 0$ (b) $x^2 - y^2 - 4x + 4 = 0$

(c) $4x^2 - 4x - y^2 + 1 = 0$ (d) $x^2 - y^2 + 2x + 1 = 0$

28. If P, Q and R are three points with coordinates (1, 4), (4, 5) and (m, m) respectively, then the value of m for which $PR + RQ$ is minimum, is :

(a) 4 (b) 3 (c) $\frac{17}{8}$ (d) $\frac{7}{2}$

29. The vertices of triangle ABC are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. The equation of the bisector of the angle ABC of ΔABC is :

(a) $y + 2x - 11 = 0$ (b) $x - 7y + 2 = 0$

(c) $y - 2x + 9 = 0$ (d) $y + 7x - 36 = 0$

30. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then $c =$

(a) −3 (b) −1 (c) 3 (d) 1

31. The equations of L_1 and L_2 are $y = mx$ and $y = nx$, respectively. Suppose L_1 make twice as large of an angle with the horizontal (measured counterclockwise from the positive x-axis) as does L_2 and that L_1 has 4 times the slope of L_2. If L_1 is not horizontal, then the value of the product (mn) equals:

(a) $\frac{\sqrt{2}}{2}$ (b) $-\frac{\sqrt{2}}{2}$

(c) 2 (d) −2

32. Given A (0, 0) and $B(x, y)$ with $x \in (0,1)$ and $y > 0$. Let the slope of the line AB equals m_1. Point C lies on the line $x = 1$ such that the slope of BC equals m_2 where $0 < m_2 < m_1$. If the area of the triangle ABC can be expressed as $(m_1 - m_2)f(x)$, then the largest possible value of $f(x)$ is:

(a) 1 (b) 1/2

(c) 1/4 (d) 1/8

33. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point, co-ordinate of fixed point is :

(a) (−1, 2) (b) (−1, −2) (c) (1, −2) (d) $\left(1, \frac{1}{2}\right)$

34. If $\frac{x^2}{a}+\frac{y^2}{b}+\frac{2xy}{h}=0$ represent pair of straight lines and slope of one line is twice the other, then $ab:h^2$ is :

(a) $9:8$ (b) $8:9$ (c) $1:2$ (d) $2:1$

35. Statement-1: A variable line drawn through a fixed point cuts the coordinate axes at A and B. The locus of mid-point of AB is a circle.

because

Statement-2: Through 3 non-collinear points in a plane, only one circle can be drawn.

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

36. A line passing through origin and is perpendicular to two parallel lines $2x+y+6=0$ and $4x+2y-9=0$, then the ratio in which the origin divides this line segment is :

(a) $1:2$ (b) $1:1$

(c) $5:4$ (d) $3:4$

37. If a vertex of a triangle is (1, 1) and the mid-points of two sides through this vertex are (−1, 2) and (3, 2), then the centroid of the triangle is :

(a) $\left(-1,\frac{7}{3}\right)$ (b) $\left(-\frac{1}{3},\frac{7}{3}\right)$ (c) $\left(1,\frac{7}{3}\right)$ (d) $\left(\frac{1}{3},\frac{7}{3}\right)$

38. The diagonals of parallelogram $PQRS$ are along the lines $x+3y=4$ and $6x-2y=7$. Then $PQRS$ must be :

(a) rectangle (b) square

(c) rhombus (d) neither rhombus nor rectangle

39. The two points on the line $x+y=4$ that lie at a unit perpendicular distance from the line $4x+3y=10$ are (a_1, b_1) and (a_2, b_2), then $a_1+b_1+a_2+b_2=$

(a) 5 (b) 6 (c) 7 (d) 8

40. The orthocentre of the triangle formed by the lines $x+y=1$, $2x+3y=6$ and $4x-y+4=0$ lies in :

(a) first quadrant (b) second quadrant

(c) third quadrant (d) fourth quadrant

41. The equation of the line passing through the intersection of the lines $3x+4y=-5$, $4x+6y=6$ and perpendicular to $7x-5y+3=0$ is :

(a) $5x+7y-2=0$ (b) $5x-7y+2=0$

(c) $7x-5y+2=0$ (d) $5x+7y+2=0$

42. The points (2, 1), (8, 5) and $(x, 7)$ lie on a straight line. Then the value of x is :

(a) 10 (b) 11 (c) 12 (d) $\frac{35}{3}$

43. In a parallelogram $PQRS$ (taken in order), P is the point $(-1, -1)$, Q is $(8, 0)$ and R is $(7, 5)$. Then S is the point :

(a) $(-1, 4)$ (b) $(-2, 2)$ (c) $\left(-2, \frac{7}{2}\right)$ (d) $(-2, 4)$

44. The area of triangle whose vertices are (a, a), $(a+1, a+1)$, $(a+2, a)$ is :

(a) a^3 (b) $2a$ (c) 1 (d) 2

45. The equation $x^2 + y^2 - 2xy - 1 = 0$ represents :

(a) two parallel straight lines (b) two perpendicular straight lines

(c) a point (d) a circle

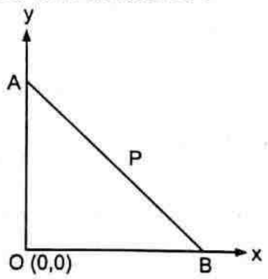
46. Let $A \equiv (-2, 0)$ and $B \equiv (2, 0)$, then the number of integral values of a, $a \in [-10, 10]$ for which line segment AB subtends an acute angle at point $C \equiv (a, a+1)$ is :

(a) 15 (b) 17 (c) 19 (d) 21

47. The angle between sides of a rhombus whose $\sqrt{2}$ times sides is mean of its two diagonal, is equal to :

(a) 300° (b) 45° (c) 60° (d) 90°

48. A rod of AB of length 3 rests on a wall as follows :



P is a point on AB such that $AP : PB = 1 : 2$. If the rod slides along the wall, then the locus of P lies on

(a) $2x + y + xy = 2$ (b) $4x^2 + xy + xy + y^2 = 4$

(c) $4x^2 + y^2 = 4$ (d) $x^2 + y^2 - x - 2y = 0$

49. If $\frac{x^2}{a} + \frac{y^2}{b} + \frac{2xy}{h} = 0$, represents pair of straight lines and slope of one line is twice the other. Then $ab : h^2$ is :

(a) 8 : 9 (b) 1 : 2 (c) 2 : 1 (d) 9 : 8

50. Locus of point of reflection of point $(a, 0)$ w.r.t. the line $yt = x + at^2$ is given by (t is parameter, $t \in R$):

(a) $x - a = 0$ (b) $y - a = 0$ (c) $x + a = 0$ (d) $y + a = 0$

51. A light ray emerging from the point source placed at $P(1, 3)$ is reflected at a point Q in the x-axis. If the reflected ray passes through $R(6, 7)$, then abscissa of Q is :

(a) $\frac{5}{2}$ (b) 3 (c) $\frac{7}{2}$ (d) 1

52. If the axes are rotated through 60° in the anticlockwise sense, find the transformed form of the equation $x^2 - y^2 = a^2$:

(a) $X^2 + Y^2 - 3\sqrt{3}\,XY = 2a^2$
(b) $X^2 + Y^2 = a^2$
(c) $Y^2 - X^2 - 2\sqrt{3}\,XY = 2a^2$
(d) $X^2 - Y^2 + 2\sqrt{3}\,XY = 2a^2$

53. The straight line $3x + y - 4 = 0$, $x + 3y - 4 = 0$ and $x + y = 0$ form a triangle which is :

(a) equilateral
(b) right-angled
(c) acute-angled and isosceles
(d) obtuse-angled and isosceles

54. If m and b are real numbers and $mb > 0$, then the line whose equation is $y = mx + b$ cannot contain the point:

(a) (0, 2008)
(b) (2008, 0)
(c) (0, – 2008)
(d) (20, – 100)

55. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is:

(a) one
(b) two
(c) three
(d) four

56. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

(a) lie on a straight line
(b) lie on a circle
(c) are vertices of a triangle
(d) None of these

57. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$; where t is a parameter is :

(a) $(3x-1)^2 + (3y)^2 = a^2 - b^2$
(b) $(3x-1)^2 + (3y)^2 = a^2 + b^2$
(c) $(3x+1)^2 + (3y)^2 = a^2 + b^2$
(d) $(3x+1)^2 + (3y)^2 = a^2 - b^2$

58. The equation of the straight line passing through (4, 3) and making intercepts on co-ordinate axes whose sum is –1 is :

(a) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(d) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

59. Let $A \equiv (3, 2)$ and $B \equiv (5, 1)$. ABP is an equilateral triangle is constructed one the side of AB remote from the origin then the orthocentre of triangle ABP is:

(a) $\left(4 - \frac{1}{2}\sqrt{3}, \frac{3}{2} - \sqrt{3}\right)$ (b) $\left(4 + \frac{1}{2}\sqrt{3}, \frac{3}{2} + \sqrt{3}\right)$

(c) $\left(4 - \frac{1}{6}\sqrt{3}, \frac{3}{2} - \frac{1}{3}\sqrt{3}\right)$ (d) $\left(4 + \frac{1}{6}\sqrt{3}, \frac{3}{2} + \frac{1}{3}\sqrt{3}\right)$

60. Area of the triangle formed by the lines through point $(6, 0)$ and at a perpendicular distance of 5 from point $(1, 3)$ and line $y = 16$ in square units is :

(a) 160 (b) 200 (c) 240 (d) 130

61. The straight lines $3x + y - 4 = 0$, $x + 3y - 4 = 0$ and $x + y = 0$ form a triangle which is :

(a) equilateral (b) right-angled

(c) acute-angled and isosceles (d) obtuse-angled and isosceles

62. The orthocentre of the triangle with vertices $(5, 0), (0, 0), \left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$ is :

(a) $(2, 3)$ (b) $\left(\frac{5}{2}, \frac{5}{2\sqrt{3}}\right)$ (c) $\left(\frac{5}{6}, \frac{5}{2\sqrt{3}}\right)$ (d) $\left(\frac{5}{2}, \frac{5}{\sqrt{3}}\right)$

63. All chords of a curve $3x^2 - y^2 - 2x + 4y = 0$ which subtends a right angle at the origin passes through a fixed point, which is :

(a) $(1, 2)$ (b) $(1, -2)$ (c) $(2, 1)$ (d) $(-2, 1)$

64. Let $P(-1, 0)$, $Q(0, 0)$, $R(3, 3\sqrt{3})$ be three points then the equation of the bisector of the angle $\angle PQR$ is :

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$ (c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

Answers

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(c)	**7.**	(b)	**8.**	(b)	**9.**	(b)	**10.**	(a)
11.	(d)	**12.**	(d)	**13.**	(d)	**14.**	(c)	**15.**	(d)	**16.**	(b)	**17.**	(c)	**18.**	(b)	**19.**	(d)	**20.**	(a)
21.	(a)	**22.**	(a)	**23.**	(c)	**24.**	(a)	**25.**	(c)	**26.**	(b)	**27.**	(d)	**28.**	(a)	**29.**	(b)	**30.**	(a)
31.	(c)	**32.**	(d)	**33.**	(c)	**34.**	(a)	**35.**	(d)	**36.**	(d)	**37.**	(c)	**38.**	(c)	**39.**	(d)	**40.**	(a)
41.	(d)	**42.**	(b)	**43.**	(d)	**44.**	(c)	**45.**	(a)	**46.**	(c)	**47.**	(d)	**48.**	(c)	**49.**	(d)	**50.**	(c)
51.	(a)	**52.**	(c)	**53.**	(d)	**54.**	(b)	**55.**	(c)	**56.**	(a)	**57.**	(b)	**58.**	(d)	**59.**	(d)	**60.**	(c)
61.	(d)	**62.**	(b)	**63.**	(b)	**64.**	(c)												

Exercise-2 : One or More than One Answer is/are Correct

1. A line makes intercepts on co-ordinate axes whose sum is 9 and their product is 20 ; then its equation is/are :

(a) $4x + 5y - 20 = 0$ (b) $5x + 4y - 20 = 0$

(c) $4x - 5y - 20 = 0$ (d) $4x + 5y + 20 = 0$

2. The equation(s) of the medians of the triangle formed by the points (4, 8), (3, 2) and (5, − 6) is/are :

(a) $x = 4$ (b) $x = 5y - 3$

(c) $2x + 3y - 12 = 0$ (d) $22x + 3y - 92 = 0$

3. The value(s) of t for which the lines $2x + 3y = 5$, $t^2x + ty - 6 = 0$ and $3x - 2y - 1 = 0$ are concurrent, can be :

(a) $t = 2$ (b) $t = -3$

(c) $t = -2$ (d) $t = 3$

4. If one of the lines given by the equation $ax^2 + 6xy + by^2 = 0$ bisects the angle between the co-ordinate axes, then value of $(a + b)$ can be :

(a) −6 (b) 3 (c) 6 (d) 12

5. Suppose *ABCD* is a quadrilateral such that the coordinates of *A*, *B* and *C* are (1, 3), (−2, 6) and (5, − 8) respectively. For what choices of coordinates of *D* will make *ABCD* a trapezium ?

(a) (3, − 6) (b) (6, − 9) (c) (0, 5) (d) (3, − 1)

6. One diagonal of a square is the portion of the line $\sqrt{3}x + y = 2\sqrt{3}$ intercepted by the axes. Then an extremity of the other diagonal is :

(a) $(1 + \sqrt{3}, \sqrt{3} - 1)$ (b) $(1 + \sqrt{3}, \sqrt{3} + 1)$

(c) $(1 - \sqrt{3}, \sqrt{3} - 1)$ (d) $(1 - \sqrt{3}, \sqrt{3} + 1)$

7. Two sides of a rhombus *ABCD* are parallel to lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at point (1, 2) and the vertex *A* is on the *y*-axis is, then the possible coordinates of *A* are:

(a) $\left(0, \frac{5}{2}\right)$ (b) (0, 0) (c) (0, 5) (d) (0, 3)

8. The equation of the sides of the triangle having (3, −1) as a vertex and $x - 4y + 10 = 0$ and $6x + 10y - 59 = 0$ as angle bisector and as median respectively drawn from different vertices, are :

(a) $6x + 7y - 13 = 0$ (b) $2x + 9y - 65 = 0$

(c) $18x + 13y - 41 = 0$ (d) $6x - 7y - 25 = 0$

9. *A*(1,3) and *C*(5, 1) are two opposite vertices of a rectangle *ABCD*. If the slope of *BD* is 2, then the coordinates of *B* can be :

(a) (4, 4) (b) (5, 4)

(c) (2, 0) (d) (1, 0)

10. All the points lying inside the triangle formed by the points (1, 3), (5, 6), and (−1, 2) satisfy:

(a) $3x + 2y \geq 0$
(b) $2x + y + 1 \geq 0$
(c) $-2x + 11 \geq 0$
(d) $2x + 3y - 12 \geq 0$

11. The slope of a median, drawn from the vertex A of the triangle ABC is −2. The co-ordinates of vertices B and C are respectively (−1, 3) and (3, 5). If the area of the triangle be 5 square units, then possible distance of vertex A from the origin is/are.

(a) 6
(b) 4
(c) $2\sqrt{2}$
(d) $3\sqrt{2}$

12. The points $A(0, 0)$, $B(\cos\alpha, \sin\alpha)$ and $C(\cos\beta, \sin\beta)$ are the vertices of a right angled triangle if:

(a) $\sin\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{\sqrt{2}}$
(b) $\cos\left(\frac{\alpha-\beta}{2}\right) = -\frac{1}{\sqrt{2}}$
(c) $\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{\sqrt{2}}$
(d) $\sin\left(\frac{\alpha-\beta}{2}\right) = -\frac{1}{\sqrt{2}}$

Answers

1.	(a, b)	**2.**	(a, c, d)	**3.**	(a, b)	**4.**	(a, c)	**5.**	(b, d)	**6.**	(b, c)
7.	(a, b)	**8.**	(b, c, d)	**9.**	(a, c)	**10.**	(a, b, c, d)	**11.**	(a, c)	**12.**	(a, b, c)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

The equations of the sides AB and CA of a ΔABC are $x + 2y = 0$ and $x - y = 3$ respectively. Given a fixed point $P(2, 3)$.

1. Let the equation of BC is $x + py = q$. Then the value of $(p + q)$ if P be the centroid of the ΔABC is:

(a) 14 (b) −14 (c) 22 (d) −22

2. If P be the orthocentre of ΔABC then equation of side BC is :

(a) $y + 5 = 0$ (b) $y - 5 = 0$ (c) $5y + 1 = 0$ (d) $5y - 1 = 0$

Paragraph for Question Nos. 3 to 4

Consider a triangle ABC with vertex $A(2, -4)$. The internal bisectors of the angles B and C are $x + y = 2$ and $x - 3y = 6$ respectively. Let the two bisectors meet at I.

3. If (a, b) is incentre of the triangle ABC then $(a + b)$ has the value equal to :

(a) 1 (b) 2 (c) 3 (d) 4

4. If (x_1, y_1) and (x_2, y_2) are the co-ordinates of the point B and C respectively, then the value of $(x_1x_2 + y_1y_2)$ is equal to :

(a) 4 (b) 5 (c) 6 (d) 8

Answers

1.	(d)	**2.**	(a)	**3.**	(b)	**4.**	(d)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If a, b, c are in A.P., then lines $ax + by + c = 0$ are concurrent at:	(P)	$(-4, -7)$
(B)	A point on the line $x + y = 4$ which lies at a unit distance from the line $4x + 3y = 10$ is :	(Q)	$(-7, 11)$
(C)	Orthocentre of triangle made by lines $x + y = 1$, $x - y + 3 = 0$, $2x + y = 7$ is	(R)	$(1, -2)$
(D)	Two vertice of a triangle are $(5, -1)$ and $(-2, 3)$. If orthocentre is the origin then coordinates of the third vertex are	(S)	$(-1, 2)$
		(T)	$(0, 0)$

2.

	Column-I		Column-II
(A)	If $\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n} {}^{k}C_{r-1}\right) = 30$, then n is equal to	(P)	1
(B)	The number of integral values of g for which atmost one member of the family of lines given by $(1 + 2\lambda)x + (1 - \lambda)y + 2 + 4\lambda = 0$ (λ is real parameter) is tangent to the circle $x^2 + y^2 + 4gx + 18x + 17y + 4g^2 = 0$ can be	(Q)	4
(C)	Number of solutions of the equation $\sin 9x + \sin 5x + 2\sin^2 x = 1$ in interval $(0, \pi)$ is	(R)	7
(D)	If the roots of the equation $x^2 + ax + b = 0$ $(a, b \in R)$ are $\tan 65°$ and $\tan 70°$, then $(a + b)$ equals.	(S)	10

3.

	Column-I		Column-II
(A)	Exact value of $\cos 40°(1 - 2\sin 10°) =$	(P)	$\frac{1}{4}$

(B)	Value of λ for which lines are concurrent $x+y+1=0$, $3x+2\lambda y+4=0$, $x+y-3\lambda=0$ can be	**(Q)**	$\frac{1}{2}$
(C)	Points $(k, 2-2k), (-k+1, 2k)$ and $(-4-k, 6-2k)$ are collinear then sum of all possible real values of 'k' is	**(R)**	$\frac{3}{2}$
(D)	Value of $\sum_{k=3}^{\infty} \sin^k\left(\frac{\pi}{6}\right) =$	**(S)**	$-\frac{1}{2}$

Answers

1.	$A \to R$; $B \to Q$; $C \to S$; $D \to P$
2.	$A \to Q$; $B \to R$; $C \to S$; $D \to P$
3.	$A \to Q$; $B \to R$; $C \to S$; $D \to P$

Exercise-5 : Subjective Type Problems

1. If the area of the quadrilateral $ABCD$ whose vertices are $A(1, 1)$, $B(7, -3)$, $C(12, 2)$ and $D(7, 21)$ is Δ. Find the sum of the digits of Δ.
2. The equation of a line through the mid-point of the sides AB and AD of rhombus $ABCD$, whose one diagonal is $3x - 4y + 5 = 0$ and one vertex is $A(3, 1)$ is $ax + by + c = 0$. Find the absolute value of $(a + b + c)$ where a, b, c are integers expressed in lowest form.
3. If the point (α, α^4) lies on or inside the triangle formed by lines $x^2y + xy^2 - 2xy = 0$, then the largest value of α is.
4. The minimum value of $[(x_1 - x_2)^2 + (12 - \sqrt{1 - x_1^2} - \sqrt{4x_2})^2]^{1/2}$ for all permissible values of x_1 and x_2 is equal to $a\sqrt{b} - c$ where $a, b, c \in N$, then find the value of $a + b - c$.
5. The number of lines that can be drawn passing through point $(2, 3)$ so that its perpendicular distance from $(-1, 6)$ is equal to 6 is :
6. The graph of $x^4 = x^2y^2$ is a union of n different lines, then the value of n is.
7. The orthocentre of triangle formed by lines $x + y - 1 = 0$, $2x + y - 1 = 0$ and $y = 0$ is (h, k), then $\frac{1}{k^2} =$
8. Find the integral value of a for which the point $(-2, a)$ lies in the interior of the triangle formed by the lines $y = x$, $y = -x$ and $2x + 3y = 6$.
9. Let $A = (-1, 0)$, $B = (3, 0)$ and PQ be any line passing through $(4, 1)$. The range of the slope of PQ for which there are two points on PQ at which AB subtends a right angle is (λ_1, λ_2), then $5(\lambda_1 + \lambda_2)$ is equal to.
10. Given that the three points where the curve $y = bx^2 - 2$ intersects the x-axis and y-axis form an equilateral triangle. Find the value of $2b$.

Answers

1.	6	**2.**	1	**3.**	1	**4.**	8	**5.**	0	**6.**	3	**7.**	4
8.	3	**9.**	6	**10.**	5								

□□□

Chapter 18 – Circle

18

CIRCLE

Exercise-1 : Single Choice Problems

1. The locus of mid-points of the chords of the circle $x^2 - 2x + y^2 - 2y + 1 = 0$ which are of unit length is :

(a) $(x-1)^2 + (y-1)^2 = \frac{3}{4}$ (b) $(x-1)^2 + (y-1)^2 = 2$

(c) $(x-1)^2 + (y-1)^2 = \frac{1}{4}$ (d) $(x-1)^2 + (y-1)^2 = \frac{2}{3}$

2. The length of a common internal tangent to two circles is 5 and a common external tangent is 15, then the product of the radii of the two circles is :

(a) 25 (b) 50 (c) 75 (d) 30

3. A circle with center (2, 2) touches the coordinate axes and a straight line AB where A and B lie on positive direction of coordinate axes such that the circle lies between origin and the line AB. If O be the origin then the locus of circumcenter of ΔOAB will be:

(a) $xy = x + y + \sqrt{x^2 + y^2}$ (b) $xy = x + y - \sqrt{x^2 + y^2}$

(c) $xy + x + y = \sqrt{x^2 + y^2}$ (d) $xy + x + y + \sqrt{x^2 + y^2} = 0$

4. Length of chord of contact of point (4, 4) with respect to the circle $x^2 + y^2 - 2x - 2y - 7 = 0$ is:

(a) $\frac{3}{\sqrt{2}}$ (b) $3\sqrt{2}$ (c) 3 (d) 6

5. Let P, Q, R, S be the feet of the perpendiculars drawn from a point (1, 1) upon the lines $x + 4y = 12$; $x - 4y + 4 = 0$ and their angle bisectors respectively; then equation of the circle which passes through Q, R, S is :

(a) $x^2 + y^2 - 5x + 3y - 6 = 0$ (b) $x^2 + y^2 - 5x - 3y + 6 = 0$

(c) $x^2 + y^2 - 5x - 3y - 6 = 0$ (d) None of these

6. From a point 'P' on the line $2x + y + 4 = 0$; which is nearest to the circle $x^2 + y^2 - 12y + 35 = 0$, tangents are drawn to given circle. The area of quadrilateral $PACB$ (where 'C' is the center of circle and PA & PB are the tangents.) is :

(a) 8 (b) $\sqrt{110}$ (c) $\sqrt{19}$ (d) None of these

7. The line $2x - y + 1 = 0$ is tangent to the circle at the point $(2, 5)$ and the centre of the circles lies on $x - 2y = 4$. The radius of the circle is:

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$

(c) $2\sqrt{5}$ (d) $5\sqrt{2}$

8. If $A(\cos\alpha, \sin\alpha)$, $B(\sin\alpha, -\cos\alpha)$, $C(1, 2)$ are the vertices of a triangle, then as α varies the locus of centroid of the ΔABC is a circle whose radius is :

(a) $\frac{2\sqrt{2}}{3}$ (b) $\sqrt{\frac{4}{3}}$ (c) $\frac{2}{3}$ (d) $\sqrt{\frac{2}{9}}$

9. Tangents drawn to circle $(x-1)^2 + (y-1)^2 = 5$ at point P meets the line $2x + y + 6 = 0$ at Q on the x-axis. Length PQ is equal to :

(a) $\sqrt{12}$ (b) $\sqrt{10}$ (c) 4 (d) $\sqrt{15}$

10. $ABCD$ is square in which A lies on positive y-axis and B lies on the positive x-axis. If D is the point $(12, 17)$, then co-ordinate of C is :

(a) $(17, 12)$ (b) $(17, 5)$ (c) $(17, 16)$ (d) $(15, 3)$

11. Statement-1: The lines $y = mx + 1 - m$ for all values of m is a normal to the circle $x^2 + y^2 - 2x - 2y = 0$.

Statement-2: The line L passes through the centre of the circle.

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

12. $A(1, 0)$ and $B(0, 1)$ are two fixed points on the circle $x^2 + y^2 = 1$. C is a variable point on this circle. As C moves, the locus of the orthocentre of the triangle ABC is :

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$ (b) $x^2 + y^2 - x - y = 0$

(c) $x^2 + y^2 = 4$ (d) $x^2 + y^2 + 2x - 2y + 1 = 0$

13. Equation of a circle passing through $(1, 2)$ and $(2, 1)$ and for which line $x + y = 2$ is a diameter ; is :

(a) $x^2 + y^2 + 2x + 2y - 11 = 0$ (b) $x^2 + y^2 - 2x - 2y - 1 = 0$

(c) $x^2 + y^2 - 2x - 2y + 1 = 0$ (d) None of these

14. The area of an equilateral triangle inscribed in a circle of radius 4 cm, is :

(a) 12 cm^2 (b) $9\sqrt{3} \text{ cm}^2$

(c) $8\sqrt{3} \text{ cm}^2$ (d) $12\sqrt{3} \text{ cm}^2$

15. Let all the points on the curve $x^2 + y^2 - 10x = 0$ are reflected about the line $y = x + 3$. The locus of the reflected points is in the form $x^2 + y^2 + gx + fy + c = 0$. The value of $(g + f + c)$ is equal to :

(a) 28 (b) −28 (c) 38 (d) −38

16. The shortest distance from the line $3x + 4y = 25$ to the circle $x^2 + y^2 = 6x - 8y$ is equal to:

(a) 7/5 (b) 9/5 (c) 11/5 (d) 32/5

17. In the xy-plane, the length of the shortest path from (0, 0) to (12, 16) that does not go inside the circle $(x-6)^2 + (y-8)^2 = 25$ is:

(a) $10\sqrt{3}$ (b) $10\sqrt{5}$

(c) $10\sqrt{3} + \frac{5\pi}{3}$ (d) $10 + 5\pi$

18. A circle is inscribed in an equilateral triangle with side lengths 6 unit. Another circle is drawn inside the triangle (but outside the first circle), tangent to the first circle and two of the sides of the triangle. The radius of the smaller circle is:

(a) $1/\sqrt{3}$ (b) 2/3

(c) 1/2 (d) 1

19. The equation of the tangent to the circle $x^2 + y^2 - 4x = 0$ which is perpendicular to the normal drawn through the origin can be :

(a) $x = 1$ (b) $x = 2$ (c) $x + y = 2$ (d) $x = 4$

20. The equation of the line parallel to the line $3x + 4y = 0$ and touching the circle $x^2 + y^2 = 9$ in the first quadrant is :

(a) $3x + 4y = 15$ (b) $3x + 4y = 45$

(c) $3x + 4y = 9$ (d) $3x + 4y = 12$

21. The centres of the three circles $x^2 + y^2 - 10x + 9 = 0$, $x^2 + y^2 - 6x + 2y + 1 = 0$, $x^2 + y^2 - 9x - 4y + 2 = 0$

(a) lie on the straight line $x - 2y = 5$ (b) lie on circle $x^2 + y^2 = 25$

(c) do not lie on straight line (d) lie on circle $x^2 + y^2 + x + y - 17 = 0$

22. The equation of the diameter of the circle $x^2 + y^2 + 2x - 4y = 4$ that is parallel to $3x + 5y = 4$ is:

(a) $3x + 5y = -7$ (b) $3x + 5y = 7$

(c) $3x + 5y = 9$ (d) $3x + 5y = 1$

23. There are two circles passing through points $A(-1, 2)$ and $B(2, 3)$ having radius $\sqrt{5}$. Then the length of intercept on x-axis of the circle intersecting x-axis is :

(a) 2 (b) 3 (c) 4 (d) 5

24. A square $OABC$ is formed by line pairs $xy = 0$ and $xy + 1 = x + y$ where 'O' is the origin. A circle with centre C_1 inside the square is drawn to touch the line pair $xy = 0$ and another circle with centre C_2 and radius twice that of C_1, is drawn to touch the circle C_1 and the other line pair. The radius of the circle with centre C_1 is:

(a) $\dfrac{\sqrt{2}}{\sqrt{3}(\sqrt{2}+1)}$ (b) $\dfrac{2\sqrt{2}}{3(\sqrt{2}+1)}$

(c) $\dfrac{\sqrt{2}}{3(\sqrt{2}+1)}$ (d) $\dfrac{\sqrt{2}+1}{3\sqrt{2}}$

25. The equation of the circle circumscribing the triangle formed by the points (3, 4), (1, 4) and (3, 2) is :

(a) $8x^2 + 8y^2 - 16x - 13y = 0$ (b) $x^2 + y^2 - 4x - 8y + 19 = 0$

(c) $x^2 + y^2 - 4x - 6y + 11 = 0$ (d) $x^2 + y^2 - 6x - 6y + 17 = 0$

26. The equation of the tangent to circle $x^2 + y^2 + 2gx + 2fy = 0$ at the origin is :

(a) $fx + gy = 0$ (b) $gx + fy = 0$ (c) $x = 0$ (d) $y = 0$

27. The line $y = x$ is tangent at (0, 0) to a circle of radius 1. The centre of the circle is :

(a) either $\left(-\frac{1}{2}, \frac{1}{2}\right)$ or $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (b) either $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(c) either $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) either (1, 0) or (−1, 0)

28. The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$:

(a) cut orthogonally (b) touch each other internally

(c) intersect in two points (d) touch each other externally

29. In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to:

(a) $\dfrac{AB \cdot AD}{\sqrt{AB^2 + AD^2}}$ (b) $\dfrac{AB \cdot AD}{AB + AD}$

(c) $\sqrt{AB \cdot AD}$ (d) $\dfrac{AB \cdot AD}{\sqrt{AB^2 - AD^2}}$

30. Radical centre of the circles drawn on the sides as a diameter of triangle formed by the lines $3x - 4y + 6 = 0$, $x - y + 2 = 0$ and $4x + 3y - 17 = 0$ is :

(a) (3, 2) (b) (3, −2) (c) (2, −3) (d) (2, 3)

31. Statement-1: A circle can be inscribed in a quadrilateral whose sides are $3x - 4y = 0$, $3x - 4y = 5$, $3x + 4y = 0$ and $3x + 4y = 7$.

Statement-2: A circle can be inscribed in a parallelogram if and only if it is a rhombus.

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

32. If $x = 3$ is the chord of contact of the circle $x^2 + y^2 = 81$, then the equation of the corresponding pair of tangents, is:

(a) $x^2 - 8y^2 + 54x + 729 = 0$ (b) $x^2 - 8y^2 - 54x + 729 = 0$

(c) $x^2 - 8y^2 - 54x - 729 = 0$ (d) $x^2 - 8y^2 = 729$

33. The shortest distance from the line $3x + 4y = 25$ to the circle $x^2 + y^2 = 6x - 8y$ is equal to :

(a) $\frac{7}{3}$ (b) $\frac{9}{5}$ (c) $\frac{11}{5}$ (d) $\frac{7}{5}$

34. The circle with equation $x^2 + y^2 = 1$ intersects the line $y = 7x + 5$ at two distinct points A and B. Let C be the point at which the positive x-axis intersects the circle. The angle ACB is :

(a) $\tan^{-1}\frac{4}{3}$ (b) $\cot^{-1}(-1)$ (c) $\tan^{-1} 1$ (d) $\cot^{-1}\frac{4}{3}$

35. The abscissae of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. The radius of the circle with AB as diameter is ::

(a) $\sqrt{a^2 + b^2 + p^2 + q^2}$ (b) $\sqrt{a^2 + p^2}$

(c) $\sqrt{b^2 + q^2}$ (d) $\sqrt{a^2 + b^2 + p^2 + 1}$

36. Let C be the circle of radius unity centred at the origin. If two positive numbers x_1 and x_2 are such that the line passing through $(x_1, -1)$ and $(x_2, 1)$ is tangent to C then:

(a) $x_1 x_2 = 1$ (b) $x_1 x_2 = -1$

(c) $x_1 + x_2 = 1$ (d) $4x_1 x_2 = 1$

37. A circle bisects the circumference of the circle $x^2 + y^2 + 2y - 3 = 0$ and touches the line $x = y$ at the point (1, 1). Its radius is :

(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{9}{\sqrt{2}}$ (c) $4\sqrt{2}$ (d) $3\sqrt{2}$

38. The distance between the chords of contact of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f) is:

(a) $\sqrt{g^2+f^2}$ (b) $\dfrac{\sqrt{g^2+f^2-c}}{2}$

(c) $\dfrac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$ (d) $\dfrac{\sqrt{g^2+f^2+c}}{2\sqrt{g^2+f^2}}$

39. If the tangents AP and AQ are drawn from the point $A(3,-1)$ to the circle $x^2+y^2-3x+2y-7=0$ and C is the centre of circle, then the area of quadrilateral $APCQ$ is :

(a) 9 (b) 4 (c) 2 (d) non-existent

40. Number of integral value(s) of k for which no tangent can be drawn from the point $(k, k+2)$ to the circle $x^2+y^2=4$ is :

(a) 0 (b) 1 (c) 2 (d) 3

41. If the length of the normal for each point on a curve is equal to the radius vector, then the curve :

(a) is a circle passing through origin

(b) is a circle having centre at origin and radius >0

(c) is a circle having centre on x-axis and touching y-axis

(d) is a circle having centre on y-axis and touching x-axis

42. A circle of radius unity is centred at origin. Two particles start moving at the same time from the point $(1, 0)$ and move around the circle in opposite direction. One of the particle moves counter clockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving $(1, 0)$, the two particles meet first at a point P, and continue until they meet next at point Q. The coordinates of the point Q are:

(a) $(1, 0)$ (b) $(0, 1)$

(c) $(0, -1)$ (d) $(-1, 0)$

43. A variable circle is drawn to touch the x-axis at the origin. The locus of the pole of the straight line $lx+my+n=0$ w.r.t the variable circle has the equation:

(a) $x(my-n)-ly^2=0$ (b) $x(my+n)-ly^2=0$

(c) $x(my-n)+ly^2=0$ (d) none of these

44. The minimum length of the chord of the circle $x^2+y^2+2x+2y-7=0$ which is passing through $(1, 0)$ is :

(a) 2 (b) 4 (c) $2\sqrt{2}$ (d) $\sqrt{5}$

45. Three concentric circles of which the biggest is $x^2+y^2=1$, have their radii in A.P. If the line $y=x+1$ cuts all the circles in real and distinct points. The interval in which the common difference of the A.P. will lie is:

(a) $\left(0, \dfrac{1}{4}\right)$ (b) $\left(0, \dfrac{1}{2\sqrt{2}}\right)$ (c) $\left(0, \dfrac{2-\sqrt{2}}{4}\right)$ (d) none

46. The locus of the point of intersection of the tangent to the circle $x^2 + y^2 = a^2$, which include an angle of 45° is the curve $(x^2 + y^2)^2 = \lambda a^2(x^2 + y^2 - a^2)$. The value of λ is:

(a) 2
(b) 4
(c) 8
(d) 16

47. A circle touches the line $y = x$ at point (4, 4) on it. The length of the chord on the line $x + y = 0$ is $6\sqrt{2}$. Then one of the possible equation of the circle is :

(a) $x^2 + y^2 + x - y + 30 = 0$
(b) $x^2 + y^2 + 2x - 18y + 32 = 0$
(c) $x^2 + y^2 + 2x + 18y + 32 = 0$
(d) $x^2 + y^2 - 2x - 22y + 32 = 0$

48. Point on the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ which is nearest to the line $y = 2x + 11$ is :

(a) $\left(1-\frac{6}{\sqrt{5}}, -2+\frac{3}{\sqrt{5}}\right)$
(b) $\left(1+\frac{6}{\sqrt{5}}, -2-\frac{3}{\sqrt{5}}\right)$
(c) $\left(1-\frac{6}{\sqrt{5}}, -2-\frac{3}{\sqrt{5}}\right)$
(d) None of these

49. A foot of the normal from the point (4, 3) to a circle is (2, 1) and a diameter of the circle has the equation $2x - y - 2 = 0$. Then the equation of the circle is:

(a) $x^2 + y^2 - 4y + 2 = 0$
(b) $x^2 + y^2 - 4y + 1 = 0$
(c) $x^2 + y^2 - 2x - 1 = 0$
(d) $x^2 + y^2 - 2x + 1 = 0$

50. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ and $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, $abcd$ is equal to:

(a) 4
(b) 1/4
(c) 1
(d) 16

Answers

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(c)	**7.**	(a)	**8.**	(d)	**9.**	(a)	**10.**	(b)
11.	(a)	**12.**	(a)	**13.**	(c)	**14.**	(d)	**15.**	(c)	**16.**	(a)	**17.**	(c)	**18.**	(a)	**19.**	(d)	**20.**	(a)
21.	(c)	**22.**	(b)	**23.**	(c)	**24.**	(c)	**25.**	(c)	**26.**	(b)	**27.**	(c)	**28.**	(d)	**29.**	(d)	**30.**	(d)
31.	(d)	**32.**	(b)	**33.**	(d)	**34.**	(c)	**35.**	(a)	**36.**	(a)	**37.**	(b)	**38.**	(c)	**39.**	(d)	**40.**	(b)
41.	(b)	**42.**	(d)	**43.**	(a)	**44.**	(b)	**45.**	(c)	**46.**	(c)	**47.**	(b)	**48.**	(a)	**49.**	(c)	**50.**	(c)

Exercise-2 : One or More than One Answer is/are Correct

1. Number of circle touching both the axes and the line $x + y = 4$ is greater than or equal to :
   (a) 1
   (b) 2
   (c) 3
   (d) 4

2. Which of the following is/are true ?
   The circles $x^2 + y^2 - 6x - 6y + 9 = 0$ and $x^2 + y^2 + 6x + 6y + 9 = 0$ are such that :
   (a) They do not intersect
   (b) They touch each other
   (c) Their exterior common tangents are parallel
   (d) Their interior common tangents are perpendicular

3. Let 'α' be a variable parameter, then the length of the chord of the curve :
   $$(x - \sin^{-1}\alpha)(x - \cos^{-1}\alpha) + (y - \sin^{-1}\alpha)(y + \cos^{-1}\alpha) = 0$$
   along the line $x = \frac{\pi}{4}$ can not be equal to :
   (a) $\frac{\pi}{3}$
   (b) $\frac{\pi}{6}$
   (c) $\frac{\pi}{4}$
   (d) $\frac{\pi}{2}$

4. If the point $(1, 4)$ lies inside the circle $x^2 + y^2 - 6x - 10y + p = 0$ and the circle does not touch or intersect the coordinate axes, then which of the following must be correct :
   (a) $p < 29$
   (b) $p > 25$
   (c) $p > 27$
   (d) $p < 27$

5. The equation of a circle $S_1 = 0$ is $x^2 + y^2 = 4$, locus of the intersection of orthogonal tangents to the circle is the curve C_1 and the locus of the intersection of perpendicular tangents to the curve C_1 is the curve C_2, then :
   (a) C_2 is a circle
   (b) C_1, C_2 are circles having different centres
   (c) C_1, C_2 are circles having same centres
   (d) area enclosed between C_1 and C_2 is 8π

6. If two distinct chords drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then :
   (a) $p^2 = q^2$
   (b) $p^2 > q^2$
   (c) $p^2 < 8q^2$
   (d) $p^2 > 8q^2$

7. If $a = \max\{(x+2)^2 + (y-3)^2\}$ and $b = \min\{(x+2)^2 + (y-3)^2\}$ where x, y satisfying $x^2 + y^2 + 8x - 10y - 40 = 0$, then :
   (a) $a + b = 18$
   (b) $a + b = 178$
   (c) $a - b = 4\sqrt{2}$
   (d) $a - b = 72\sqrt{2}$

8. The locus of points of intersection of the tangents to $x^2 + y^2 = a^2$ at the extremeties of a chord of circle $x^2 + y^2 = a^2$ which touches the circle $x^2 + y^2 - 2ax = 0$ is/are :

(a) $y^2 = a(a - 2x)$ (b) $x^2 = a(a - 2y)$

(c) $x^2 + y^2 = (x - a)^2$ (d) $x^2 + y^2 = (y - a)^2$

9. A circle passes through the points $(-1, 1)$, $(0, 6)$ and $(5, 5)$. The point(s) on this circle, the tangent(s) at which is/are parallel to the straight line joining the origin to its centre is/are

(a) $(1, -5)$ (b) $(5, 1)$ (c) $(-5, -1)$ (d) $(-1, 5)$

10. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y - 93 = 0$ with the sides parallel to the co-ordinate axes. The co-ordinate of the vertices are :

(a) $(8, 5)$ (b) $(8, 9)$ (c) $(-6, 5)$ (d) $(-6, -9)$

Answers

1.	(a, b, c, d)	**2.**	(a, c, d)	**3.**	(a, b, c)	**4.**	(a, b)	**5.**	(a, c, d)	**6.**	(b, d)
7.	(b, d)	**8.**	(a, c)	**9.**	(b, d)	**10.**	(a, c)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Let each of the circles,

$S_1 \equiv x^2 + y^2 + 4y - 1 = 0,$ $S_2 \equiv x^2 + y^2 + 6x + y + 8 = 0,$

$S_3 \equiv x^2 + y^2 - 4x - 4y - 37 = 0$

touches the other two. Let P_1, P_2, P_3 be the points of contact of S_1 and S_2, S_2 and S_3, S_3 and S_1 respectively and C_1, C_2, C_3 be the centres of S_1, S_2, S_3 respectively.

1. The co-ordinates of P_1 are :

(a) $(2, -1)$ (b) $(2, 1)$ (c) $(-2, 1)$ (d) $(-2, -1)$

2. The ratio $\dfrac{\text{area }(\Delta P_1P_2P_3)}{\text{area }(\Delta C_1C_2C_3)}$ is equal to :

(a) $3:2$ (b) $2:5$ (c) $5:3$ (d) $2:3$

3. P_2 and P_3 are image of each other with respect to line :

(a) $y = x + 1$ (b) $y = -x$ (c) $y = x$ (d) $y = -x + 2$

Paragraph for Question Nos. 4 to 6

Let $A(3, 7)$ and $B(6, 5)$ are two points. $C : x^2 + y^2 - 4x - 6y - 3 = 0$ is a circle.

4. The chords in which the circle C cuts the members of the family S of circle passing through A and B are concurrent at :

(a) $(2, 3)$ (b) $\left(2, \dfrac{23}{3}\right)$ (c) $\left(3, \dfrac{23}{2}\right)$ (d) $(3, 2)$

5. Equation of the member of the family of circles S that bisects the circumference of C is :

(a) $x^2 + y^2 - 5x - 1 = 0$ (b) $x^2 + y^2 - 5x + 6y - 1 = 0$

(c) $x^2 + y^2 - 5x - 6y - 1 = 0$ (d) $x^2 + y^2 + 5x - 6y - 1 = 0$

6. If O is the origin and P is the center of C, then absolute value of difference of the squares of the lengths of the tangents from A and B to the circle C is equal to :

(a) $(AB)^2$ (b) $(OP)^2$ (c) $|(AP)^2 - (BP)^2|$ (d) $(AP)^2 + (BP)^2$

Paragraph for Question Nos. 7 to 8

Let the diameter of a subset S of the plane be defined as the maximum of the distance between arbitrary pairs of points of S.

7. Let $S = \{(x, y) : (y - x) \le 0, x + y \ge 0, x^2 + y^2 \le 2\}$, then the diameter of S is :

(a) 2 (b) 4 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

8. Let $S=\{(x,y):(\sqrt{5}-1)x-\sqrt{10+2\sqrt{5}}\ y\geq 0,\ (\sqrt{5}-1)\ x+\sqrt{10+12\sqrt{5}}\ y\geq 0,\ x^2+y^2\leq 9\}$ then the diameter of S is :

(a) $\frac{3}{2}(\sqrt{5}-1)$ (b) $3(\sqrt{5}-1)$ (c) $3\sqrt{2}$ (d) 3

Paragraph for Question Nos. 9 to 10

Let L_1, L_2 and L_3 be the lengths of tangents drawn from a point P to the circles $x^2+y^2=4$, $x^2+y^2-4x=0$ and $x^2+y^2-4y=0$ respectively. If $L_1^4=L_2^2L_3^2+16$ then the locus of P are the curves, C_1 (a straight line) and C_2 (a circle).

9. Circum centre of the triangle formed by C_1 and two other lines which are at angle of 45° with C_1 and tangent to C_2 is :

(a) $(1,1)$ (b) $(0,0)$ (c) $(-1,-1)$ (d) $(2,2)$

10. If S_1, S_2 and S_3 are three circles congruent to C_2 and touch both C_1 and C_2; then the area of triangle formed by joining centres of the circles S_1, S_2 and S_3 is (in square units)

(a) 2 (b) 4 (c) 8 (d) 16

Answers

1.	(d)	**2.**	(b)	**3.**	(c)	**4.**	(b)	**5.**	(c)	**6.**	(c)	**7.**	(a)	**8.**	(d)	**9.**	(b)	**10.**	(c)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	The triangle PQR is inscribed in the circle $x^2 + y^2 = 169$. If $Q(5, 12)$ and $R(-12, 5)$ then $\angle QPR$ is	**(P)**	$\pi / 6$
(B)	The angle between the lines joining the origin to the points of intersection of the line $4x + 3y = 24$ with circle $(x-3)^2 + (y-4)^2 = 25$	**(Q)**	$\pi / 4$
(C)	Two parallel tangents drawn to given circle are cut by a third tangent. The angle subtended by the portion of third tangent between the given tangents at the centre is	**(R)**	$\pi / 3$
(D)	A chord is drawn joining the point of contact of tangents drawn from a point P to the circle. If the chord subtends an angle $\pi / 2$ at the centre then the angle included between the tangents at P is	**(S)**	$\pi / 2$
		(T)	π

2.

	Column-I		Column-II
(A)	A ray of light coming from the point $(1, 2)$ is reflected at a point A on the x-axis then passes through the point $(5, 3)$. The coordinates of the point A are :	**(P)**	$\left(\frac{13}{5}, 0\right)$
(B)	The equation of three sides of triangle ABC are $x + y = 3$, $x - y = 5$ and $3x + y = 4$. Considering the sides as diameter, three circles S_1, S_2, S_3 are drawn whose radical centre is at :	**(Q)**	$(4, -1)$
(C)	If the straight line $x - 2y + 1 = 0$ intersects the circle $x^2 + y^2 = 25$ at the points P and Q, then the coordinate of the point of intersection of tangents drawn at P and Q to the circle is	**(R)**	$(-25, 50)$
(D)	The equation of three sides of a triangle are $4x + 3y + 9 = 0$, $2x + 3 = 0$ and $3y - 4 = 0$. The circum centre of the triangle is :	**(S)**	$\left(\frac{-19}{8}, \frac{1}{6}\right)$
		(T)	$(-1, 2)$

Answers

1. A $\to$ Q; B $\to$ S; C $\to$ S; D $\to$ S

2. A $\to$ P; B $\to$ Q; C $\to$ R; D $\to$ S

Exercise-5 : Subjective Type Problems

1. Tangents are drawn to circle $x^2+y^2=1$ at its intersection points (distinct) with the circle $x^2+y^2+(\lambda-3)x+(2\lambda+2)y+2=0$. The locus of intersection of tangents is a straight line, then the slope of that straight line is.
2. The radical centre of the three circles is at the origin. The equations of the two of the circles are $x^2+y^2=1$ and $x^2+y^2+4x+4y-1=0$. If the third circle passes through the points $(1, 1)$ and $(-2, 1)$; and its radius can be expressed in the form of $\frac{p}{q}$, where p and q are relatively prime positive integers. Find the value of $(p+q)$.
3. Let $S=\{(x, y) \mid x, y \in R, x^2+y^2-10x+16=0\}$. The largest value of $\frac{y}{x}$ can be put in the form $\frac{m}{n}$ where m, n are relatively prime natural numbers, then $m^2+n^2=$
4. In the above problem, the complete range of the expression $x^2+y^2-26x+12y+210$ is $[a, b]$, then $b-2a=$
5. If the line $y=2-x$ is tangent to the circle S at the point $P(1, 1)$ and circle S is orthogonal to the circle $x^2+y^2+2x+2y-2=0$, then find the length of tangent drawn from the point $(2, 2)$ to circle S.
6. Two circles having radii r_1 and r_2 passing through vertex A of a triangle ABC. One of the circle touches the side BC at B and other circle touches the side BC at C. If $a=5$ and $A=30°$; find $\sqrt{r_1r_2}$.
7. A circle S of radius 'a' is the director circle of another circle S_1. S_1 is the director circle of S_2 and so on. If the sum of radius of S, S_1, S_2, S_3 .... circles is '2' and $a=(k-\sqrt{k})$, then the value of k is ......
8. If r_1 and r_2 be the maximum and minimum radius of the circle which pass through the point $(4, 3)$ and touch the circle $x^2+y^2=49$, then $\frac{r_1}{r_2}$ is ......
9. Let C be the circle $x^2+y^2-4x-4y-1=0$. The number of points common to C and the sides of the rectangle determined by the lines $x=2$, $x=5$, $y=-1$ and $y=5$ is P then find P.
10. Two congruent circles with centres at $(2, 3)$ and $(5, 6)$ intersects at right angle; find the radius of the circle.
11. The sum of abscissa and ordinate of a point on the circle $x^2+y^2-4x+2y-20=0$ which is nearest to $\left(2, \frac{3}{2}\right)$ is :
12. AB is any chord of the circle $x^2+y^2-6x-8y-11=0$ which subtends an angle $\frac{\pi}{2}$ at $(1, 2)$. If locus of midpoint of AB is a circle $x^2+y^2-2ax-2by-c=0$; then find the value of $(a+b+c)$.

13. If circles $x^2 + y^2 = c$ with radius $\sqrt{3}$ and $x^2 + y^2 + ax + by + c = 0$ with radius $\sqrt{6}$ intersect at two points A and B. If length of $AB = \sqrt{l}$. Find l.

Answers

1.	2	**2.**	5	**3.**	25	**4.**	66	**5.**	2	**6.**	5	**7.**	2
8.	6	**9.**	3	**10.**	3	**11.**	6	**12.**	8	**13.**	8		

❑❑❑

19 PARABOLA

Exercise-1 : Single Choice Problems

1. Let PQ be the latus rectum of the parabola $y^2 = 4x$ with vertex A. Minimum length of the projection of PQ on a tangent drawn in portion of parabola PAQ is :

(a) 2 (b) 4
(c) $2\sqrt{3}$ (d) $2\sqrt{2}$

2. A normal is drawn to the parabola $y^2 = 9x$ at the point $P(4, 6)$. A circle is described on SP as diameter; where S is the focus. The length of the intercept made by the circle on the normal at point P is :

(a) $\frac{17}{4}$ (b) $\frac{15}{4}$ (c) 4 (d) 5

3. A trapezium is inscribed in the parabola $y^2 = 4x$, such that its diagonal pass through the point $(1, 0)$ and each has length $\frac{25}{4}$. If the area of the trapezium be P, then $4P$ is equal to :

(a) 70 (b) 71 (c) 80 (d) 75

4. The length of normal chord of parabola $y^2 = 4x$, which subtends an angle of $90°$ at the vertex is :

(a) $6\sqrt{3}$ (b) $7\sqrt{2}$ (c) $8\sqrt{2}$ (d) $9\sqrt{2}$

5. If b and c are the lengths of the segments of any focal chord of a parabola $y^2 = 4ax$. Then the length of semi-latus rectum is :

(a) $\frac{bc}{b+c}$ (b) $\frac{2bc}{b+c}$
(c) $\frac{b+c}{2}$ (d) $\sqrt{bc}$

6. The length of the shortest path that begins at the point $(-1, 1)$, touches the x-axis and then ends at a point on the parabola $(x-y)^2 = 2(x+y-4)$, is :

(a) $3\sqrt{2}$ (b) 5 (c) $4\sqrt{10}$ (d) 13

7. If the normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in a point O' and S be its focus, then $|SP| \cdot |SQ| \cdot |SR|$ is equal to :

(a) a^3 (b) $a^2(SO')$

(c) $a(SO')^2$ (d) None of these

8. Let P and Q are points on the parabola $y^2 = 4ax$ with vertex O, such that OP is perpendicular to OQ and have lengths r_1 and r_2 respectively, then the value of $\dfrac{r_1^{4/3} r_2^{4/3}}{r_1^{2/3} + r_2^{2/3}}$ is :

(a) $16a^2$ (b) a^2 (c) $4a$ (d) None of these

9. Length of the shortest chord of the parabola $y^2 = 4x + 8$, which belongs to the family of lines $(1 + \lambda)y + (\lambda - 1)x + 2(1 - \lambda) = 0$, is :

(a) 6 (b) 5 (c) 8 (d) 2

10. If locus of mid-point of any normal chord of the parabola :

$$y^2 = 4x \text{ is } x - a = \frac{b}{y^2} + \frac{y^2}{c};$$

where $a, b, c \in N$, then $(a + b + c)$ equals to :

(a) 5 (b) 8 (c) 10 (d) None of these

11. Let tangents at P and Q to curve $y^2 - 4x - 2y + 5 = 0$ intersect at T. If $S(2, 1)$ is a point such that $(SP)(SQ) = 16$, then the length ST is equal to :

(a) 3 (b) 4 (c) 5 (d) None of these

12. Abscissa of two points P and Q on parabola $y^2 = 8x$ are roots of equation $x^2 - 17x + 11 = 0$. Let Tangents at P and Q meet at point T, then distance of T from the focus of parabola is :

(a) 7 (b) 6 (c) 5 (d) 4

13. If $Ax + By = 1$ is a normal to the curve $ay = x^2$, then :

(a) $4A^2(1 - aB) = aB^3$ (b) $4A^2(2 + aB) = aB^3$

(c) $4A^2(1 + aB) + aB^3 = 0$ (d) $2A^2(2 - aB) = aB^3$

14. The equation of a curve which passes through the point (3, 1), such that the segment of any tangent between the point of tangency and the x-axis is bisected at its point of intersection with y-axis, is :

(a) $x = 3y^2$ (b) $x^2 = 9y$ (c) $x = y^2 + 2$ (d) $2x = 3y^2 + 3$

15. The parabola $y = 4 - x^2$ has vertex P. It intersects x-axis at A and B. If the parabola is translated from its initial position to a new position by moving its vertex along the line $y = x + 4$, so that it intersects x-axis at B and C, then abscissa of C will be :

(a) 3 (b) 4 (c) 6 (d) 8

16. A focal chord for parabola $y^2 = 8(x+2)$ is inclined at an angle of 60° with positive x-axis and intersects the parabola at P and Q. Let perpendicular bisector of the chord PQ intersects the x-axis at R ; then the distance of R from focus is :

(a) $\frac{8}{3}$ (b) $\frac{16\sqrt{3}}{3}$ (c) $\frac{16}{3}$ (d) $8\sqrt{3}$

17. The Director circle of the parabola $(y-2)^2 = 16(x+7)$ touches the circle $(x-1)^2 + (y+1)^2 = r^2$, then r is equal to :

(a) 10 (b) 11 (c) 12 (d) None of these

18. The chord of contact of a point $A(x_A, y_A)$ of $y^2 = 4x$ passes through (3, 1) and point A lies on $x^2 + y^2 = 5^2$. Then :

(a) $5x_A^2 + 24x_A + 11 = 0$ (b) $13x_A^2 + 8x_A - 21 = 0$

(c) $5x_A^2 + 24x_A + 61 = 0$ (d) $13x_A^2 + 21x_A - 31 = 0$

Answers

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(a)	**5.**	(b)	**6.**	(a)	**7.**	(c)	**8.**	(a)	**9.**	(c)	**10.**	(b)
11.	(b)	**12.**	(a)	**13.**	(d)	**14.**	(a)	**15.**	(d)	**16.**	(c)	**17.**	(c)	**18.**	(a)				

Exercise-2 : One or More than One Answer is/are Correct

1. PQ is a double ordinate of the parabola $y^2 = 4ax$. If the normal at P intersect the line passing through Q and parallel to x-axis at G; then locus of G is a parabola with :

(a) vertex at $(4a, 0)$

(b) focus at $(5a, 0)$

(c) directrix as the line $x - 3a = 0$

(d) length of latus rectum equal to $4a$

Answers

1.	(a, b, c, d)									

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Consider the following lines :

$$L_1 : x - y - 1 = 0$$
$$L_2 : x + y - 5 = 0$$
$$L_3 : y - 4 = 0$$

Let L_1 is axis to a parabola, L_2 is tangent at the vertex to this parabola and L_3 is another tangent to this parabola at some point P.

Let 'C' be the circle circumscribing the triangle formed by tangent and normal at point P and axis of parabola. The tangent and normals at the extremities of latus rectum of this parabola forms a quadrilateral $ABCD$.

1. The equation of the circle 'C' is :

(a) $x^2 + y^2 - 2x - 31 = 0$
(b) $x^2 + y^2 - 2y - 31 = 0$
(c) $x^2 + y^2 - 2x - 2y - 31 = 0$
(d) $x^2 + y^2 + 2x + 2y = 31$

2. The given parabola is equal to which of the following parabola ?

(a) $y^2 = 16\sqrt{2}x$
(b) $x^2 = -4\sqrt{2}y$
(c) $y^2 = -\sqrt{2}x$
(d) $y^2 = 8\sqrt{2}x$

3. The area of the quadrilateral $ABCD$ is :

(a) 16 (b) 8 (c) 64 (d) 32

Answers

1.	(a)	**2.**	(d)	**3.**	(c)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	The equation of tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which cuts off equal intercepts on axes is $x-y=a$ where $\lvert a \rvert$ equal to	**(P)**	$\sqrt{2}$
(B)	The normal $y=mx-2am-am^2$ to the parabola $y^2=4ax$ subtends a right angle at the vertex if $\lvert m \rvert$ equal to	**(Q)**	$\sqrt{3}$
(C)	The equation of the common tangent to parabola $y^2=4x$ and $x^2=4y$ is $x+y+\frac{k}{\sqrt{3}}=0$, then k is equal to	**(R)**	$\sqrt{8}$
(D)	An equation of common tangent to parabola $y^2=8x$ and the hyperbola $3x^2-y^2=3$ is $4x-2y+\frac{k}{\sqrt{2}}=0$, then k is equal to	**(S)**	$\sqrt{41}$
		(T)	2

2.

	Column-I		Column-II
(A)	Area of ΔPQR is equal to	**(P)**	2
(B)	Radius of circumcircle of ΔPQR is equal to	**(Q)**	$\frac{5}{2}$
(C)	Distance of the vertex from the centroid of ΔPQR is equal to	**(R)**	$\frac{3}{2}$
(D)	Distance of the centroid from the circumcentre of ΔPQR is equal to	**(S)**	$\frac{2}{3}$
		(T)	$\frac{11}{6}$

Answers

1. A $\rightarrow$ S; B $\rightarrow$ P; C $\rightarrow$ Q; D $\rightarrow$ R

2. A $\rightarrow$ P; B $\rightarrow$ Q; C $\rightarrow$ S; D $\rightarrow$ T

Exercise-5 : Subjective Type Problems

1. Points A and B lie on the parabola $y = 2x^2 + 4x - 2$, such that origin is the mid-point of the line segment AB. If 'l' be the length of the line segment AB, then find the unit digit of l^2.
2. For the parabola $y = -x^2$, let $a < 0$ and $b > 0$; $P(a, -a^2)$ and $Q(b, -b^2)$. Let M be the mid-point of PQ and R be the point of intersection of the vertical line through M, with the parabola. If the ratio of the area of the region bounded by the parabola and the line segment PQ to the area of the triangle PQR be $\frac{\lambda}{\mu}$; where λ and μ are relatively prime positive integers, then find the value of $(\lambda + \mu)$:
3. The chord AC of the parabola $y^2 = 4ax$ subtends an angle of 90° at points B and D on the parabola. If points A, B, C and D are represented by $(at_i^2, 2at_i)$, $i = 1, 2, 3, 4$ respectively, then find the value of $\left|\frac{t_2 + t_4}{t_1 + t_3}\right|$.

Answers

1.	8	2.	7	3.	1										

□□□

Chapter 20 – Ellipse

20

ELLIPSE

Exercise-1 : Single Choice Problems

1. If CF be the perpendicular from the centre C of the ellipse $\frac{x^2}{12}+\frac{y^2}{8}=1$, on the tangent at any point P and G is the point where the normal at P meets the major axis, then the value of $(CF \cdot PG)$ equals to :

(a) 5 (b) 6 (c) 8 (d) None of these

2. The minimum length of intercept on any tangent to the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ cut by the circle $x^2+y^2=25$ is :

(a) 8 (b) 9 (c) 2 (d) 11

3. The point on the ellipse $x^2+2y^2=6$, whose distance from the line $x+y=7$ is minimum is :

(a) (2, 3) (b) (2, 1) (c) (1, 0) (d) None of these

4. If lines $2x+3y=10$ and $2x-3y=10$ are tangents at the extremities of a latus rectum of an ellipse; whose centre is origin, then the length of the latus rectum is :

(a) $\frac{110}{27}$ (b) $\frac{98}{27}$ (c) $\frac{100}{27}$ (d) $\frac{120}{27}$

5. The area bounded by the circle $x^2+y^2=a^2$ and the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the area of another ellipse having semi-axes :

(a) $a+b$ and b (b) $a-b$ and a (c) a and b (d) None of these

6. If F_1 and F_2 are the feet of the perpendiculars from foci S_1 and S_2 of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ on the tangent at any point P of the ellipse, then :

(a) $S_1F_1+S_2F_2 \geq 2$ (b) $S_1F_1+S_2F_2 \geq 3$ (c) $S_1F_1+S_2F_2 \geq 6$ (d) $S_1F_1+S_2F_2 \geq 8$

7. Consider the ellipse $\dfrac{x^2}{f(k^2+2k+5)}+\dfrac{y^2}{f(k+11)}=1$, where $f(x)$ is a positive decreasing function, then the value of k for which major axis coincides with x-axis is :

(a) $k \in (-7, -5)$ (b) $k \in (-5, -3)$ (c) $k \in (-3, 2)$ (d) None of these

8. If area of the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{b^2}=1$ inscribed in a square of side length $5\sqrt{2}$ is A, then $\dfrac{A}{\pi}$ equals to :

(a) 12 (b) 10 (c) 8 (d) 11

9. Any chord of the conic $x^2+y^2+xy=1$ passing through origin is bisected at a point (p, q), then $(p+q+12)$ equals to :

(a) 13 (b) 14 (c) 11 (d) 12

10. Tangents are drawn from the point (4, 2) to the curve $x^2+9y^2=9$, the tangent of angle between the tangents :

(a) $\dfrac{3\sqrt{3}}{5\sqrt{17}}$ (b) $\dfrac{\sqrt{43}}{10}$ (c) $\dfrac{\sqrt{43}}{5}$ (d) $\sqrt{\dfrac{3}{17}}$

Answers

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(c)	**5.**	(b)	**6.**	(d)	**7.**	(c)	**8.**	(a)	**9.**	(d)	**10.**	(c)

Exercise-2 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

An ellipse has semi-major axis of length 2 and semi-minor axis of length 1. It slides between the co-ordinate axes in the first quadrant, while maintaining contact with both x-axis and y-axis.

1. The locus of the centre of ellipse is :

(a) $x^2 + y^2 = 3$ (b) $x^2 + y^2 = 5$

(c) $(x-2)^2 + (y-1)^2 = 5$ (d) $(x-2)^2 + (y-1)^2 = 3$

2. The locus of the foci of the ellipse is :

(a) $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 16$ (b) $x^2 + y^2 + \frac{1}{x^2} - \frac{1}{y^2} = 2\sqrt{3} + 4$

(c) $x^2 + y^2 - \frac{1}{x^2} - \frac{1}{y^2} = 2\sqrt{3} + 4$ (d) $x^2 - y^2 + \frac{1}{x^2} - \frac{1}{y^2} = 2\sqrt{3} + 4$

Paragraph for Question Nos. 3 to 5

A coplanar beam of light emerging from a point source have the equation $\lambda x - y + 2(1+\lambda) = 0$, $\forall\ \lambda \in R$; the rays of the beam strike an elliptical surface and get reflected inside the ellipse. The reflected rays form another convergent beam having the equation $\mu x - y + 2(1-\mu) = 0$, $\forall\ \mu \in R$. Further it is found that the foot of the perpendicular from the point (2, 2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$

3. The eccentricity of the ellipse is equal to :

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

4. The area of the largest triangle that an incident ray and corresponding reflected ray can enclose with the major axis of the ellipse is equal to :

(a) $4\sqrt{5}$ (b) $\sqrt{5}$

(c) $3\sqrt{5}$ (d) $2\sqrt{5}$

5. The least value of total distance travelled by an incident ray and the corresponding reflected ray is equal to :

(a) 6 (b) 3

(c) $\sqrt{5}$ (d) $2\sqrt{5}$

Answers

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(d)	**5.**	(a)

Exercise-3 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point $P(4\cos\phi, 2\sin\phi)$ is a normal to the circle $x^2 + y^2 - 8x - 4y = 0$ then $\frac{\phi}{2}$ may be	**(P)**	0
(B)	The eccentric angle(s) of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance 2 units from the centre of the ellipse is/are	**(Q)**	$\cos^{-1}\left(-\frac{2}{3}\right)$
(C)	The eccentric angle of point of intersection of the ellipse $x^2 + 4y^2 = 4$ and the parabola $x^2 + 1 = y$ is	**(R)**	$\frac{\pi}{4}$
(D)	If the normal at the point $P(\sqrt{14}\cos\theta, \sqrt{5}\sin\theta)$ to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ intersect it again at the point $Q(\sqrt{14}\cos 2\theta, \sqrt{5}\sin 2\theta)$, then θ is	**(S)**	$\frac{5\pi}{4}$
		(T)	$\frac{\pi}{2}$

Answers

1. $A \to P, R$; $B \to R, S$; $C \to P$; $D \to Q$

Exercise-4 : Subjective Type Problems

1. For the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let O be the centre and S and S' be the foci. For any point P on the ellipse the value of $PS.\ PS'd^2$ (where d is the distance of O from the tangent at P) is equal to
2. Number of perpendicular tangents that can be drawn on the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ from point (6, 7) is

Answers

1.	4	**2.**	0										

□□□

Chapter 21 – Hyperbola

21

HYPERBOLA

Exercise-1 : Single Choice Problems

1. The normal to curve $xy = 4$ at the point (1, 4) meets the curve again at :

(a) $(-4, -1)$ (b) $\left(-8, -\frac{1}{2}\right)$ (c) $\left(-16, -\frac{1}{4}\right)$ (d) $(-1, -4)$

2. Let $PQ : 2x + y + 6 = 0$ is a chord of the curve $x^2 - 4y^2 = 4$. Coordinates of the point $R(\alpha, \beta)$ that satisfy $\alpha^2 + \beta^2 - 1 \le 0$; such that area of trianlge PQR is minimum; are given by :

(a) $\left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ (b) $\left(\frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

(c) $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

3. If $y = mx + c$ be a tangent to hyperbola $\frac{x^2}{\lambda^2} - \frac{y^2}{(\lambda^3 + \lambda^2 + \lambda)^2} = 1$, then least value of $16\,m^2$ equals to :

(a) 0 (b) 1 (c) 4 (d) 9

4. Let the double ordinate PP' of the hyperbola $\frac{x^2}{4} - \frac{y^2}{3} = 1$ is produced both sides to meet asymptotes of hyperbola in Q and Q'. The product $(PQ)(PQ')$ is equal to :

(a) 3 (b) 4 (c) 1 (d) 5

5. If eccentricity of conjugate hyperbola of the given hyperbola :

$$\left|\sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2}\right| = 3$$

is e', then value of $8e'$ is :

(a) 12 (b) 14 (c) 17 (d) 10

6. A normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ has equal intercepts on positive x and positive y-axes. If this normal touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $3(a^2+b^2)$ is equal to :

(a) 5 (b) 25 (c) 16 (d) None of these

7. Locus of a point, whose chord of contact with respect to the circle $x^2+y^2=4$ is a tangent to the hyperbola $xy=1$ is a/an :

(a) ellipse (b) circle

(c) hyperbola (d) parabola

8. Let the chord $x\cos\alpha+y\sin\alpha=p$ of the hyperbola $\frac{x^2}{16}-\frac{y^2}{18}=1$ subtends a right angle at the centre. Let diameter of the circle, concentric with the hyperbola, to which the given chord is a tangent is d, then $\frac{d}{4}$ is equal to :

(a) 4 (b) 5 (c) 6 (d) 7

9. If the tangent and normal at a point on rectangular hyperbola cut-off intercept a_1, a_2 on x-axis and b_1, b_2 on the y-axis, then $a_1a_2+b_1b_2$ is equal to :

(a) 2 (b) $\frac{1}{2}$ (c) 0 (d) -1

Answers

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(a)	**5.**	(d)	**6.**	(b)	**7.**	(c)	**8.**	(c)	**9.**	(c)	

Exercise-2 : One or More than One Answer is/are Correct

1. A common tangent to the hyperbola $9x^2 - 16y^2 = 144$ and the circle $x^2 + y^2 = 9$ is/are :

(a) $y = \frac{3}{\sqrt{7}}x + \frac{15}{\sqrt{7}}$ (b) $y = 3\sqrt{\frac{2}{\sqrt{7}}}x + \frac{25}{\sqrt{7}}$

(c) $y = 2\sqrt{\frac{3}{7}}x + 15\sqrt{7}$ (d) $y = -3\sqrt{\frac{2}{\sqrt{7}}}x + \frac{25}{\sqrt{7}}$

2. Tangents are drawn to the hyperbola $x^2 - y^2 = 3$ which are parallel to the line $2x + y + 8 = 0$. Then their points of contact is/are :

(a) (2, 1) (b) (2, –1)

(c) (–2, –1) (d) (–2, 1)

3. If the line $ax + by + c = 0$ is normal to the curve $xy = 1$, then:

(a) $a > 0, b > 0$ (b) $a > 0, b < 0$

(c) $b < 0, a < 0$ (d) $a < 0, b > 0$

4. A circle cuts rectangular hyperbola $xy = 1$ in the points (x_r, y_r), $r = 1, 2, 3, 4$ then :

(a) $y_1 y_2 y_3 y_4 = 1$ (b) $x_1 x_2 x_3 x_4 = 1$

(c) $x_1 x_2 x_3 x_4 = y_1 y_2 y_3 y_4 = -1$ (d) $y_1 y_2 y_3 y_4 = 0$

Answers

1.	(b, d)	**2.**	(b, d)	**3.**	(b, d)	**4.**	(a, b)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

A point P moves such that sum of the slopes of the normals drawn from it to the hyperbola $xy = 16$ is equal to the sum of the ordinates of the feet of the normals. Let 'P' lies on the curve C, then :

1. The equation of 'C' is :
   (a) $x^2 = 4y$ (b) $x^2 = 16y$
   (c) $x^2 = 12y$ (d) $y^2 = 8x$
2. If tangents are drawn to the curve C, then the locus of the midpoint of the portion of tangent intercepted between the co-ordinate axes, is :
   (a) $x^2 = 4y$ (b) $x^2 = 2y$
   (c) $x^2 + 2y = 0$ (d) $x^2 + 4y = 0$
3. Area of the equilateral triangle, inscribed in the curve C, and having one vertex same as the vertex of C is :
   (a) $768\sqrt{3}$ (b) $776\sqrt{3}$
   (c) $760\sqrt{3}$ (d) None of these

Answers

1.	(b)	2.	(c)	3.	(a)														

Exercise-4 : Subjective Type Problems

1. Let $y = mx + c$ be a common tangent to $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and $\frac{x^2}{4} + \frac{y^2}{3} = 1$, then find the value of $m^2 + c^2$.

2. The maximum number of normals that can be drawn to an ellipse/hyperbola passing through a given point is :

3. Tangent at P to rectangular hyperbola $xy = 2$ meets coordinate axes at A and B, then area of triangle OAB (where O is origin) is :

Answers

1.	8	**2.**	4	**3.**	4								

❑❑❑

Trigonometry

Chapter 22 – Compound Angles

22

Compound Angles

Exercise-1 : Single Choice Problems

1. $\left(\cos^4\frac{\pi}{24}-\sin^4\frac{\pi}{24}\right)$ equals :

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{6}-\sqrt{2}}{4}$ (c) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (d) $\frac{\sqrt{3}+1}{2}$

2. If $a\sin x+b\cos(c+x)+b\cos(c-x)=\alpha,\ \alpha>a$, then the minimum value of $|\cos c|$ is :

(a) $\sqrt{\frac{\alpha^2-a^2}{b^2}}$ (b) $\sqrt{\frac{\alpha^2-a^2}{2b^2}}$ (c) $\sqrt{\frac{\alpha^2-a^2}{3b^2}}$ (d) $\sqrt{\frac{\alpha^2-a^2}{4b^2}}$

3. If all values of $x\in(a,b)$ satisfy the inequality $\tan x\tan 3x<-1,\ x\in\left(0,\frac{\pi}{2}\right)$, then the maximum value $(b-a)$ is :

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

4. $\sum_{r=1}^{8}\tan(rA)\tan((r+1)A)$ where $A=36^\circ$ is :

(a) $-10-\tan A$ (b) $-10+\tan A$ (c) -10 (d) -9

5. Let $f(x)=2\operatorname{cosec}2x+\sec x+\operatorname{cosec}x$, then minimum value of $f(x)$ for $x\in\left(0,\frac{\pi}{2}\right)$ is :

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{2}-1}$ (c) $\frac{1}{\sqrt{2}+1}$ (d) $\frac{2}{\sqrt{2}+1}$

6. The exact value of $\operatorname{cosec}10^\circ+\operatorname{cosec}50^\circ-\operatorname{cosec}70^\circ$ is :

(a) 4 (b) 5 (c) 6 (d) 8

7. If $u=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}+\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by :

(a) $2(a^2+b^2)$ (b) $2\sqrt{a^2+b^2}$ (c) $(a+b)^2$ (d) $(a-b)^2$

8. If $u_n = \sin(n\theta)\sec^n\theta$, $v_n = \cos(n\theta)\sec^n\theta$, $n \in N$, $n \neq 1$, then $\dfrac{v_n - v_{n-1}}{u_{n-1}} + \dfrac{1}{n}\dfrac{u_n}{v_n} =$

(a) $-\cot\theta + \dfrac{1}{n}\tan(n\theta)$ (b) $\cot\theta + \dfrac{1}{n}\tan(n\theta)$

(c) $\tan\theta + \dfrac{1}{n}\tan(n\theta)$ (d) $-\tan\theta + \dfrac{\tan(n\theta)}{n}$

9. If $a\cos^2 3\alpha + b\cos^4\alpha = 16\cos^6\alpha + 9\cos^2\alpha$ is an identity, then

(a) $a = 1, b = 24$ (b) $a = 3, b = 24$ (c) $a = 4, b = 2$ (d) $a = 7, b = 18$

10. Maximum value of $\cos x(\sin x + \cos x)$ is equal to :

(a) $\sqrt{2}$ (b) 2 (c) $\dfrac{\sqrt{2}+1}{2}$ (d) $\sqrt{2}+1$

11. If $\dfrac{\sin A}{\sin B} = \dfrac{\sqrt{3}}{2}$ and $\dfrac{\cos A}{\cos B} = \dfrac{\sqrt{5}}{2}$, $0 < A, B < \dfrac{\pi}{2}$ then $\tan A + \tan B$ is equal to :

(a) $\sqrt{\dfrac{3}{5}}$ (b) $\sqrt{\dfrac{5}{3}}$ (c) $\dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$ (d) $\dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{3}}$

12. Let $0 \le \alpha, \beta, \gamma, \delta \le \pi$ where β and γ are not complementary such that

$$2\cos\alpha + 6\cos\beta + 7\cos\gamma + 9\cos\delta = 0$$

and $$2\sin\alpha - 6\sin\beta + 7\sin\gamma - 9\sin\delta = 0$$

If $\dfrac{\cos(\alpha+\delta)}{\cos(\beta+\gamma)} = \dfrac{m}{n}$ where m and n are relatively prime positive numbers, then the value of $(m+n)$ is equal to :

(a) 11 (b) 10 (c) 9 (d) 7

13. If $-\pi < \theta < -\dfrac{\pi}{2}$, then $\left|\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}\right|$ is equal to :

(a) $2\sec\theta$ (b) $-2\sec\theta$ (c) $2\sec\dfrac{\theta}{2}$ (d) $-\sec\dfrac{\theta}{2}$

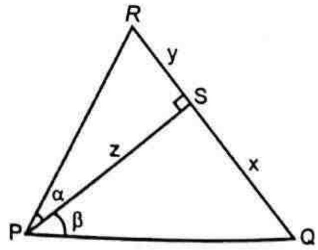
14. If $A = \sum_{r=1}^{3}\cos\dfrac{2r\pi}{7}$ and $B = \sum_{r=1}^{3}\cos\dfrac{2^r\pi}{7}$, then :

(a) $A + B = 0$ (b) $2A + B = 0$ (c) $A + 2B = 0$ (d) $A = B$

15. In a ΔPQR (as shown in figure) if $x : y : z = 2 : 3 : 6$, then the value of $\angle QPR$ is :

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$



16. If $A = \sum_{r=1}^{3} \cos \frac{2r\pi}{7}$ and $B = \sum_{r=1}^{3} \cos \frac{2^r \pi}{7}$, then :

(a) $A + B = 0$ (b) $2A + B = 0$ (c) $A + 2B = 0$ (d) $A - B = 0$

17. Let $f(x) = \sin x + 2\cos^2 x; \frac{\pi}{6} \le x \le \frac{2\pi}{3}$, then maximum value of $f(x)$ is :

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$

18. In ΔABC, $\angle C = \frac{2\pi}{3}$ then the value of $\cos^2 A + \cos^2 B - \cos A \cdot \cos B$ is equal to :

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

19. The number of solutions of the equation $4\sin^2 x + \tan^2 x + \cot^2 x + \text{cosec}^2 x = 6$ in $[0, 2\pi]$:

(a) 1 (b) 2 (c) 3 (d) 4

20. If $\sin A, \cos A$ and $\tan A$ are in G.P., then $\cos^3 A + \cos^2 A$ is equal to :

(a) 1 (b) 2 (c) 4 (d) none

21. Range of function $f(x) = \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{6}\right)$ is :

(a) $[-\sqrt{2}, \sqrt{2}]$ (b) $[-\sqrt{2}(\sqrt{3}+1), \sqrt{2}(\sqrt{3}+1)]$

(c) $\left[-\frac{\sqrt{3}+1}{\sqrt{2}}, \frac{\sqrt{3}+1}{\sqrt{2}}\right]$ (d) $\left[-\frac{\sqrt{3}-1}{\sqrt{2}}, \frac{\sqrt{3}-1}{\sqrt{2}}\right]$

22. The value of

$\tan(\log_2 6) \cdot \tan(\log_2 3) \cdot \tan 1$ is always equal to :

(a) $\tan(\log_2 6) + \tan(\log_2 3) + \tan 1$ (b) $\tan(\log_2 6) - \tan(\log_2 3) - \tan 1$

(c) $\tan(\log_2 6) - \tan(\log_2 3) + \tan 1$ (d) $\tan(\log_2 6) + \tan(\log_2 3) - \tan 1$

23. In a triangle ABC, side $BC = 3, AC = 4$ and $AB = 5$. The value of $\sin A + \sin 2B + \sin 3C$ is equal to:

(a) $\frac{24}{25}$ (b) $\frac{14}{25}$ (c) $\frac{64}{25}$ (d) none

24. If $A + B + C = 180°$, then $\frac{\cos A \cos C + \cos(A+B)\cos(B+C)}{\cos A \sin C - \sin(A+B)\cos(B+C)}$ simplifies to :

(a) $-\cot C$ (b) 0 (c) $\tan C$ (d) $\cot C$

25. If $\alpha + \gamma = 2\beta$ then the expression $\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$ simplifies to :

(a) $\tan\beta$ (b) $-\tan\beta$ (c) $\cot\beta$ (d) $-\cot\beta$

26. The product $\left(\cos\frac{x}{2}\right)\cdot\left(\cos\frac{x}{4}\right)\cdot\left(\cos\frac{x}{8}\right)\ldots\ldots\left(\cos\frac{x}{256}\right)$ is equal to :

(a) $\frac{\sin x}{128\sin\frac{x}{256}}$ (b) $\frac{\sin x}{256\sin\frac{x}{256}}$ (c) $\frac{\sin x}{128\sin\frac{x}{128}}$ (d) $\frac{\sin x}{512\sin\frac{x}{512}}$

27. The value of the expression

$\frac{\sin 7\alpha + 6\sin 5\alpha + 17\sin 3\alpha + 12\sin\alpha}{\sin 6\alpha + 5\sin 4\alpha + 12\sin 2\alpha}$, where $\alpha = \frac{\pi}{5}$ is equal to :

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{4}$ (c) $\frac{\sqrt{5}+1}{2}$ (d) $\frac{\sqrt{5}-1}{2}$

28. In a triangle ABC if $\sum\tan^2 A = \sum\tan A\tan B$, then largest angle of the triangle in radian will be :

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

29. Which one of the following values is not the solution of the equation

$\log_{|\sin x|}(|\cos x|) + \log_{|\cos x|}(|\sin x|) = 2$

(a) $\frac{7\pi}{4}$ (b) $\frac{11\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{8}$

30. Range of $f(x) = \sin^6 x + \cos^6 x$ is :

(a) $\left[\frac{1}{4}, 1\right]$ (b) $\left[\frac{1}{4}, \frac{3}{4}\right]$ (c) $\left[\frac{3}{4}, 1\right]$ (d) $[1, 2]$

31. If $y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$, then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is equal to :

(a) $\frac{1}{y}$ (b) y (c) $1-y$ (d) $1+y$

32. If $\frac{\tan^3 A}{1+\tan^2 A} + \frac{\cot^3 A}{1+\cot^2 A} = p\sec A\,\mathrm{cosec}\,A + q\sin A\cos A$, then :

(a) $p=2, q=1$ (b) $p=1, q=2$ (c) $p=1, q=-2$ (d) $p=2, q=-1$

33. If θ lies in the second quadrant. Then the value of $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to :

(a) $2\sec\theta$ (b) $-2\sec\theta$ (c) $2\,\mathrm{cosec}\,\theta$ (d) 2

34. If $y = (\sin\theta + \mathrm{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$, then minimum value of y is :

(a) 7 (b) 8 (c) 9 (d) none of these

35. If $\log_3 \sin x - \log_3 \cos x - \log_3(1-\tan x) - \log_3(1+\tan x) = -1$, then $\tan 2x$ is equal to (wherever defined)

(a) -2 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) 6

36. If $\sin\theta + \operatorname{cosec}\theta = 2$, then the value of $\sin^8\theta + \operatorname{cosec}^8\theta$ is equal to :

(a) 2 (b) 2^4 (c) 2^8 (d) more than 2^8

37. If $\tan^3\theta + \cot^3\theta = 52$, then the value of $\tan^2\theta + \cot^2\theta$ is equal to :

(a) 14 (b) 15
(c) 16 (d) 17

38. The maximum value of $\log_{20}(3\sin x - 4\cos x + 15)$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

39. If $x^2 + y^2 = 9$ and $4a^2 + 9b^2 = 16$, then maximum value of $4a^2x^2 + 9b^2y^2 - 12abxy$ is :

(a) 81 (b) 100 (c) 121 (d) 144

40. If $A = \sqrt{\sin 2 - \sin\sqrt{3}}$, $B = \sqrt{\cos 2 - \cos\sqrt{3}}$, then which of the following statement is true ?

(a) A and B both are real numbers and $A > B$
(b) A and B both are real numbers and $A < B$
(c) Exactly one of A and B is not real number
(d) Both A and B are not real numbers

41. The number of real values of x such that

$(2^x + 2^{-x} - 2\cos x)(3^{x+\pi} + 3^{-x-\pi} + 2\cos x)(5^{\pi-x} + 5^{x-\pi} - 2\cos x) = 0$ is :

(a) 1 (b) 2 (c) 3 (d) infinite

42. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

(a) infinite number of real roots (b) no real roots
(c) exactly one real root (d) exactly four real roots

43. If $\pi < \alpha < \frac{3\pi}{2}$, then the expression $\sqrt{4\sin^4\alpha + \sin^2 2\alpha} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$ is equal to :

(a) $2 + 4\sin\alpha$ (b) $2 - 4\cos\alpha$ (c) 2 (d) $2 - 4\sin\alpha$

44. $\left(\cos\frac{\pi}{12} - \sin\frac{\pi}{12}\right)\left(\tan\frac{\pi}{12} + \cot\frac{\pi}{12}\right) =$

(a) $\frac{1}{\sqrt{2}}$ (b) $4\sqrt{2}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

45. $\tan(100°) + \tan(125°) + \tan(100°)\tan(125°) =$

(a) 0 (b) $\frac{1}{2}$ (c) -1 (d) 1

46. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x =$

(a) 2 (b) 1 (c) 3 (d) $\frac{1}{2}$

47. The maximum value of $\log_5(3x + 4y)$, if $x^2 + y^2 = 25$ is :

(a) 1 (b) 2 (c) 3 (d) 4

48. The number of values of θ between $-\pi$ and $\frac{3\pi}{2}$ that satisfies the equation $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$ is :

(a) 3 (b) 4 (c) 5 (d) 6

49. Given that $\sin\beta = \frac{4}{5}, 0 < \beta < \pi$ and $\tan\beta > 0$, then $((3\sin(\alpha+\beta) - 4\cos(\alpha+\beta))\operatorname{cosec}\alpha$ is equal to:

(a) 2 (b) 3 (c) 4 (d) 5

50. The maximum value of $\sin\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)$ for $x \in \left[0, \frac{\pi}{2}\right]$ is attained at $x =$

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

51. The values of 'a' for which the equation $\sin x(\sin x + \cos x) = a$ has a real solution are

(a) $1-\sqrt{2} \le a \le 1+\sqrt{2}$ (b) $2-\sqrt{3} \le a \le 2+\sqrt{3}$

(c) $0 \le a \le 2+\sqrt{3}$ (d) $\frac{1-\sqrt{2}}{2} \le a \le \frac{1+\sqrt{2}}{2}$

52. The value of $\cos 12° \cos 24° \cos 36° \cos 48° \cos 60° \cos 72° \cos 84°$ is :

(a) $\frac{1}{64}$ (b) $\frac{1}{128}$ (c) $\frac{1}{256}$ (d) $\frac{1}{512}$

53. The ratio of the maximum value to minimum value of $2\cos^2\theta + \cos\theta + 1$ is :

(a) 32 : 7 (b) 32 : 9 (c) 4 : 1 (d) 2 : 1

54. If all values of $x \in (a, b)$ satisfy the inequality $\tan x \tan 3x < -1$, $x \in \left(0, \frac{\pi}{2}\right)$, then the maximum value $(b-a)$ is :

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

55. If a regular polygon of 'n' sides has circum radius $= R$ and inradius $= r$; then each side of polygon is :

(a) $(R+r)\tan\left(\frac{\pi}{2n}\right)$ (b) $2(R+r)\tan\left(\frac{\pi}{2n}\right)$

(c) $(R+r)\sin\left(\frac{\pi}{2n}\right)$ (d) $2(R+r)\cot\left(\frac{\pi}{2n}\right)$

56. The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is:

(a) $\frac{1}{8}$ (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{1}{2}$

57. $\frac{\sin\theta}{\cos(3\theta)} + \frac{\sin(3\theta)}{\cos(9\theta)} + \frac{\sin(9\theta)}{\cos(27\theta)} + \frac{\sin(27\theta)}{\cos(81\theta)} =$

(a) $\dfrac{\sin(81\theta)}{2\cos(80\theta)\cos\theta}$ (b) $\dfrac{\sin(80\theta)}{2\cos(81\theta)\cos\theta}$

(c) $\dfrac{\sin(81\theta)}{\cos(80\theta)\cos\theta}$ (d) $\dfrac{\sin(80\theta)}{\cos(81\theta)\cos\theta}$

58. The value of $\left(\sin\dfrac{\pi}{9}\right)\left(4+\sec\dfrac{\pi}{9}\right)$ is :

(a) $\dfrac{1}{2}$ (b) $\sqrt{2}$ (c) 1 (d) $\sqrt{3}$

59. If $\dfrac{dy}{dx}=\sin\left(\dfrac{x\pi}{2}\right)\cos(x\pi)$, then y is strictly increasing in :

(a) (3, 4) (b) $\left(\dfrac{5}{2},\dfrac{7}{2}\right)$ (c) (2, 3) (d) $\left(\dfrac{1}{2},\dfrac{3}{2}\right)$

60. Smallest positive value of θ satisfying the equation $8\sin\theta\cos2\theta\sin3\theta\cos4\theta=\cos6\theta$; is :

(a) $\dfrac{\pi}{18}$ (b) $\dfrac{\pi}{22}$ (c) $\dfrac{\pi}{24}$ (d) None of these

61. If an angle A of a triangle ABC is given by $3\tan A+1=0$, then $\sin A$ and $\cos A$ are the roots of the equation

(a) $10x^2-2\sqrt{10}x+3=0$ (b) $10x^2-2\sqrt{10}x-3=0$

(c) $10x^2+2\sqrt{10}x+3=0$ (d) $10x^2+2\sqrt{10}x-3=0$

62. If θ is an acute angle and $\tan\theta=\dfrac{1}{\sqrt{7}}$, then the value of $\dfrac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}$ is :

(a) 3/4 (b) 1/2 (c) 2 (d) 5/4

63. If $2\cos\theta+\sin\theta=1$, then $7\cos\theta+6\sin\theta$ equals

(a) 1 or 2 (b) 2 or 3 (c) 2 or 4 (d) 2 or 6

64. If $\sin\theta+\operatorname{cosec}\theta=2$, then the value of $\sin^8\theta+\operatorname{cosec}^8\theta$ is equal to :

(a) 2 (b) 2^4 (c) 2^8 (d) more than 2^8

65. If $\tan^3\theta+\cot^3\theta=52$, then the value of $\tan^2\theta+\cot^2\theta$ is equal to :

(a) 14 (b) 15 (c) 16 (d) 17

66. If $ABCD$ is a cyclic quadrilateral such that $12\tan A-5=0$ and $5\cos B+3=0$ then $\tan C+\tan D$ is equal to :

(a) $\dfrac{21}{12}$ (b) $\dfrac{11}{12}$ (c) $-\dfrac{11}{12}$ (d) $-\dfrac{21}{12}$

67. If $\dfrac{\pi}{2}<\theta<\dfrac{3\pi}{2}$ then $\sqrt{\tan^2\theta-\sin^2\theta}$ is equal to :

(a) $\tan\theta\sin\theta$ (b) $-\tan\theta\sin\theta$ (c) $\tan\theta-\sin\theta$ (d) $\sin\theta-\tan\theta$

68. The value of $\dfrac{\sin 10°+\sin 20°}{\cos 10°+\cos 20°}$ equals

(a) $2+\sqrt{3}$ (b) $\sqrt{2}-1$ (c) $2-\sqrt{3}$ (d) $\sqrt{2}+1$

69. The expression $\cos^6\theta+\sin^6\theta+3\sin^2\theta\cos^2\theta$ simplifies to :

(a) 0 (b) 1 (c) 2 (d) 3

70. $\dfrac{\sin x+\cos x}{\sin x-\cos x}-\dfrac{\sec^2 x+2}{\tan^2 x-1}=$, where $x\in\left(0,\dfrac{\pi}{2}\right)$

(a) $\dfrac{1}{\tan x+1}$ (b) $\dfrac{2}{1+\tan x}$ (c) $\dfrac{2}{1+\cot x}$ (d) $\dfrac{2}{1-\tan x}$

71. If $\dfrac{\cot\alpha+\cot(270°+\alpha)}{\cot\alpha-\cot(270°+\alpha)}-2\cos(135°+\alpha)\cos(315°-\alpha)=\lambda\cos 2\alpha$, where $\alpha\in\left(0,\dfrac{\pi}{2}\right)$, then $\lambda=$

(a) 0 (b) 1 (c) 2 (d) 4

72. The expression $\dfrac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}\tan\left(\dfrac{\pi}{4}+\alpha\right)+1$, $\alpha\in\left(-\dfrac{\pi}{4},\dfrac{\pi}{4}\right)$ simplifies to :

(a) $\operatorname{cosec}^2\left(\dfrac{\pi}{4}-\alpha\right)$ (b) $\sec^2\left(\dfrac{\pi}{4}-\alpha\right)$ (c) $\tan^2\left(\dfrac{\pi}{4}-\alpha\right)$ (d) $\cot^2\left(\dfrac{\pi}{4}-\alpha\right)$

73. The value of expression $\dfrac{\tan\alpha+\sin\alpha}{2\cos^2\dfrac{\alpha}{2}}$ for $\alpha=\dfrac{\pi}{4}$ is :

(a) 4 (b) 3 (c) 2 (d) 1

74. $\cos 2\alpha-\cos 3\alpha-\cos 4\alpha+\cos 5\alpha$ simplifies to :

(a) $-4\sin\dfrac{\alpha}{2}\sin\alpha\cos\dfrac{7\alpha}{2}$ (b) $4\sin\dfrac{\alpha}{2}\sin\alpha\cos\dfrac{7\alpha}{2}$

(c) $-4\sin\dfrac{\alpha}{2}\sin\dfrac{7\alpha}{2}\cos\alpha$ (d) $-4\sin\alpha\cos\dfrac{\alpha}{2}\sin\dfrac{7\alpha}{2}$

75. If $\tan\gamma=\sec\alpha\sec\beta+\tan\alpha\tan\beta$, then the least value of $\cos 2\gamma$ is :

(a) -1 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) 0

76. If $\operatorname{cosec} x=\dfrac{2}{\sqrt{3}}$, $\cot x=-\dfrac{1}{\sqrt{3}}$, $x\in[0,2\pi]$, then $\cos x+\cos 2x+\cos 3x+\ldots\ldots+\cos 100x=$

(a) $\dfrac{1}{2}$ (b) $-\dfrac{1}{2}$ (c) $-\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{\sqrt{3}}{2}$

77. The value of $\displaystyle\sum_{r=0}^{10}\cos^3\left(\frac{\pi r}{3}\right)$ is equal to :

(a) $-\dfrac{7}{8}$ (b) $-\dfrac{9}{8}$ (c) $-\dfrac{3}{8}$ (d) $-\dfrac{1}{8}$

78. The value of the expression $\frac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$ is :

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

79. If $x, y \in R$ and satisfy $(x+5)^2+(y-12)^2=14^2$, then the minimum value of x^2+y^2 is :

(a) 2 (b) 1 (c) $\sqrt{3}$ (d) $\sqrt{2}$

80. If θ_1, θ_2 and θ_3 are the three values of $\theta \in [0, 2\pi]$ for which $\tan\theta = \lambda$ then the value of $\tan\frac{\theta_1}{3}\tan\frac{\theta_2}{3}+\tan\frac{\theta_2}{3}\tan\frac{\theta_3}{3}+\tan\frac{\theta_3}{3}\tan\frac{\theta_1}{3}$ is equal to (λ is a constant)

(a) -3 (b) -2 (c) 2 (d) 3

81. If $\tan\alpha = \frac{b}{a}$, $a > b > 0$ and if $0 < \alpha < \frac{\pi}{4}$, then $\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}$ is equal to :

(a) $\frac{2\sin\alpha}{\sqrt{\cos 2\alpha}}$ (b) $\frac{2\cos\alpha}{\sqrt{\cos 2\alpha}}$ (c) $\frac{2\sin\alpha}{\sqrt{\sin 2\alpha}}$ (d) $\frac{2\cos\alpha}{\sqrt{\sin 2\alpha}}$

82. Minimum value of $3\sin\theta + 4\cos\theta$ in the interval $\left[0, \frac{\pi}{2}\right]$ is :

(a) -5 (b) 3 (c) 4 (d) $\frac{7}{\sqrt{2}}$

83. If $f(n) = \prod_{r=1}^{n} \cos r, n \in N$, then

(a) $|f(n)|>|f(n+1)|$ (b) $f(5)>0$ (c) $f(4)>0$ (d) $|f(n)|<|f(n+1)|$

84. If $\tan A + \sin A = p$ and $\tan A - \sin A = q$, then the value of $\frac{(p^2-q^2)^2}{pq}$ is :

(a) 16 (b) 22 (c) 18 (d) 42

85. Let $t_1 = (\sin\alpha)^{\cos\alpha}$, $t_2 = (\sin\alpha)^{\sin\alpha}$, $t_3 = (\cos\alpha)^{\cos\alpha}$, $t_4 = (\cos\alpha)^{\sin\alpha}$, where $\alpha \in \left(0, \frac{\pi}{4}\right)$, then which of the following is correct

(a) $t_3 > t_1 > t_2$ (b) $t_4 > t_2 > t_1$ (c) $t_4 > t_1 > t_2$ (d) $t_1 > t_3 > t_2$

86. If $\cos A = \frac{3}{4}$, then the value of expression $32\sin\frac{A}{2}\sin\frac{5A}{2}$ is equal to :

(a) 11 (b) -11 (c) 12 (d) 4

87. If $\cos(\alpha+\beta)+\sin(\alpha-\beta)=0$ and $\tan\beta = \frac{1}{2009}$; then $\tan 3\alpha$ is :

(a) 2 (b) 1 (c) 3 (d) 4

88. If $2^x = 3^y = 6^{-z}$, the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is equal to :

(a) 0 (b) 1 (c) 2 (d) 3

89. Let α, β be such that $\pi < \alpha - \beta < 3\pi$

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$ then the value of $\cos\left(\frac{\alpha-\beta}{2}\right)$ is :

(a) $\frac{-3}{\sqrt{130}}$ (b) $\frac{3}{\sqrt{130}}$ (c) $\frac{6}{65}$ (d) $-\frac{6}{65}$

90. If $\mu = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between maximum and minimum values of μ^2 is :

(a) $2(a^2+b^2)$ (b) $(a+b)^2$ (c) $2\sqrt{a^2+b^2}$ (d) $(a-b)^2$

91. If $P = (\tan(3^{n+1}\theta) - \tan\theta)$ and $Q = \sum_{r=0}^{n} \frac{\sin(3^r\theta)}{\cos(3^{r+1}\theta)}$, then

(a) $P = 2Q$ (b) $P = 3Q$ (c) $2P = Q$ (d) $3P = Q$

92. If $270° < \theta < 360°$, then find $\sqrt{2+\sqrt{2(1+\cos\theta)}}$

(a) $-2\sin\left(\frac{\theta}{4}\right)$ (b) $2\sin\left(\frac{\theta}{4}\right)$ (c) $\pm 2\sin\frac{\theta}{4}$ (d) $2\cos\frac{\theta}{4}$

93. If $y = (\sin x + \cos x) + (\sin 4x + \cos 4x)^2$, then :

(a) $y > 0 \ \forall\, x \in R$ (b) $y \geq 0 \ \forall\, x \in R$

(c) $y < 2+\sqrt{2} \ \forall\, x \in R$ (d) $y = 2+\sqrt{2}$ for some $x \in R$

94. If $\cos x + \cos y + \cos z = \sin x + \sin y + \sin z = 0$ then $\cos(x-y) =$

(a) 0 (b) $-\frac{1}{2}$ (c) 2 (d) 1

95. The exact value of $\operatorname{cosec} 10° + \operatorname{cosec} 50° - \operatorname{cosec} 70°$ is :

(a) 4 (b) 5 (c) 6 (d) 8

96. If $270° < \theta < 360°$, then find $\sqrt{2+\sqrt{2(1+\cos\theta)}}$:

(a) $-2\sin\left(\frac{\theta}{4}\right)$ (b) $2\sin\left(\frac{\theta}{4}\right)$ (c) $\pm 2\sin\frac{\theta}{4}$ (d) $2\cos\frac{\theta}{4}$

Answers

1.	(c)	**2.**	(d)	**3.**	(a)	**4.**	(c)	**5.**	(b)	**6.**	(c)	**7.**	(d)	**8.**	(d)	**9.**	(a)	**10.**	(c)
11.	(c)	**12.**	(b)	**13.**	(b)	**14.**	(d)	**15.**	(b)	**16.**	(d)	**17.**	(c)	**18.**	(a)	**19.**	(d)	**20.**	(a)
21.	(c)	**22.**	(b)	**23.**	(b)	**24.**	(d)	**25.**	(c)	**26.**	(b)	**27.**	(c)	**28.**	(b)	**29.**	(d)	**30.**	(a)
31.	(b)	**32.**	(c)	**33.**	(b)	**34.**	(c)	**35.**	(c)	**36.**	(a)	**37.**	(a)	**38.**	(a)	**39.**	(d)	**40.**	(d)
41.	(b)	**42.**	(b)	**43.**	(c)	**44.**	(d)	**45.**	(d)	**46.**	(b)	**47.**	(b)	**48.**	(c)	**49.**	(d)	**50.**	(a)
51.	(d)	**52.**	(b)	**53.**	(a)	**54.**	(a)	**55.**	(b)	**56.**	(b)	**57.**	(b)	**58.**	(d)	**59.**	(b)	**60.**	(a)
61.	(d)	**62.**	(a)	**63.**	(d)	**64.**	(a)	**65.**	(a)	**66.**	(b)	**67.**	(b)	**68.**	(c)	**69.**	(b)	**70.**	(b)
71.	(c)	**72.**	(a)	**73.**	(d)	**74.**	(a)	**75.**	(d)	**76.**	(b)	**77.**	(d)	**78.**	(a)	**79.**	(b)	**80.**	(a)
81.	(b)	**82.**	(b)	**83.**	(a)	**84.**	(a)	**85.**	(b)	**86.**	(a)	**87.**	(b)	**88.**	(a)	**89.**	(a)	**90.**	(d)
91.	(a)	**92.**	(b)	**93.**	(c)	**94.**	(b)	**95.**	(c)	**96.**	(b)								

Exercise-2 : One or More than One Answer is/are Correct

1. $\cot 12^\circ \cdot \cot 24^\circ \cdot \cot 28^\circ \cdot \cot 32^\circ \cdot \cot 48^\circ \cdot \cot 88^\circ = \ldots\ldots$
   (a) $\tan 45^\circ$
   (b) 2
   (c) $2\tan 15^\circ \cdot \tan 45^\circ \cdot \tan 75^\circ$
   (d) $\tan 15^\circ \cdot \tan 45^\circ \cdot \tan 75^\circ$

2. If the equation $\cot^4 x - 2\operatorname{cosec}^2 x + a^2 = 0$ has at least one solution then possible integral values of a can be :
   (a) -1
   (b) 0
   (c) 1
   (d) 2

3. Which of the following is/are true ?
   (a) $\tan 1 > \tan^{-1} 1$
   (b) $\sin 1 > \cos 1$
   (c) $\tan 1 < \sin 1$
   (d) $\cos(\cos 1) > \frac{1}{\sqrt{2}}$

4. Which of the following is/are +ve ?
   (a) $\log_{\sin 1} \tan 1$
   (b) $\log_{\cos 1}(1 + \tan 3)$
   (c) $\log_{\log_{10} 5}(\cos\theta + \sec\theta)$
   (d) $\log_{\tan 15^\circ}(2\sin 18^\circ)$

5. If $\sin\alpha + \cos\alpha = \frac{\sqrt{3}+1}{2}$, $0 < \alpha < 2\pi$, then possible values $\tan\frac{\alpha}{2}$ can take is/are :
   (a) $2-\sqrt{3}$
   (b) $\frac{1}{\sqrt{3}}$
   (c) 1
   (d) $\sqrt{3}$

6. If $3\sin\beta = \sin(2\alpha + \beta)$, then :
   (a) $(\cot\alpha + \cot(\alpha+\beta))(\cot\beta - 3\cot(2\alpha+\beta)) = 6$
   (b) $\sin\beta = \cos(\alpha+\beta)\sin\alpha$
   (c) $\tan(\alpha+\beta) = 2\tan\alpha$
   (d) $2\sin\beta = \sin(\alpha+\beta)\cos\alpha$

7. If $\sin(x + 20^\circ) = 2\sin x \cos 40^\circ$ where $x \in (0, 90^\circ)$, then which of the following hold good ?
   (a) $\sec\frac{x}{2} = \sqrt{6}-\sqrt{2}$
   (b) $\cot\frac{x}{2} = 2+\sqrt{3}$
   (c) $\tan 4x = \sqrt{3}$
   (d) $\operatorname{cosec} 4x = 2$

8. If $2(\cos(x-y) + \cos(y-z) + \cos(z-x)) = -3$, then :
   (a) $\cos x \cos y \cos z = 1$
   (b) $\cos x + \cos y + \cos z = 0$
   (c) $\sin x + \sin y + \sin z = 1$
   (d) $\cos 3x + \cos 3y + \cos 3z = 12\cos x \cos y \cos z$

9. If $0 < x < \frac{\pi}{2}$ and $\sin^n x + \cos^n x \geq 1$, then '$n$' may belong to interval :
   (a) $[1, 2)$
   (b) $[3, 4]$
   (c) $(-\infty, 2]$
   (d) $[-1, 1]$

10. If $x = \sin(\alpha-\beta)\cdot\sin(\gamma-\delta)$, $y = \sin(\beta-\gamma)\cdot\sin(\alpha-\delta)$, $z = \sin(\gamma-\alpha)\cdot\sin(\beta-\delta)$, then :
    (a) $x+y+z=0$
    (b) $x^3+y^3+z^3 = 3xyz$
    (c) $x+y-z=0$
    (d) $x^3+y^3-z^3 = 3xyz$

11. If $X = x\cos\theta - y\sin\theta, Y = x\sin\theta + y\cos\theta$ and $X^2 + 4XY + Y^2 = Ax^2 + By^2, 0 \le \theta \le \pi/2$, then : (where A and B are constants)

(a) $\theta = \frac{\pi}{6}$ (b) $\theta = \frac{\pi}{4}$ (c) $A = 3$ (d) $B = -1$

12. If $2a = 2\tan 10° + \tan 50°$; $2b = \tan 20° + \tan 50°$

$2c = 2\tan 10° + \tan 70°$; $2d = \tan 20° + \tan 70°$

Then which of the following is/are correct ?

(a) $a + d = b + c$ (b) $a + b = c$ (c) $a > b < c > d$ (d) $a < b < c < d$

13. Which of the following real numbers when simplified are neither terminating nor repeating decimal ?

(a) $\sin 75° \cdot \cos 75°$ (b) $\log_2 28$ (c) $\log_3 5 \cdot \log_5 6$ (d) $8^{-(\log_{27} 3)}$

14. If $\alpha = \sin x \cos^3 x$ and $\beta = \cos x \sin^3 x$, then :

(a) $\alpha - \beta > 0$; for all x in $\left(0, \frac{\pi}{4}\right)$ (b) $\alpha - \beta < 0$; for all x in $\left(0, \frac{\pi}{4}\right)$

(c) $\alpha + \beta > 0$; for all x in $\left(0, \frac{\pi}{2}\right)$ (d) $\alpha + \beta < 0$; for all x in $\left(0, \frac{\pi}{2}\right)$

15. If $\frac{\pi}{2} < \theta < \pi$, then possible answers of $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}}$ is/are :

(a) $2\cos\theta$ (b) $2\sin\theta$ (c) $-2\sin\theta$ (d) $-2\cos\theta$

16. If $\cot^3\alpha + \cot^2\alpha + \cot\alpha = 1$ then which of the following is/are correct:

(a) $\cos 2\alpha \tan\alpha = 1$ (b) $\cos 2\alpha \cdot \tan\alpha = -1$

(c) $\cos 2\alpha - \tan 2\alpha = -1$ (d) $\cos 2\alpha - \tan 2\alpha = 1$

17. All values of $x \in \left(0, \frac{\pi}{2}\right)$ such that $\frac{\sqrt{3}-1}{\sin x} + \frac{\sqrt{3}+1}{\cos x} = 4\sqrt{2}$ are :

(a) $\frac{\pi}{15}$ (b) $\frac{\pi}{12}$ (c) $\frac{11\pi}{36}$ (d) $\frac{3\pi}{10}$

18. If $\alpha > \frac{1}{\sin^6 x + \cos^6 x}\ \forall x \in R$, then α can be :

(a) 3 (b) 4 (c) 5 (d) 6

19. If $x \in \left(0, \frac{\pi}{2}\right)$ and $\sin x = \frac{3}{\sqrt{10}}$;

Let $k = \log_{10}\sin x + \log_{10}\cos x + 2\log_{10}\cot x + \log_{10}\tan x$ then the value of k satisfies

(a) $k = 0$ (b) $k + 1 = 0$ (c) $k - 1 = 0$ (d) $k^2 - 1 = 0$

20. If A, B, C are angles of a triangle ABC and $\tan A \tan C = 3$; $\tan B \tan C = 6$ then which is(are) correct :

(a) $A = \frac{\pi}{4}$ (b) $\tan A \tan B = 2$ (c) $\frac{\tan A}{\tan C} = 3$ (d) $\tan B = 2\tan A$

21. The value of $\frac{\sin x - \cos x}{\sin^3 x}$ is equal to :

(a) $\operatorname{cosec}^2 x(1-\cot x)$ (b) $1-\cot x+\cot^2 x-\cot^3 x$

(c) $\operatorname{cosec}^2 x-\cot x-\cot^3 x$ (d) $\frac{1-\cot x}{\sin^2 x}$

22. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{2\pi}{3}\right)+\sin^2\left(x+\frac{4\pi}{3}\right)$ then :

(a) $f\left(\frac{\pi}{15}\right)=\frac{3}{2}$ (b) $f\left(\frac{15}{\pi}\right)=\frac{2}{3}$ (c) $f\left(\frac{\pi}{10}\right)=\frac{3}{2}$ (d) $f\left(\frac{10}{\pi}\right)=\frac{2}{3}$

23. The range of $y=\frac{\sin 4x-\sin 2x}{\sin 4x+\sin 2x}$ satisfies

(a) $y\in\left(-\infty,\frac{1}{3}\right)$ (b) $y\in\left(\frac{1}{3},1\right)$ (c) $y\in(1,3)$ (d) $y\in(3,\infty)$

24. If $\sqrt{2}\cos A=\cos B+\cos^3 B$ and $\sqrt{2}\sin A=\sin B-\sin^3 B$, then the possible value of $\sin(A-B)$ is/are

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $-\frac{1}{2}$ (d) $-\frac{1}{3}$

25. If $\alpha>\frac{1}{\sin^6 x+\cos^6 x}\ \forall\, x\in R$, then α can be

(a) 3 (b) 4 (c) 5 (d) 6

26. If $\cot^3\alpha+\cot^2\alpha+\cot\alpha=1$ then which of the following is/are correct

(a) $\cos 2\alpha\tan\alpha=1$ (b) $\cos 2\alpha\cdot\tan\alpha=-1$

(c) $\cos 2\alpha-\tan 2\alpha=-1$ (d) $\cos 2\alpha-\tan 2\alpha=1$

Answers

1.	(a, d)	**2.**	(a, b, c)	**3.**	(a, b, d)	**4.**	(b, d)	**5.**	(a, b)	**6.**	(a, b, c, d)
7.	(a, b)	**8.**	(b, d)	**9.**	(a, c, d)	**10.**	(a, b)	**11.**	(b, c, d)	**12.**	(a, b, d)
13.	(b, c)	**14.**	(a, c)	**15.**	(b, d)	**16.**	(b, d)	**17.**	(b, c)	**18.**	(b, d)
19.	(b, d)	**20.**	(a, b, d)	**21.**	(a, b, c, d)	**22.**	(a, c)	**23.**	(a, d)	**24.**	(b, d)
25.	(c, d)	**26.**	(b, d)								

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Let $l = \sin\theta, m = \cos\theta$ and $n = \tan\theta$.

1. If $\theta = 5$ radian, then :

(a) $l > m$ (b) $l < m$ (c) $l = m$ (d) none of these

2. If $\theta = -1042°$, then :

(a) $n > 1$ (b) $n < 1$ (c) $n = 1$ (d) nothing can be said

3. If $\theta = 7$ radian, then :

(a) $l + m > 0$ (b) $l + m < 0$ (c) $l + m = 0$ (d) nothing can be said

Paragraph for Question Nos. 4 to 6

Let a, b, c are respectively the sines and p, q, r are respectively the cosines of $\alpha, \alpha + \frac{2\pi}{3}$ and $\alpha + \frac{4\pi}{3}$, then :

4. The value of $(a + b + c)$ is :

(a) 0 (b) $\frac{3}{4}$ (c) 1 (d) none of these

5. The value of $(ab + bc + ca)$ is :

(a) 0 (b) $-\frac{3}{4}$ (c) $-\frac{1}{2}$ (d) -1

6. The value of $(qc - rb)$ is :

(a) 0 (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) depends on α

Paragraph for Question Nos. 7 to 8

Consider a right angle triangle ABC right angle at B such that $AC = \sqrt{8 + 4\sqrt{3}}$ and $AB = 1$. A line through vertex A meet BC at D such that $AB = BD$. An arc DE of radius AD is drawn from vertex A to meet AC at E and another arc DF of radius CD is drawn from vertex C to meet AC at F. On the basis of above information, answer the following questions.

7. $\sqrt{\tan A + \cot C}$ is equal to :

(a) $\sqrt{3}$ (b) 1 (c) $2 + \sqrt{3}$ (d) $\sqrt{3} + 1$

8. $\log_{AE}\left(\frac{AC}{CD}\right)$ is equal to :

(a) $\sqrt{2}$ (b) 1 (c) 0 (d) -1

Paragraph for Question Nos. 9 to 10

Consider a triangle ABC such that $\cot A + \cot B + \cot C = \cot\theta$. Now answer the following :

9. The possible value of θ is :

(a) 60° (b) 25° (c) 35° (d) 45°

10. $\sin(A-\theta)\sin(B-\theta)\sin(C-\theta) =$:

(a) $\tan^3\theta$ (b) $\cot^3\theta$ (c) $\sin^3\theta$ (d) $\cos^3\theta$

Paragraph for Question Nos. 11 to 12

Consider the function $f(x) = \dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}$ then

11. If $x \in (\pi, 2\pi)$ then $f(x)$ is :

(a) $\cot\left(\frac{\pi}{2}+\frac{x}{2}\right)$ (b) $\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ (c) $\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)$ (d) $\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

12. If the value of $f\left(\frac{\pi}{3}\right) = a + b\sqrt{c}$ where $a, b, c \in N$ then the value of $a+b+c$ is :

(a) 4 (b) 5 (c) 6 (d) 7

Answers

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(b)	**6.**	(c)	**7.**	(d)	**8.**	(b)	**9.**	(b)	**10.**	(c)
11.	(d)	**12.**	(c)																

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If $(1+\tan 5°)(1+\tan 10°)......(1+\tan 45°)=2^{k+1}$ then 'k' equals	**(P)**	0
(B)	Sum of positive integral values of 'a' for which $a^2-6\sin x-5a\le 0\ \forall\ x\in R$ is	**(Q)**	2
(C)	The minimum value of $\dfrac{\left(a+\frac{1}{a}\right)^4-\left(a^4+\frac{1}{a^4}\right)-2}{\left(a+\frac{1}{a}\right)^2+a^2+\frac{1}{a^2}}$ is	**(R)**	5
(D)	Number of real roots of the equation $\sum_{k=1}^{3}(x-k)^2=0$ is	**(S)**	4
		(T)	5

2.

	Column-I		Column-II
(A)	Maximum value of $y=\dfrac{1-\tan^2(\pi/4-x)}{1+\tan^2(\pi/4-x)}$	**(P)**	1
(B)	Minimum value of $\log_3\left(\dfrac{5\sin x-12\cos x+26}{13}\right)$	**(Q)**	0
(C)	Minimum value of $y=-2\sin^2 x+\cos x+3$	**(R)**	$\frac{7}{8}$
(D)	Maximum value of $y=4\sin^2\theta+4\sin\theta\cos\theta+\cos^2\theta$	**(S)**	5
		(T)	6

3.

	Column-I		Column-II
(A)	The value of $\dfrac{\cos 68°}{\sin 56°\sin 34°\tan 22°}$ equals to	**(P)**	16
(B)	The value of $(\cos 65°+\sqrt{3}\sin 5°+\cos 5°)^2=\lambda\cos^2 25°$; then value of λ be	**(Q)**	3

(C)	If $\cos A = \frac{3}{4}$; then the value of $\frac{32}{11}\sin\frac{A}{2}\sin\frac{5A}{2}$ is equal to	**(R)**	4
(D)	If $7\log_a\frac{16}{15} + 5\log_a\frac{25}{24} + 3\log_a\frac{81}{80} = 8$ then the value of a^{16} equals to	**(S)**	2
		(T)	1

4.

	Column-I		Column-II
(A)	If $\sin x + \cos x = \frac{1}{5}$; then $\lvert 12\tan x\rvert$ is equal to	**(P)**	2
(B)	Number of values of θ lying in $(-2\pi, \pi)$ and satisfying $\cot\frac{\theta}{2} = (1 + \cot\theta)$ is	**(Q)**	6
(C)	If $2 - \sin^4 x + 8\sin^2 x = \alpha$ has solution, then α can be	**(R)**	9
(D)	Number of integral values of x satisfying $\log_4(2x^2 + 5x + 27) - \log_2(2x - 1) \geq 0$	**(S)**	14
		(T)	16

5. Match the function given in **Column-I** to the number of integers in its range given in **Column-II.**

	Column-I		Column-II
(A)	$f(x) = 2\cos^2 x + \sin x - 8$	**(P)**	5
(B)	$f(x) = \sin^2 x + 3\cos^2 x + 5$	**(Q)**	4
(C)	$f(x) = 4\sin x\cos x - \sin^2 x + 3\cos^2 x$	**(R)**	3
(D)	$f(x) = \cos(\sin x) + \sin(\sin x)$	**(S)**	2

Answers

1. $A \to S$; $B \to R$; $C \to Q$; $D \to P$

2. $A \to P$; $B \to Q$; $C \to R$; $D \to S$

3. $A \to S$; $B \to Q$; $C \to T$; $D \to R$

4. $A \to R, T$; $B \to P$; $C \to P, Q, R$; $D \to Q$

5. $A \to Q$; $B \to R$; $C \to P$; $D \to S$

Exercise-5 : Subjective Type Problems

1. Let $P = \dfrac{\sin 80^\circ \sin 65^\circ \sin 35^\circ}{\sin 20^\circ + \sin 50^\circ + \sin 110^\circ}$, then the value of $24P$ is :
2. The value of expression $(1 - \cot 23^\circ)(1 - \cot 22^\circ)$ is equal to :
3. If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $4x^2 - 7x + 1 = 0$ then evaluate $4\sin^2(A+B) - 7\sin(A+B)\cdot\cos(A+B) + \cos^2(A+B)$.
4. $A_1A_2A_3 \ldots\ldots A_{18}$ is a regular 18 sided polygon. B is an external point such that A_1A_2B is an equilateral triangle. If $A_{18}A_1$ and A_1B are adjacent sides of a regular n sided polygon, then $n =$
5. If $10\sin^4\alpha + 15\cos^4\alpha = 6$ and the value of $9\operatorname{cosec}^4\alpha + \beta\sec^4\alpha$ is S, then find the value of $\dfrac{S}{25}$.
6. The value of $\left(1+\tan\dfrac{3\pi}{8}\tan\dfrac{\pi}{8}\right)+\left(1+\tan\dfrac{5\pi}{8}\tan\dfrac{3\pi}{8}\right)+\left(1+\tan\dfrac{7\pi}{8}\tan\dfrac{5\pi}{8}\right)+\left(1+\tan\dfrac{9\pi}{8}\tan\dfrac{7\pi}{8}\right)$
7. If $\alpha = \dfrac{\pi}{7}$ then find the value of $\left(\dfrac{1}{\cos\alpha} + \dfrac{2\cos\alpha}{\cos 2\alpha}\right)$.
8. Given that for $a, b, c, d \in R$, if $a\sec(200^\circ) - c\tan(200^\circ) = d$ and $b\sec(200^\circ) + d\tan(200^\circ) = c$, then find the value of $\left(\dfrac{a^2+b^2+c^2+d^2}{bd-ac}\right)\sin 20^\circ$.
9. The expression $2\cos\dfrac{\pi}{17}\cdot\cos\dfrac{9\pi}{17} + \cos\dfrac{7\pi}{17} + \cos\dfrac{9\pi}{17}$ simplifies to an integer P. Find the value of P.
10. If the expression $\dfrac{\sin\theta\sin 2\theta + \sin 3\theta\sin 6\theta + \sin 4\theta\sin 13\theta}{\sin\theta\cos 2\theta + \sin 3\theta\cos 6\theta + \sin 4\theta\cos 13\theta} = \tan k\theta$, where $k \in N$. Find the value of k.
11. Let $a = \sin 10^\circ$, $b = \sin 50^\circ$, $c = \sin 70^\circ$, then $8abc\left(\dfrac{a+b}{c}\right)\left(\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{1}{c}\right)$ is equal to
12. If $\sin^3\theta + \sin^3\left(\theta + \dfrac{2\pi}{3}\right) + \sin^3\left(\theta + \dfrac{4\pi}{3}\right) = a\sin b\theta$. Find the value of $\left|\dfrac{b}{a}\right|$.
13. If $\displaystyle\sum_{r=1}^{n}\left(\dfrac{\tan 2^{r-1}}{\cos 2^r}\right) = \tan p^n - \tan q$, then find the value of $(p+q)$.
14. If $x = \sec\theta - \tan\theta$ and $y = \operatorname{cosec}\theta + \cot\theta$, then $y - x - xy =$
15. If $\cos 18^\circ - \sin 18^\circ = \sqrt{n}\sin 27^\circ$, then $n =$
16. The value of $3(\sin 1 - \cos 1)^4 + 6(\sin 1 + \cos 1)^2 + 4(\sin^6 1 + \cos^6 1)$ is equal to
17. If $x = \alpha$ satisfy the equation $3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28$, then $(\sin 2\alpha - \cos 2\alpha)^2 + 8\sin 4\alpha$ is equal to :
18. The least value of the expression $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 \ \forall\ \theta \in R$ is
19. If $\tan 20^\circ + \tan 40^\circ + \tan 80^\circ - \tan 60^\circ = \lambda\sin 40^\circ$, then λ is equal to

20. If $K°$ lies between $360°$ and $540°$ and $K°$ satisfies the equation
$1+\cos 10x\cos 6x = 2\cos^2 8x+\sin^2 8x$, then $\frac{K}{10}=$

21. If $\cos 20°+2\sin^2 55° = 1+\sqrt{2}\sin K°, K\in(0,90)$, then $K=$

22. The exact value of $\operatorname{cosec}10°+\operatorname{cosec}50°-\operatorname{cosec}70°$ is :

23. Let α be the smallest integral value of x, $x>0$ such that $\tan 19x=\frac{\cos 96°+\sin 96°}{\cos 96°-\sin 96°}$. The last digit of α is :

24. Find the value of the expression $\frac{\sin 20°(4\cos 20°+1)}{\cos 20°\cos 30°}$

25. If the value of $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}+\cos\frac{7\pi}{7}=-\frac{l}{2}$. Find the value of l.

26. If $\cos A=\frac{3}{4}$ and $k\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)=\frac{11}{8}$. Find k.

27. Find the least value of the expression $3\sin^2 x+4\cos^2 x$.

28. If $\tan\alpha$ and $\tan\beta$ are the roots of equation $x^2-12x-3=0$, then the value of $\sin^2(\alpha+\beta)+2\sin(\alpha+\beta)\cos(\alpha+\beta)+5\cos^2(\alpha+\beta)$ is :

29. The value of $\frac{\cos 24°}{2\tan 33°\sin^2 57°}+\frac{\sin 162°}{\sin 18°-\cos 18°\tan 9°}+\cos 162°$ is equal to :

30. Find the value of $\tan\theta(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)$, when $\theta=\frac{\pi}{32}$.

31. If λ be the minimum value of $y=(\sin x+\operatorname{cosex} x)^2+(\cos x+\sec x)^2+(\tan x+\cot x)^2$ where $x\in R$. Find $\lambda-6$.

Answers

1.	6	**2.**	2	**3.**	1	**4.**	9	**5.**	3	**6.**	0	**7.**	4
8.	2	**9.**	0	**10.**	9	**11.**	6	**12.**	4	**13.**	3	**14.**	1
15.	2	**16.**	13	**17.**	1	**18.**	9	**19.**	8	**20.**	45	**21.**	65
22.	6	**23.**	9	**24.**	2	**25.**	3	**26.**	4	**27.**	3	**28.**	2
29.	2	**30.**	1	**31.**	7								

□□□

Chapter 23 – Trigonometric Equations

23 TRIGONOMETRIC EQUATIONS

Exercise-1 : Single Choice Problems

1. Let x and y be 2 real numbers which satisfy the equations
$(\tan^2 x - \sec^2 y) = \frac{5a}{6} - 3$ and $(-\sec^2 x + \tan^2 y) = a^2$, then the product of all possible value's of a can be equal to :

(a) 0 (b) $\frac{-2}{3}$ (c) -1 (d) $\frac{-3}{2}$

2. The general solution of the equation $\tan^2(x+y) + \cot^2(x+y) = 1 - 2x - x^2$ lie on the line is :

(a) $x = -1$ (b) $x = -2$ (c) $y = -1$ (d) $y = -2$

3. General solution of the equation
$\sin x + \cos x = \min_{a \in R} \{1, a^2 - 4a + 6\}$ is :

(a) $\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$ (b) $2n\pi + (-1)^n \frac{\pi}{4}$

(c) $n\pi + (-1)^{n+1} \frac{\pi}{4}$ (d) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

(where $n \in I$, I represent set of integers)

4. The number of solutions of the equation
$\left(2\sin\left(\frac{\sin x}{2}\right)\right)\left(\cos\left(\frac{\sin x}{2}\right)\right)\left(\sin\left(2\tan\frac{x}{2}\cos^2\frac{x}{2}\right) - 3\right) + 2 = 0$ in $[0, 2\pi]$ is :

(a) 0 (b) 1 (c) 2 (d) 4

5. Number of solution of $\tan(2x) = \tan(6x)$ in $(0, 3\pi)$ is :

(a) 4 (b) 5 (c) 3 (d) None of these

6. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is :

(a) 0 (b) 2 (c) 6 (d) 8

7. The number of different values of θ satisfying the equation $\cos\theta + \cos 2\theta = -1$, and at the same time satisfying the condition $0 < \theta < 360°$ is :

(a) 1 (b) 2 (c) 3 (d) 4

8. The total number of solutions of the equation $\max(\sin x, \cos x) = \frac{1}{2}$ for $x \in (-2\pi, 5\pi)$ is equal to:

(a) 3 (b) 6 (c) 7 (d) 8

9. The general value of x satisfying the equation
$2\cot^2 x + 2\sqrt{3}\cot x + 4\operatorname{cosec} x + 8 = 0$ is : (where $n \in I$)

(a) $n\pi - \frac{\pi}{6}$ (b) $n\pi + \frac{\pi}{6}$ (c) $2n\pi - \frac{\pi}{6}$ (d) $2n\pi + \frac{\pi}{6}$

10. The general solution of the equation $\sin^2 x + \cos^2 3x = 1$ is equal to :

(a) $x = \frac{n\pi}{2}$ (b) $x = n\pi + \frac{\pi}{4}$ (c) $x = \frac{n\pi}{4}$ (d) $x = n\pi + \frac{\pi}{2}$

(where $n \in I$)

11. Values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in A.P. whose common difference is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

12. Number of solutions of $\sum_{r=1}^{5} \cos rx = 5$ in the interval $[0, 4\pi]$ is :

(a) 0 (b) 2 (c) 3 (d) 7

13. General solution of $4\sin^2 x + \tan^2 x + \operatorname{cosec}^2 x + \cot^2 x - 6 = 0$ is :

(a) $n\pi \pm \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{4}$ (c) $n\pi + \frac{\pi}{3}$ (d) $n\pi - \frac{\pi}{6}$

[where $n \in I$]

14. Smallest positive x satisfying the equation $\cos^3 3x + \cos^3 5x = 8\cos^3 4x \cdot \cos^3 x$ is :

(a) 15° (b) 18° (c) 22.5° (d) 30°

15. The general solution of the equation $\sin^{100} x - \cos^{100} x = 1$ is (where $n \in I$):

(a) $2n\pi + \frac{\pi}{2}$ (b) $n\pi + \frac{\pi}{2}$ (c) $2n\pi - \frac{\pi}{2}$ (d) $n\pi$

16. Number of solution(s) of equation $\sin\theta = \sec^2 4\theta$ in $[0, \pi]$ is/are:

(a) 0 (b) 1 (c) 2 (d) 3

17. The number of solutions of the equation $4\sin^2 x + \tan^2 x + \cot^2 x + \operatorname{cosec}^2 x = 6$ in $[0, 2\pi]$

(a) 1 (b) 2 (c) 3 (d) 4

18. The number of solutions of the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$ which lie between 0 and 2π is :

(a) 0 (b) 2 (c) 4 (d) 8

19. The smallest positive value of p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has solution in $0 \le x \le 2\pi$ is :

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{3\pi}{2\sqrt{2}}$

20. The total number of ordered pairs (x, y) satisfying $|x|+|y| = 2$ and $\sin\left(\frac{\pi x^2}{3}\right) = 1$ is :

(a) 2 (b) 4 (c) 6 (d) 8

21. The complete set of values of x, $x \in \left(-\frac{\pi}{2}, \pi\right)$ satisfying the inequality $\cos 2x > |\sin x|$ is :

(a) $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(c) $\left(-\frac{\pi}{2}, -\frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$ (d) $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$

22. The total number of solution of the equation $\sin^4 x + \cos^4 x = \sin x \cos x$ in $[0, 2\pi]$ is :

(a) 2 (b) 4 (c) 6 (d) 8

23. Number of solution of the equation $\sin\frac{5x}{2} - \sin\frac{x}{2} = 2$ in the interval $[0, 2\pi]$, is :

(a) 1 (b) 2 (c) 0 (d) Infinite

24. In the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. The equation $\log_{\sin\theta} \cos 2\theta = 2$ has

(a) No solution (b) One solution (c) Two solution (d) Infinite solution

25. If α and β are 2 distinct roots of equation $a\cos\theta + b\sin\theta = C$ then $\cos(\alpha + \beta) =$

(a) $\frac{2ab}{a^2 + b^2}$ (b) $\frac{2ab}{a^2 - b^2}$ (c) $\frac{a^2 + b^2}{a^2 - b^2}$ (d) $\frac{a^2 - b^2}{a^2 + b^2}$

Answers

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(a)	**5.**	(b)	**6.**	(c)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(c)
11.	(a)	**12.**	(c)	**13.**	(a)	**14.**	(b)	**15.**	( )	**16.**	(b)	**17.**	(d)	**18.**	(a)	**19.**	(c)	**20.**	(b)
21.	(d)	**22.**	(a	**23.**	(c)	**24.**	(b)	**25.**	(d)										

Exercise-2 : One or More than One Answer is/are Correct

1. If $2\cos\theta + 2\sqrt{2} = 3\sec\theta$ where $\theta \in (0, 2\pi)$ then which of the following can be correct ?
   (a) $\cos\theta = \frac{1}{\sqrt{2}}$ (b) $\tan\theta = 1$ (c) $\sin\theta = -\frac{1}{\sqrt{2}}$ (d) $\cot\theta = -1$

2. In a triangle ABC if $\tan C < 0$ then :
   (a) $\tan A \tan B < 1$
   (b) $\tan A \tan B > 1$
   (c) $\tan A + \tan B + \tan C < 0$
   (d) $\tan A + \tan B + \tan C > 0$

3. The inequality $4\sin 3x + 5 \geq 4\cos 2x + 5\sin x$ is true for $x \in$
   (a) $\left[-\pi, \frac{3\pi}{2}\right]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left[\frac{5\pi}{8}, \frac{13\pi}{8}\right]$ (d) $\left[\frac{23\pi}{14}, \frac{41\pi}{14}\right]$

4. The least difference between the roots of the equation $4\cos x(2 - 3\sin^2 x) + \cos 2x + 1 = 0$ $\forall\, x \in R$ is :
   (a) equal to $\frac{\pi}{2}$ (b) $> \frac{\pi}{10}$ (c) $< \frac{\pi}{2}$ (d) $< \frac{\pi}{3}$

5. The equation $\cos x \cos 6x = -1$:
   (a) has 50 solutions in $[0, 100\pi]$
   (b) has 3 solutions in $[0, 3\pi]$
   (c) has even number of solutions in $(3\pi, 13\pi)$
   (d) has one solution in $\left[\frac{\pi}{2}, \pi\right]$

6. Identify the correct options :
   (a) $\frac{\sin 3\alpha}{\cos 2\alpha} > 0$ for $\alpha \in \left(\frac{3\pi}{8}, \frac{23\pi}{48}\right)$
   (b) $\frac{\sin 3\alpha}{\cos 2\alpha} < 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$
   (c) $\frac{\sin 2\alpha}{\cos\alpha} < 0$ for $\alpha \in \left(-\frac{\pi}{2}, 0\right)$
   (d) $\frac{\sin 2\alpha}{\cos\alpha} > 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

7. The equation $\sin^4 x + \cos^4 x + \sin 2x + k = 0$ must have real solutions if :
   (a) $k = 0$
   (b) $|k| \leq \frac{1}{2}$
   (c) $-\frac{3}{2} \leq k \leq \frac{1}{2}$
   (d) $-\frac{1}{2} \leq k \leq \frac{3}{2}$

8. Let $f(\theta) = \left(\cos\theta - \cos\frac{\pi}{8}\right)\left(\cos\theta - \cos\frac{3\pi}{8}\right)\left(\cos\theta - \cos\frac{5\pi}{8}\right)\left(\cos\theta - \cos\frac{7\pi}{8}\right)$ then :
   (a) maximum value of $f(\theta)\ \forall\, \theta \in R$ is $\frac{1}{4}$
   (b) maximum value of $f(\theta)\ \forall\, \theta \in R$ is $\frac{1}{8}$
   (c) $f(0) = \frac{1}{8}$
   (d) Number of principle solutions of $f(\theta) = 0$ is 8

9. If $\dfrac{\sin^2 2x + 4\sin^4 x - 4\sin^2 x \cdot \cos^2 x}{4 - \sin^2 2x - 4\sin^2 x} = \dfrac{1}{9}$ and $0 < x < \pi$. Then the value of x is :

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{6}$ (c) $\dfrac{2\pi}{3}$ (d) $\dfrac{5\pi}{6}$

10. The possible value(s) of 'θ' satisfying the equation

$$\sin^2 \theta \tan \theta + \cos^2 \theta \cot \theta - \sin 2\theta = 1 + \tan \theta + \cot \theta$$

where $\theta \in [0, \pi]$ is/are :

(a) $\dfrac{\pi}{4}$ (b) π (c) $\dfrac{7\pi}{12}$ (d) None of these

11. If $\sin \theta + \sqrt{3} \cos \theta = 6x - x^2 - 11$, $0 \le \theta \le 4\pi$, $x \in R$ holds for

(a) no values of x and θ

(b) one value of x and two values of θ

(c) two values of x and two values of θ

(d) two pairs of values of (x, θ)

Answers

1.	(a, b, c, d)	**2.**	(a, c)	**3.**	(a, b, c, d)	**4.**	(b, c, d)	**5.**	(a, c, d)	**6.**	(a, b, c, d)
7.	(a, b, c)	**8.**	(b, c, d)	**9.**	(b, d)	**10.**	(c)	**11.**	(b, d)		

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Consider f, g and h be three real valued function defined on R.
Let $f(x) = \sin 3x + \cos x$, $g(x) = \cos 3x + \sin x$ and $h(x) = f^2(x) + g^2(x)$

1. The length of a longest interval in which the function $y = h(x)$ is increasing, is :

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

2. General solution of the equation $h(x) = 4$, is :

(a) $(4n+1)\frac{\pi}{8}$ (b) $(8n+1)\frac{\pi}{8}$ (c) $(2n+1)\frac{\pi}{4}$ (d) $(7n+1)\frac{\pi}{4}$

[where $n \in I$]

3. Number of point(s) where the graphs of the two function, $y = f(x)$ and $y = g(x)$ intersects in $[0, \pi]$, is :

(a) 2 (b) 3 (c) 4 (d) 5

Answers

1.	(b)	**2.**	(a)	**3.**	(c)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	If $\sin x + \cos x = \frac{1}{5}$; then $\lvert 12\tan x\rvert$ is equal to	**(P)**	2
(B)	Number of values of θ lying in $(-2\pi, \pi)$ and satisfying $\cot\frac{\theta}{2} = (1 + \cot\theta)$ is	**(Q)**	6
(C)	If $2 - \sin^4 x + 8\sin^2 x = \alpha$ has solution, then α can be	**(R)**	9
(D)	Number of integral values of x satisfying $\log_4(2x^2 + 5x + 27) - \log_2(2x - 1) \ge 0$	**(S)**	14
		(T)	16

2.

	Column-I		Column-II
(A)	If $x, y \in [0, 2\pi]$, then total number of ordered pair (x, y) satisfying $\sin x \cos y = 1$ is	**(P)**	4
(B)	If $f(x) = \sin x - \cos x - kx + b$ decreases for all real values of x, then $2\sqrt{2}\,k$ may be	**(Q)**	0
(C)	The number of solution of the equation $\sin^{-1}(\lvert x^2 - 1\rvert) + \cos^{-1}(\lvert 2x^2 - 5\rvert) = \frac{\pi}{2}$ is	**(R)**	2
(D)	The number of ordered pair (x, y) satisfying the equation $\sin x + \sin y = \sin(x + y)$ and $\lvert x\rvert + \lvert y\rvert = 1$ is	**(S)**	3
		(T)	6

3.

	Column-I		Column-II
(A)	Minimum value of $y = 4\sec^2 x + \cos^2 x$ for permissible real values of x is equal to	**(P)**	2
(B)	If m, n are positive integers and $m + n\sqrt{2} = \sqrt{41 + 24\sqrt{2}}$ then $(m + n)$ is equal to :	**(Q)**	7

(C)	Number of solutions of the equation : $\log_{\left(\frac{9x-x^2-14}{7}\right)}(\sin 3x - \sin x) = \log_{\left(\frac{9x-x^2-14}{7}\right)} \cos 2x$ is equal to :	(R)	4
(D)	Consider an arithmetic sequence of positive integers. If the sum of the first ten terms is equal to the 58th term, then the least possible value of the first term is equal to :	(S)	5
		(T)	3

Answers

1. $A \to R, T$; $B \to P$; $C \to P, Q, R$; $D \to Q$
2. $A \to S$; $B \to P, T$; $C \to R$; $D \to T$
3. $A \to S$; $B \to Q$; $C \to P$; $D \to R$

Exercise-5 : Subjective Type Problems

1. Find the number of solutins of the equations
$(\sin x-1)^3+(\cos x-1)^3+(\sin x)^3=(2\sin x+\cos x-2)^3$ in $[0, 2\pi]$.

2. If $x+\sin y=2014$ and $x+2014\cos y=2013$, $0\le y\le\frac{\pi}{2}$, then find the value of $[x+y]-2005$ (where $[\cdot]$ denotes greatest integer function)

3. The complete set of values of x satisfying $\frac{2\sin 6x}{\sin x-1}<0$ and $\sec^2 x-2\sqrt{2}\tan x\le 0$ in $\left(0,\frac{\pi}{2}\right)$ is $[a, b)\cup(c, d]$, then find the value of $\left(\frac{cd}{ab}\right)$.

4. The range of value's of k for which the equation $2\cos^4 x-\sin^4 x+k=0$ has atleast one solution is $[\lambda, \mu]$. Find the value of $(9\mu+\lambda)$.

5. The number of points in interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, where the graphs of the curves $y=\cos x$ and $y=\sin 3x$, $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ intersects is

6. The number of solutions of the system of equations :
$$2\sin^2 x+\sin^2 2x=2$$
$$\sin 2x+\cos 2x=\tan x$$
in $[0, 4\pi]$ satisfying $2\cos^2 x+\sin x\le 2$ is :

7. If the sum of all the solutions of the equation $3\cot^2\theta+10\cot\theta+3=0$ in $[0, 2\pi]$ is $k\pi$ where $k\in I$, then find the value of k.

8. If the sum of all values of θ, $0\le\theta\le 2\pi$ satisfying the equation
$(8\cos 4\theta-3)(\cot\theta+\tan\theta-2)(\cot\theta+\tan\theta+2)=12$ is $k\pi$, then k is equal to :

9. Find the number of solutions of the equation $2\sin^2 x+\sin^2 2x=2$; $\sin 2x+\cos 2x=\tan x$ in $[0, 4\pi]$ satisfying the condition $2\cos^2 x+\sin x\le 2$.

Answers

1.	5	**2.**	9	**3.**	6	**4.**	7	**5.**	3	**6.**	8	**7.**	5
8.	8	**9.**	8										

□□□

Chapter 24 – Solution of Triangles

24 SOLUTION OF TRIANGLES

Exercise-1 : Single Choice Problems

1. In a ΔABC if $9(a^2+b^2)=17c^2$ then the value of the expression $\dfrac{\cot A+\cot B}{\cot C}$ is :

(a) $\dfrac{13}{4}$ (b) $\dfrac{7}{4}$ (c) $\dfrac{5}{4}$ (d) $\dfrac{9}{4}$

2. Let H be the orthocenter of triangle ABC, then angle subtended by side BC at the centre of incircle of ΔCHB is :

(a) $\dfrac{A}{2}+\dfrac{\pi}{2}$ (b) $\dfrac{B+C}{2}+\dfrac{\pi}{2}$ (c) $\dfrac{B-C}{2}+\dfrac{\pi}{2}$ (d) $\dfrac{B+C}{2}+\dfrac{\pi}{4}$

3. Circum radius of a ΔABC is 3 units; let O be the circum centre and H be the orthocentre then the value of $\dfrac{1}{64}(AH^2+BC^2)(BH^2+AC^2)(CH^2+AB^2)$ equals :

(a) 3^4 (b) 9^3 (c) 27^6 (d) 81^4

4. The angles A, B and C of a triangle ABC are in arithmetic progression. If $2b^2=3c^2$ then the angle A is :

(a) $15°$ (b) $60°$ (c) $75°$ (d) $90°$

5. In a triangle ABC, if $\tan\dfrac{A}{2}\tan\dfrac{C}{2}=\dfrac{1}{3}$ and $ac=4$, then the least value of b is :

(notation have their usual meaning)

(a) 1 (b) 2 (c) 4 (d) 6

6. In a triangle ABC the expression $a\cos B\cos C+b\cos C\cos A+c\cos A\cos B$ equals to :

(a) $\dfrac{rs}{R}$ (b) $\dfrac{r}{sR}$ (c) $\dfrac{R}{rs}$ (d) $\dfrac{Rs}{r}$

7. The set of real numbers a such that a^2+2a, $2a+3$, a^2+3a+8 are the sides of a triangle, is :

(a) $(0,\infty)$ (b) $(5,8)$ (c) $\left(-\dfrac{11}{3},\infty\right)$ (d) $(5,\infty)$

8. In a ΔABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$ let D divide BC internally in the ratio 1 : 3, then $\frac{\sin(\angle BAD)}{\sin(\angle CAD)}$ is equal to :

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{2}}{3}$

9. Let AD, BE, CF be the lengths of internal bisectors of angles A, B, C respectively of triangle ABC. Then the harmonic mean of $AD \sec\frac{A}{2}, BE \sec\frac{B}{2}, CF \sec\frac{C}{2}$ is equal to :

(a) Harmonic mean of sides of ΔABC
(b) Geometric mean of sides of ΔABC
(c) Arithmetic mean of sides of ΔABC
(d) Sum of reciprocals of the sides of ΔABC

10. In triangle ABC, if $2b = a + c$ and $A - C = 90°$, then $\sin B$ equals :
[**Note :** All symbols used have usual meaning in triangle ABC.]

(a) $\frac{\sqrt{7}}{5}$ (b) $\frac{\sqrt{5}}{8}$ (c) $\frac{\sqrt{7}}{4}$ (d) $\frac{\sqrt{5}}{3}$

11. In a triangle ABC, if $2a\cos\left(\frac{B-C}{2}\right) = b + c$, then $\sec A$ is equal to :

(All symbols used have usual meaning in a triangle.)

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{2}$ (c) 2 (d) 3

12. Triangle ABC has $BC = 1$ and $AC = 2$, then maximum possible value of $\angle A$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

13. $\Delta I_1 I_2 I_3$ is an excentral triangle of an equilateral triangle ΔABC such that $I_1 I_2 = 4$ unit, if ΔDEF is pedal triangle of ΔABC, then $\frac{Ar(\Delta I_1 I_2 I_3)}{Ar(\Delta DEF)} =$

(a) 16 (b) 4 (c) 2 (d) 1

14. Let ABC be a triangle with $\angle BAC = \frac{2\pi}{3}$ and $AB = x$ such that $(AB)(AC) = 1$. If x varies then the longest possible length of the internal angle bisector AD equals :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{\sqrt{2}}{3}$

15. In an equilateral triangle r, R and r_1 form (where symbols used have usual meaning)

(a) an A.P. (b) a G.P. (c) an H.P. (d) none of these

16. In ΔABC if $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, then a^2, b^2, c^2 are in :

(a) A.P. (b) G.P. (c) H.P. (d) none of these

17. In ΔABC, $\tan A = 2$, $\tan B = \frac{3}{2}$ and $c = \sqrt{65}$, then circumradius of the triangle is :

(a) 65 (b) $\frac{65}{7}$ (c) $\frac{65}{14}$ (d) none of these

18. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3 - 13x^2 + 54x - 72 = 0$, then the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is equal to :

(a) $\frac{61}{144}$ (b) $\frac{61}{72}$ (c) $\frac{169}{144}$ (d) $\frac{59}{144}$

19. In ΔABC, if $\angle C = 90°$, then $\frac{a+c}{b} + \frac{b+c}{a}$ is equal to :

(a) $\frac{c}{r}$ (b) $\frac{1}{2Rr}$ (c) 2 (d) $\frac{R}{r}$

20. In a ΔABC, if $a^2 \sin B = b^2 + c^2$, then :

(a) $\angle A$ is obtuse (b) $\angle A$ is acute (c) $\angle B$ is obtuse (d) $\angle A$ is right angle

21. If R and R' are the circumradii of triangles ABC and OBC, where O is the orthocenter of triangle ABC, then :

(a) $R' = \frac{R}{2}$ (b) $R' = 2R$ (c) $R' = R$ (d) $R' = 3R$

22. The acute angle of a rhombus whose side is geometric mean between its diagonals, is :

(a) 15° (b) 20° (c) 30° (d) 60°

23. In a ΔABC right angled at A, a line is drawn through A to meet BC at D dividing BC in $2 : 1$. If $\tan(\angle ADC) = 3$ then $\angle BAD$ is :

(a) 30° (b) 45° (c) 60° (d) 75°

24. A circle is cirumscribed in an equilateral triangle of side 'l'. The area of any square inscribed in the circle is :

(a) $\frac{4}{3}l^2$ (b) $\frac{2}{3}l^2$ (c) $\frac{1}{3}l^2$ (d) l^2

25. If the sides of a triangle are in the ratio $2 : \sqrt{6} : (\sqrt{3} + 1)$, then the largest angle of the triangle will be :

(a) 60° (b) 72° (c) 75° (d) 90°

26. In a triangle ABC if a, b, c are in A.P. and $C - A = 120°$, then $\frac{s}{r} =$

(where notations have their usual meaning)

(a) $\sqrt{15}$ (b) $2\sqrt{15}$ (c) $3\sqrt{15}$ (d) $6\sqrt{15}$

27. In a triangle ABC, $a = 5, b = 4$ and $\cos(A - B) = \frac{31}{32}$, then the third side is equal to :

(where symbols used have usual meanings)

(a) $\sqrt{6}$ **(b)** $6\sqrt{6}$ **(c)** 6 **(d)** $(216)^{1/4}$

28. If semiperimeter of a triangle is 15, then the value of $(b+c)\cos(B+C)+(c+a)\cos(C+A)+(a+b)\cos(A+B)$ is equal to :
(where symbols used have usual meanings)
(a) -60 (b) -15
(c) -30 (d) can not be determined

29. Let triangle ABC be an isosceles triangle with $AB = AC$. Suppose that the angle bisector of its angle B meets the side AC at a point D and that $BC = BD + AD$. Measure of the angle A in degrees, is :
(a) 80 (b) 100 (c) 110 (d) 130

30. In triangle ABC if $A:B:C = 1:2:4$, then $(a^2-b^2)(b^2-c^2)(c^2-a^2) = \lambda a^2b^2c^2$, where $\lambda =$
(where notations have their usual meaning)
(a) 1 (b) 2 (c) 4 (d) 9

31. In a triangle ABC with altitude AD, $\angle BAC = 45°$, $DB = 3$ and $CD = 2$. The area of the triangle ABC is :
(a) 6 (b) 15 (c) 15/4 (d) 12

32. A triangle has base 10 cm long and the base angles of 50° and 70°. If the perimeter of the triangle is $x + y\cos z°$ where $z \in (0, 90)$ then the value of $x + y + z$ equals :
(a) 60 (b) 55 (c) 50 (d) 40

33. Let H be the orthocenter of triangle ABC, then angle subtended by side BC at the centre of incircle of ΔCHB is :
(a) $\frac{A}{2}+\frac{\pi}{2}$ (b) $\frac{B+C}{2}+\frac{\pi}{2}$ (c) $\frac{B-C}{2}+\frac{\pi}{2}$ (d) $\frac{B+C}{2}+\frac{\pi}{4}$

34. Triangle ABC is right angled at A. The points P and Q are on the hypotenuse BC such that $BP = PQ = QC$. If $AP = 3$ and $AQ = 4$ then the length BC is equal to :
(a) $\sqrt{27}$ (b) $\sqrt{36}$ (c) $\sqrt{45}$ (d) $\sqrt{54}$

35. In a ΔABC if $b = a(\sqrt{3}-1)$ and $\angle C = 30°$ then the measure of the angle A is :
(a) 15° (b) 45° (c) 75° (d) 105°

36. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the perpendicular distance of P from the base of the triangle. Let h_1 and h_2 be the perpendicular distance of P from the other two sides of the triangle. Then :
(a) $h = \frac{h_1+h_2}{2}$ (b) $h = \sqrt{h_1h_2}$
(c) $h = \frac{2h_1h_2}{h_1+h_2}$ (d) $h = \frac{(h_1+h_2)\sqrt{3}}{4}$

37. The angles A, B and C of a triangle ABC are in arithmetic progression. $AB = 6$ and $BC = 7$. Then AC is :
(a) $\sqrt{41}$ (b) $\sqrt{39}$ (c) $\sqrt{42}$ (d) $\sqrt{43}$

38. In ΔABC, If $A - B = 120°$ and $R = 8r$, then the value of $\frac{1+\cos C}{1-\cos C}$ equals :

(All symbols used have their usual meaning in a triangle)

(a) 12 (b) 15 (c) 21 (d) 31

39. The lengths of the sides CB and CA of a triangle ABC are given by a and b and the angle C is $\frac{2\pi}{3}$.

The line CD bisects the angle C and meets AB at D. Then the length of CD is :

(a) $\frac{1}{a+b}$ (b) $\frac{a^2+b^2}{a+b}$ (c) $\frac{ab}{2(a+b)}$ (d) $\frac{ab}{a+b}$

40. In ΔABC, angle A is $120°$, $BC + CA = 20$ and $AB + BC = 21$, then the length of the side BC, equals :

(a) 13 (b) 15 (c) 17 (d) 19

41. A triangle has sides 6, 7, 8. The line through its incentre parallel to the shortest side is drawn to meet the other two sides at P and Q. The length of the segment PQ is :

(a) $\frac{12}{5}$ (b) $\frac{15}{4}$ (c) $\frac{30}{7}$ (d) $\frac{33}{9}$

42. The perimeter of a ΔABC is 48 cm and one side is 20 cm. Then remaining sides of ΔABC must be greater than :

(a) 8 cm (b) 9 cm (c) 12 cm (d) 4 cm

43. In an equilateral ΔABC, (where symbols used have usual meanings), then r, R and r_1 form :

(a) an A.P. (b) a G.P.

(c) an H.P. (d) neither an A.P., G.P. nor H.P.

44. The expression $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$ is equal to :

(a) $\cos^2 A$ (b) $\sin^2 A$ (c) $\cos A \cos B \cos C$ (d) $\sin A \sin B \sin C$

(where symbols used have usual meanings)

45. Circumradius of an isosceles ΔABC with $\angle A = \angle B$ is 4 times its in radius, then $\cos A$ is root of the equation :

(a) $x^2 - x - 8 = 0$ (b) $8x^2 - 8x + 1 = 0$ (c) $x^2 - x - 4 = 0$ (d) $4x^2 - 4x + 1 = 0$

46. A is the orthocentre of ΔABC and D is reflection point of A w.r.t. perpendicular bisector of BC, then orthocenter of ΔDBC is :

(a) D (b) C (c) B (d) A

47. If a, b, c are sides of a scalene triangle, then the value of determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always:

(a) ≥ 0 (b) > 0 (c) ≤ -1 (d) < 0

48. In a triangle ABC if $A : B : C = 1 : 2 : 4$, then $(a^2 - b^2)(b^2 - c^2)(c^2 - a^2) = \lambda a^2 b^2 c^2$, where $\lambda =$:

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{3}$

49. The minimum value of $\frac{r_1 r_2 r_3}{r^3}$ in a triangle is (symbols have their usual meaning)

(a) 1 (b) 3 (c) 8 (d) 27

50. In a triangle ABC, $BC = 3$, $AC = 4$ and $AB = 5$. The value of $\sin A + \sin 2B + \sin 3C$ equals

(a) $\frac{24}{25}$ (b) $\frac{14}{25}$ (c) $\frac{64}{25}$ (d) None

51. In any triangle ABC, the value of $\frac{r_1 + r_2}{1 + \cos C}$ is equal to (where notation have their usual meaning) :

(a) $2R$ (b) $2r$ (c) R (d) $\frac{2R^2}{r}$

52. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$; $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$ then the area of the triangle ABC is :

(a) $\frac{8}{3\sqrt{3}}$ (b) $\frac{16}{3\sqrt{3}}$ (c) $\frac{32}{3\sqrt{3}}$ (d) $\frac{64}{3\sqrt{3}}$

53. The sides of a triangle are $\sin\alpha$, $\cos\alpha$, $\sqrt{1 + \sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$ then the greatest angle of the triangle is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

54. Let ABC be a right triangle with $\angle BAC = \frac{\pi}{2}$, then $\left(\frac{r^2}{2R^2} + \frac{r}{R}\right)$ is equal to :

(where symbols used have usual meaning in a triangle)

(a) $\sin B \sin C$ (b) $\tan B \tan C$ (c) $\sec B \sec C$ (d) $\cot B \cot C$

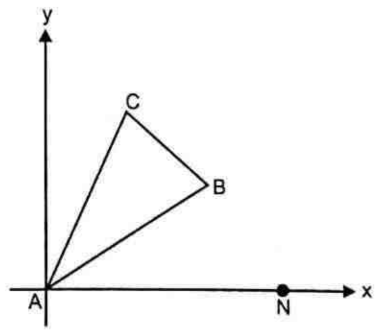
55. Find the radius of the circle escribed to the triangle ABC (Shown in the figure below) on the side BC if $\angle NAB = 30°$; $\angle BAC = 30°$; $AB = AC = 5$.

(a) $\frac{(10\sqrt{2} + 5\sqrt{3} - 5)(2 - \sqrt{3})}{2\sqrt{2}}$

(b) $\frac{(10\sqrt{2} + 5\sqrt{3} + 5)}{2\sqrt{2}}(2 - \sqrt{3})$

(c) $\frac{(10\sqrt{2} + 5\sqrt{3} - 5)}{2\sqrt{2}}(2 + \sqrt{3})$

(d) $\frac{(10\sqrt{2} + 5\sqrt{2} + 1)}{2\sqrt{3}}(\sqrt{3} - 1)$



56. In a ΔABC, with usual notations, if $b > c$ then distance between foot of median and foot of altitude both drawn from vertex A on BC is :

(a) $\frac{a^2 - b^2}{2c}$ (b) $\frac{b^2 - c^2}{2a}$ (c) $\frac{b^2 + c^2 - a^2}{2a}$ (d) $\frac{b^2 + c^2 - a^2}{2c}$

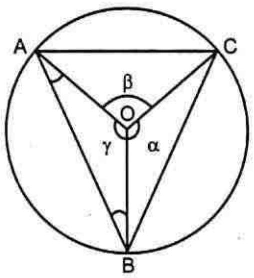
57. In a triangle ABC the expression $a \cos B \cos C + b \cos C \cos A + c \cos A \cos B$ equals to :

(a) $\frac{rs}{R}$ (b) $\frac{r}{sR}$ (c) $\frac{R}{rs}$ (d) $\frac{Rs}{r}$

58. In an acute triangle ABC, altitudes from the vertices A, B and C meet the opposite sides at the points D, E and F respectively. If the radius of the circumcircle of ΔAFE, ΔBFD, ΔCED, ΔABC be respectively R_1, R_2, R_3 and R. Then the maximum value of $R_1 + R_2 + R_3$ is :

(a) $\frac{3R}{8}$ (b) $\frac{2R}{3}$ (c) $\frac{4R}{3}$ (d) $\frac{3R}{2}$

59. A circle of area 20 sq. units is centered at the point O. Suppose ΔABC is inscribed in that circle and has area 8 sq. units. The central angles α, β and γ are as shown in the figure. The value of $(\sin \alpha + \sin \beta + \sin \gamma)$ is equal to :



(a) $\frac{4\pi}{5}$ (b) $\frac{3\pi}{4}$ (c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{4}$

Answers

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(b)	**6.**	(a)	**7.**	(d)	**8.**	(a)	**9.**	(a)	**10.**	(c)
11.	(c)	**12.**	(a)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(c)	**18.**	(a)	**19.**	(a)	**20.**	(a)
21.	(c)	**22.**	(c)	**23.**	(b)	**24.**	(b)	**25.**	(c)	**26.**	(c)	**27.**	(c)	**28.**	(c)	**29.**	(b)	**30.**	(a)
31.	(b)	**32.**	(d)	**33.**	(b)	**34.**	(c)	**35.**	(d)	**36.**	(a)	**37.**	(d)	**38.**	(b)	**39.**	(d)	**40.**	(a)
41.	(c)	**42.**	(d)	**43.**	(a)	**44.**	(b)	**45.**	(b)	**46.**	(a)	**47.**	(d)	**48.**	(a)	**49.**	(d)	**50.**	(b)
51.	(a)	**52.**	(c)	**53.**	(c)	**54.**	(a)	**55.**	(a)	**56.**	(b)	**57.**	(a)	**58.**	(d)	**59.**	(a)		

Exercise-2 : One or More than One Answer is/are Correct

1. If r_1, r_2, r_3 are radii of the escribed circles of a triangle ABC and r is the radius of its incircle, then the root(s) of the equation $x^2 - r(r_1r_2 + r_2r_3 + r_3r_1)x + (r_1r_2r_3 - 1) = 0$ is/are :

(a) r_1 (b) $r_2 + r_3$ (c) 1 (d) $r_1r_2r_3 - 1$

2. In ΔABC, $\angle A = 60°$, $\angle B = 90°$, $\angle C = 30°$. Let H be its orthocentre, then :
(where symbols used have usual meanings)

(a) $AH = c$ (b) $CH = a$ (c) $AH = a$ (d) $BH = 0$

3. In an equilateral triangle, if inradius is a rational number then which of the following is/are correct ?

(a) circumradius is always rational (b) exradii are always rational
(c) area is always ir-rational (d) perimeter is always rational

4. Let A, B, C be angles of a triangle ABC and let $D = \frac{5\pi + A}{32}, E = \frac{5\pi + B}{32}, F = \frac{5\pi + C}{32}$, then :

$\left(\text{where } D, E, F \neq \frac{n\pi}{2}, n \in I, I \text{ denote set of integers}\right)$

(a) $\cot D \cot E + \cot E \cot F + \cot D \cot F = 1$ (b) $\cot D + \cot E + \cot F = \cot D \cot E \cot F$
(c) $\tan D \tan E + \tan E \tan F + \tan F \tan D = 1$ (d) $\tan D + \tan E + \tan F = \tan D \tan E \tan F$

5. In a triangle ABC, if $a = 4, b = 8$ and $\angle C = 60°$, then :
(where symbols used have usual meanings)

(a) $c = 6$ (b) $c = 4\sqrt{3}$ (c) $\angle A = 30°$ (d) $\angle B = 90°$

6. In a ΔABC if $\frac{r}{r_1} = \frac{r_2}{r_3}$, then which of the following is/are true ?
(where symbols used have usual meanings)

(a) $a^2 + b^2 + c^2 = 8R^2$ (b) $\sin^2 A + \sin^2 B + \sin^2 C = 2$
(c) $a^2 + b^2 = c^2$ (d) $\Delta = s(s + c)$

7. ABC is a triangle whose circumcentre, incentre and orthocentre are O, I and H respectively which lie inside the triangle, then :

(a) $\angle BOC = A$ (b) $\angle BIC = \frac{\pi}{2} + \frac{A}{2}$
(c) $\angle BHC = \pi - A$ (d) $\angle BHC = \pi - \frac{A}{2}$

8. In a triangle ABC, $\tan A$ and $\tan B$ satisfy the inequality $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then which of the following must be correct ?
(where symbols used have usual meanings)

(a) $a^2 + b^2 - ab < c^2$ (b) $a^2 + b^2 > c^2$
(c) $a^2 + b^2 + ab > c^2$ (d) $a^2 + b^2 < c^2$

9. If in a ΔABC ; $\angle C = \frac{\pi}{8}$; $a = \sqrt{2}$; $b = \sqrt{2+\sqrt{2}}$ then the measure of $\angle A$ can be :

(a) 45° (b) 135° (c) 30° (d) 150°

10. In triangle ABC, $a = 3, b = 4, c = 2$. Point D and E trisect the side BC. If $\angle DAE = \theta$, then $\cot^2\theta$ is divisible by :

(a) 2 (b) 3 (c) 5 (d) 7

11. In a ΔABC if $3\sin A + 4\cos B = 6$; $4\sin B + 3\cos A = 1$ then possible value(s) of $\underline{|C}$ be:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{5\pi}{6}$

12. If the line joining the incentre to the centroid of a triangle ABC is parallel to the side BC. Which of the following are correct ?

(a) $2b = a + c$ (b) $2a = b + c$ (c) $\cot\frac{A}{2}\cot\frac{C}{2} = 3$ (d) $\cot\frac{B}{2}\cot\frac{C}{2} = 3$

13. In a triangle the length of two larger sides are 10 and 9 respectively. It the angles are in A.P., the length of third side can be :

(a) $5-\sqrt{6}$ (b) $5+\sqrt{6}$ (c) $6-\sqrt{5}$ (d) $6+\sqrt{5}$

14. If area of ΔABC, Δ and angle C are given and if the side c opposite to given angle is minimum, then

(a) $a = \sqrt{\frac{2\Delta}{\sin C}}$ (b) $b = \sqrt{\frac{2\Delta}{\sin C}}$ (c) $a = \frac{4\Delta}{\sin C}$ (d) $b = \frac{4\Delta}{\sin^2 C}$

15. In a triangle ABC, if $\tan A = 2\sin 2C$ and $3\cos A = 2\sin B\sin C$ then possible values of C is/are

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

Answers

1.	(c, d)	**2.**	(a, b, d)	**3.**	(a, b, c)	**4.**	(b, c)	**5.**	(b, c, d)	**6.**	(a, b, c)
7.	(b, c)	**8.**	(a, c)	**9.**	(a)	**10.**	(b, c)	**11.**	(b)	**12.**	(b, d)
13.	(a, b)	**14.**	(a, b)	**15.**	(c, d)						

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $\angle A = 23°$, $\angle B = 75°$ and $\angle C = 82°$ be the angles of ΔABC.
The incircle of ΔABC touches the sides BC, CA, AB at points D, E, F respectively. Let r', r_1' respectively be the inradius, exradius opposite to vertex D of ΔDEF and r be the inradius of ΔABC, then

1. $\dfrac{r'}{r} =$

(a) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1$
(b) $1 - \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$
(c) $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} - 1$
(d) $1 - \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$

2. $\dfrac{r_1'}{r} =$

(a) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1$
(b) $1 - \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$
(c) $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} - 1$
(d) $1 - \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$

Paragraph for Question Nos. 3 to 4

Internal angle bisectors of ΔABC meets its circum circle at D, E and F where symbols have usual meaning.

3. Area of ΔDEF is :

(a) $2R^2\cos^2\left(\frac{A}{2}\right)\cos^2\left(\frac{B}{2}\right)\cos^2\left(\frac{C}{2}\right)$
(b) $2R^2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$
(c) $2R^2\sin^2\left(\frac{A}{2}\right)\sin^2\left(\frac{B}{2}\right)\sin^2\left(\frac{C}{2}\right)$
(d) $2R^2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$

4. The ratio of area of triangle ABC and triangle DEF is :

(a) ≥ 1 (b) ≤ 1 (c) $\geq 1/2$ (d) $\leq 1/2$

Paragraph for Question Nos. 5 to 6

Let triangle ABC is right triangle right angled at C such that $A < B$ and $r = 8, R = 41$.

5. Area of ΔABC is :

(a) 720 (b) 1440 (c) 360 (d) 480

6. $\tan\frac{A}{2} =$

(a) $\frac{1}{18}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

[where notations have their usual meaning]

Paragraph for Question Nos. 7 to 8

Let the incircle of ΔABC touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of $\Delta A_1B_1C_1$ touches its sides of B_1C_1, C_1A_1 and A_1B_1 at A_2, B_2, C_2 respectively and so on.

7. $\lim\limits_{n\to\infty} \angle A_n =$

(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

8. In $\Delta A_4B_4C_4$, the value of $\angle A_4$ is :

(a) $\frac{3\pi + A}{6}$ (b) $\frac{3\pi - A}{8}$ (c) $\frac{5\pi - A}{16}$ (d) $\frac{5\pi + A}{16}$

Paragraph for Question Nos. 9 to 10

Let ABC be a given triangle. Points D and E are on sides AB and AC respectively and point F is on line segment DE. Let $\frac{AD}{AB} = x, \frac{AE}{AC} = y, \frac{DF}{DE} = z$. Let area of $\Delta BDF = \Delta_1$, area of $\Delta CEF = \Delta_2$ and area of $\Delta ABC = \Delta$.

9. $\frac{\Delta_1}{\Delta}$ is equal to :

(a) xyz (b) $(1-x)y(1-z)$ (c) $(1-x)yz$ (d) $x(1-y)z$

10. $\frac{\Delta_2}{\Delta}$ is equal to :

(a) $(1-x)y(1-z)$ (b) $(1-x)(1-y)z$ (c) $x(1-y)(1-z)$ (d) $(1-x)yz$

Paragraph for Question Nos. 11 to 13

a, b, c are the length of sides BC, CA, AB respectively of ΔABC satisfying $\log\left(1+\frac{c}{a}\right) + \log a - \log b = \log 2$.

Also the quadratic equation $a(1-x^2) + 2bx + c(1+x^2) = 0$ has two equal roots.

11. a, b, c are in :

(a) A.P. (b) G.P. (c) H.P. (d) None

12. Measure of angle C is :

(a) 30° (b) 45° (c) 60° (d) 90°

13. The value of $(\sin A + \sin B + \sin C)$ is equal to :

(a) $\frac{5}{2}$ (b) $\frac{12}{5}$

(c) $\frac{8}{3}$ (d) 2

Paragraph for Question Nos. 14 to 16

Let ABC be a triangle inscribed in a circle and let $l_a = \frac{m_a}{M_a}$; $l_b = \frac{m_b}{M_b}$; $l_c = \frac{m_c}{M_c}$ where m_a, m_b, m_c are the lengths of the angle bisectors of angles A, B and C respectively, internal to the triangle and M_a, M_b and M_c are the lengths of these internal angle bisectors extended until they meet the circumcircle.

14. l_a equals :

(a) $\frac{\sin A}{\sin\left(B + \frac{A}{2}\right)}$ (b) $\frac{\sin B \sin C}{\sin^2\left(\frac{B+C}{2}\right)}$ (c) $\frac{\sin B \sin C}{\sin^2\left(B + \frac{A}{2}\right)}$ (d) $\frac{\sin B + \sin C}{\sin^2\left(B + \frac{A}{2}\right)}$

15. The maximum value of the product $(l_a l_b l_c) \times \cos^2\left(\frac{B-C}{2}\right) \times \cos^2\left(\frac{C-A}{2}\right) \times \cos^2\left(\frac{A-B}{2}\right)$ is equal to :

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{27}{64}$ (d) $\frac{27}{32}$

16. The minimum value of the expression $\frac{l_a}{\sin^2 A} + \frac{l_b}{\sin^2 B} + \frac{l_c}{\sin^2 C}$ is :

(a) 2 (b) 3 (c) 4 (d) none of these

Answers

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(d)	**7.**	(d)	**8.**	(d)	**9.**	(c)	**10.**	(c)
11.	(a)	**12.**	(d)	**13.**	(b)	**14.**	(c)	**15.**	(c)	**16.**	(b)								

Exercise-4 : Matching Type Problems

1. Consider a right angled triangle ***ABC* right angled at *C*** with integer sides. List-I gives inradius. List-II gives the number of triangles.

	Column-I		Column-II
(A)	3	**(P)**	6
(B)	4	**(Q)**	7
(C)	6	**(R)**	8
(D)	9	**(S)**	10
		(T)	12

2.

	Column-I		Column-II
(A)	Find the sum of the series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{12}+\ldots\ldots\infty$, where the terms are the reciprocals of the positive integers whose only prime factors are two's and three's	**(P)**	7
(B)	The length of the sides of ΔABC are a, b and c and A is the angle opposite to side a. If $b^2+c^2=a^2+54$ and $bc=\frac{a^3}{\cos A}$ then the value of $\left(\frac{b^2+c^2}{9}\right)$, is	**(Q)**	10
(C)	The equations of perpendicular bisectors of two sides AB and AC of a triangle ABC are $x+y+1=0$ and $x-y+1=0$ respectively. If circumradius of ΔABC is 2 units and the locus of vertex A is $x^2+y^2+gx+c=0$, then (g^2+c^2), is equal to	**(R)**	13
(D)	Number of solutions of the equation $\cos\theta\sin\theta+6(\cos\theta-\sin\theta)+6=0$ in $[0, 30]$, is equal to	**(S)**	3

3. In ΔABC, if $r_1=21, r_2=24, r_3=28$, then

	Column-I		Column-II
(A)	$a=$	**(P)**	8
(B)	$b=$	**(Q)**	12
(C)	$s=$	**(R)**	26

(D)	$r =$	**(S)**	28
		(T)	42

(Where notations have their usual meaning)

4.

	Column-I		Column-II
(A)	$\dfrac{r_1(r_2+r_3)}{\sqrt{r_2r_3+r_3r_1+r_1r_2}}$	**(P)**	$\sin\dfrac{A}{2}$
(B)	$\dfrac{r_1}{\sqrt{(r_1+r_2)(r_1+r_3)}}$	**(Q)**	$4R$
(C)	$r_1+r_2+r_3-r$	**(R)**	0
(D)	$\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}-\dfrac{1}{r}$	**(S)**	$2R\sin A$

Answers

1. A → P; B → P; C → T; D → S

2. A → S; B → P; C → R; D → Q

3. A → R; B → S; C → T; D → P

4. A → S; B → P; C → Q; D → R

Exercise-5 : Subjective Type Problems

1. If the median AD of ΔABC makes an angle $\angle ADC = \frac{\pi}{4}$. Find the value of $|\cot B - \cot C|$.

2. In a $\Delta ABC, a = \sqrt{3}, b = 3$ and $\angle C = \frac{\pi}{3}$. Let internal angle bisector of angle C intersects side AB at D and altitude from B meets the angle bisector CD at E. If O_1 and O_2 are incentres of ΔBEC and ΔBED. Find the distance between the vertex B and orthocentre of ΔO_1EO_2.

3. In a ΔABC; inscribed circle with centre I touches sides AB, AC and BC at D, E, F respectively. Let area of quadrilateral $ADIE$ is 5 units and area of quadrilteral $BFID$ is 10 units. Find the value of $\dfrac{\cos\left(\frac{C}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}$.

4. If Δ be area of incircle of a triangle ABC and $\Delta_1, \Delta_2, \Delta_3$ be the area of excircles then find the least value of $\dfrac{\Delta_1\Delta_2\Delta_3}{729\Delta^3}$.

5. In ΔABC, $b = c$, $\angle A = 106°$, M is an interior point such that $\angle MBA = 7°$, $\angle MAB = 23°$ and $\angle MCA = \theta°$, then $\frac{\theta}{2}$ is equal to

(where notations have their usual meaning)

6. In an acute angled triangle ABC, $\angle A = 20°$, let DEF be the feet of altitudes through A, B, C respectively and H is the orthocentre of ΔABC. Find $\dfrac{AH}{AD} + \dfrac{BH}{BE} + \dfrac{CH}{CF}$.

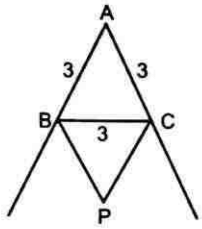
7. Let ΔABC be inscribed in a circle having radius unity. The three internal bisectors of the angles A, B and C are extended to intersect the circumcircle of ΔABC at A_1, B_1 and C_1 respectively. Then $\dfrac{AA_1 \cos\frac{A}{2} + BB_1 \cos\frac{B}{2} + CC_1 \cos\frac{C}{2}}{\sin A + \sin B + \sin C} =$

8. If the quadratic equation $ax^2 + bx + c = 0$ has equal roots where a, b, c denotes the lengths of the sides opposite to vertex A, B and C of the ΔABC respectively. Find the number of integers in the range of $\dfrac{\sin A}{\sin C} + \dfrac{\sin C}{\sin A}$.

9. If in the triangle ABC, $\tan\frac{A}{2}$, $\tan\frac{B}{2}$ and $\tan\frac{C}{2}$ are in harmonic progression then the least value of $\cot^2\frac{B}{2}$ is equal to :

10. In ΔABC, if circumradius 'R' and inradius 'r' are connected by relation $R^2 - 4Rr + 8r^2 - 12r + 9 = 0$, then the greatest integer which is less than the semiperimeter of ΔABC is :

11. Sides AB and AC in an equilateral triangle ABC with side length 3 is extended to form two rays from point A as shown in the figure. Point P is chosen outside the triangle ABC and between the two rays such that $\angle ABP + \angle BCP = 180°$. If the maximum length of CP is M, then $M^2/2$ is equal to :



12. Let a, b, c be sides of a triangle ABC and Δ denotes its area.
If $a = 2; \Delta = \sqrt{3}$ and $a\cos C + \sqrt{3}\, a\sin C - b - c = 0$; then find the value of $(b + c)$.
(symbols used have usual meaning in ΔABC).

13. If circumradius of ΔABC is 3 units and its area is 6 units and ΔDEF is formed by joining foot of perpendiculars drawn from A, B, C on sides BC, CA, AB respectively. Find the perimeter of ΔDEF.

Answers

1.	2	**2.**	1	**3.**	3	**4.**	1	**5.**	7	**6.**	2	**7.**	2
8.	3	**9.**	3	**10.**	7	**11.**	6	**12.**	4	**13.**	4		

□□□

Chapter 25 – Inverse Trigonometric Functions

25 INVERSE TRIGONOMETRIC FUNTIONS

Exercise-1 : Single Choice Problems

1. If $\sin^{-1} x \in \left(0, \frac{\pi}{2}\right)$, then the value of $\tan\left(\frac{\cos^{-1}(\sin(\cos^{-1} x)) + \sin^{-1}(\cos(\sin^{-1} x))}{2}\right)$ is :

(a) 1 (b) 2 (c) 3 (d) 4

2. The solution set of $(\cot^{-1} x)(\tan^{-1} x) + \left(2 - \frac{\pi}{2}\right)\cot^{-1} x - 3\tan^{-1} x - 3\left(2 - \frac{\pi}{2}\right) > 0$, is :

(a) $x \in (\tan 2, \tan 3)$ (b) $x \in (\cot 3, \cot 2)$

(c) $x \in (-\infty, \tan 2) \cup (\tan 3, \infty)$ (d) $x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$

3. The value of $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$ is :

(a) 14 (b) 15 (c) 16 (d) 17

4. Sum the series :

$\tan^{-1}\left(\frac{4}{1+3\cdot 4}\right) + \tan^{-1}\left(\frac{6}{1+8\cdot 9}\right) + \tan^{-1}\left(\frac{8}{1+15\cdot 16}\right) + \ldots\ldots \infty$ is :

(a) $\cot^{-1}(2)$ (b) $\tan^{-1}(2)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

5. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x =$

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$ (c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

6. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \cot^{-1}\left(\frac{67}{4}\right) + \ldots\ldots \infty$ is :

(a) $\frac{\pi}{4} - \cot^{-1}(3)$ (b) $\frac{\pi}{4} - \tan^{-1}(3)$ (c) $\frac{\pi}{4} + \cot^{-1}(3)$ (d) $\frac{\pi}{4} + \tan^{-1}(3)$

7. The number of solutions of equation $\cos^{-1}(1-x) + m\cos^{-1} x = \frac{n\pi}{2}$ is : (where $m > 0; n \le 0$)

(a) 0 (b) 1 (c) 2 (d) none of these

8. Number of solution(s) of the equation $2\tan^{-1}(2x-1)=\cos^{-1}(x)$ is :

(a) 1 (b) 2 (c) 3 (d) infinitely many

9. $\sin^{-1}\left(\frac{x^2}{4}+\frac{y^2}{9}\right)+\cos^{-1}\left(\frac{x}{2\sqrt{2}}+\frac{y}{3\sqrt{2}}-2\right)$ equals to :

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{\sqrt{2}}$ (d) $\frac{3\pi}{2}$

10. The complete solution set of the inequality $(\cos^{-1}x)^2-(\sin^{-1}x)^2>0$ is :

(a) $\left[0,\frac{1}{\sqrt{2}}\right)$ (b) $\left[-1,\frac{1}{\sqrt{2}}\right)$ (c) $(-1,1)$ (d) $\left[-1,\frac{1}{2}\right)$

11. Let α,β are the roots of the equation $x^2+7x+k(k-3)=0$, where $k\in(0,3)$ and k is a constant. Then the value of $\tan^{-1}\alpha+\tan^{-1}\beta+\tan^{-1}\frac{1}{\alpha}+\tan^{-1}\frac{1}{\beta}$ is :

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) $-\frac{\pi}{2}$

12. Let $f(x)=a+2b\cos^{-1}x$, $b>0$. If domain and range of $f(x)$ are the same set, then $(b-a)$ is equal to :

(a) $1-\frac{1}{\pi}$ (b) $\frac{2}{\pi}$

(c) $\frac{2}{\pi}+1$ (d) $1+\frac{1}{\pi}$

13. If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$, then x equals to :

(a) -1 (b) 1 (c) 0 (d) $\sqrt{3}$

14. The total number of ordered pairs (x,y) satisfying $|y|=\cos x$ and $y=\sin^{-1}(\sin x)$, where $x\in[-2\pi,3\pi]$ is equal to :

(a) 2 (b) 4 (c) 5 (d) 6

15. If $[\sin^{-1}(\cos^{-1}(\sin^{-1}(\tan^{-1}x)))]=1$, where $[\cdot]$ denotes greatest integer function, then complete set of values of x is :

(a) $[\tan(\sin(\cos 1)),\tan(\cos(\sin 1))]$ (b) $[\tan(\sin(\cos 1)),\tan(\sin(\cos(\sin 1)))]$

(c) $[\tan(\cos(\sin 1)),\tan(\sin(\cos(\sin 1)))]$ (d) $[\tan(\sin(\cos 1)),1]$

16. The number of ordered pair(s) (x,y) of real numbers satisfying the equation $1+x^2+2x\sin(\cos^{-1}y)=0$, is :

(a) 0 (b) 1 (c) 2 (d) 3

17. The value of $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3$ is :

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{8}$

18. The complete set of values of x for which $2\tan^{-1}x+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is independent of x is :

(a) $(-\infty, 0]$ (b) $[0, \infty)$ (c) $(-\infty, -1]$ (d) $[1, \infty)$

19. The number of ordered pair(s) (x, y) which satisfy $y=\tan^{-1}\tan x$ and $16(x^2+y^2)-48\pi x+16\pi y+31\pi^2=0$, is :

(a) 0 (b) 1 (c) 2 (d) 3

20. Domain (D) and range (R) of $f(x)=\sin^{-1}(\cos^{-1}[x])$ where [ ] denotes the greatest integer function is

(a) $D\equiv[1, 2), R\equiv\{0\}$ (b) $D\equiv[0, 1), R\equiv\{-1, 0, 1\}$

(c) $D\equiv[-1, 1), R\equiv\left\{0, \frac{\pi}{2}, \pi\right\}$ (d) $D\equiv[-1, 1), R\equiv\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

21. If $2\sin^{-1}x+\{\cos^{-1}x\}>\frac{\pi}{2}+\{\sin^{-1}x\}$, then $x\in$: (where $\{\cdot\}$ denotes fractional part function)

(a) $(\cos 1, 1]$ (b) $[\sin 1, 1]$ (c) $(\sin 1, 1]$ (d) None of these

22. Let $f(x)=x^{11}+x^9-x^7+x^3+1$ and $f(\sin^{-1}(\sin 8))=\alpha$, ($\alpha$ is constant). If $f(\tan^{-1}(\tan 8))=\lambda-\alpha$, then the value of λ is :

(a) 2 (b) 3 (c) 4 (d) 1

23. The number of real values of x satisfying the equation $3\sin^{-1}x+\pi x-\pi=0$ is/are :

(a) 0 (b) 1 (c) 2 (d) 3

24. Range of $f(x)=\sin^{-1}x+x^2+4x+1$ is :

(a) $\left[-\frac{\pi}{2}-2, \frac{\pi}{2}+6\right]$ (b) $\left[0, \frac{\pi}{2}+6\right]$ (c) $\left[-\frac{\pi}{2}-2, \infty\right)$ (d) $[-3, \infty)$

25. The solution set of the inequality $(\text{cosec}^{-1}x)^2-2\text{cosec}^{-1}x\geq\frac{\pi}{6}(\text{cosec}^{-1}x-2)$ is $(-\infty, a]\cup[b, \infty)$, then $(a+b)$ equals :

(a) 0 (b) 1 (c) 2 (d) -3

26. Number of solution of the equation $2\sin^{-1}(x+2)=\cos^{-1}(x+3)$ is :

(a) 0 (b) 1 (c) 2 (d) None of these

27. $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\ldots..\infty=$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

28. If $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}x$ then x is equal to :

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) none of these

29. The set of value of x, satisfying the equation $\tan^2(\sin^{-1} x) > 1$ is :

(a) $(-1, 1)$ (b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $[-1, 1] - \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) $(-1, 1) - \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

30. The sum of the series $\cot^{-1}\left(\frac{9}{2}\right) + \cot^{-1}\left(\frac{33}{4}\right) + \cot^{-1}\left(\frac{129}{8}\right) + \ldots\ldots \infty$ is equal to :

(a) $\cot^{-1}(2)$ (b) $\cot^{-1}(3)$ (c) $\cot^{-1}(-1)$ (d) $\cot^{-1}(1)$

31. If $\int \frac{\ln(\cot x)}{\sin x \cos x} dx = -\frac{1}{k} \ln^2(\cot x) + C$

(where C is a constant); then the value of k is :

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

32. The number of solutions of $\sin^{-1} x + \sin^{-1}(1 + x) = \cos^{-1} x$ is/are :

(a) 0 (b) 1 (c) 2 (d) infinite

33. The value of x satisfying the equation

$$(\sin^{-1} x)^3 - (\cos^{-1} x)^3 + (\sin^{-1} x)(\sin^{-1} x - \cos^{-1} x) = \frac{\pi^3}{16} \text{ is :}$$

(a) $\cos\frac{\pi}{5}$ (b) $\cos\frac{\pi}{4}$ (c) $\cos\frac{\pi}{8}$ (d) $\cos\frac{\pi}{12}$

34. The complete solution set of the equation

$$\sin^{-1}\sqrt{\frac{1+x}{2}} - \sqrt{2-x} = \cot^{-1}(\tan\sqrt{2-x}) - \sin^{-1}\sqrt{\frac{1-x}{2}} \text{ is :}$$

(a) $\left[2-\frac{\pi^2}{4}, 1\right]$ (b) $\left[1-\frac{\pi^2}{4}, 1\right]$ (c) $\left[2-\frac{\pi^2}{4}, 0\right]$ (d) $[-1, 1]$

35. Let $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ then which of the following is correct :

(a) $f(x)$ has only one integer in its range (b) Range of $f(x)$ is $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right) - \{0\}$

(c) Range of $f(x)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (d) Range of $f(x)$ is $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right] - \{0\}$

36. If $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1} x$ then x is equal to :

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) None of these

37. The set of values of x, satisfying the equation $\tan^2(\sin^{-1} x) > 1$ is :

(a) $(-1, 1)$ (b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $[-1, 1] - \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) $(-1, 1) - \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

38. The sum of the series $\cot^{-1}\left(\frac{9}{2}\right) + \cot^{-1}\left(\frac{33}{4}\right) + \cot^{-1}\left(\frac{129}{8}\right) + \ldots\ldots \infty$ is equal to

(a) $\cot^{-1}(2)$ (b) $\cot^{-1}(3)$ (c) $\cot^{-1}(-1)$ (d) $\cot^{-1}(1)$

39. The number of real values of x satisfying $\tan^{-1}\left(\frac{x}{1-x^2}\right) + \tan^{-1}\left(\frac{1}{x^3}\right) = \frac{3\pi}{4}$ is :

(a) 0 (b) 1 (c) 2 (d) infinitely many

40. Number of integral values of λ such that the equation $\cos^{-1} x + \cot^{-1} x = \lambda$ possesses solution is:

(a) 2 (b) 8 (c) 5 (d) 10

41. If the equation $x^3 + bx^2 + cx + 1 = 0\,(b < c)$ has only one real root α.
Then the value of $2\tan^{-1}(\operatorname{cosec}\alpha) + \tan^{-1}(2\sin\alpha\sec^2\alpha)$ is :

(a) $-\frac{\pi}{2}$ (b) $-\pi$ (c) $\frac{\pi}{2}$ (d) π

42. Range of the function $f(x) = \cot^{-1}\{-x\} + \sin^{-1}\{x\} + \cos^{-1}\{x\}$, where $\{\cdot\}$ denotes fractional part function

(a) $\left(\frac{3\pi}{4}, \pi\right)$ (b) $\left[\frac{3\pi}{4}, \pi\right)$ (c) $\left[\frac{3\pi}{4}, \pi\right]$ (d) $\left(\frac{3\pi}{4}, \pi\right]$

43. If $3 \le a < 4$ then the value of $\sin^{-1}(\sin[a]) + \tan^{-1}(\tan[a]) + \sec^{-1}(\sec[a])$, where $[x]$ denotes greatest integer function less than or equal to x, is equal to :

(a) 3 (b) $2\pi - 9$ (c) $2\pi - 3$ (d) $9 - 2\pi$

44. The number of real solutions of $y + y^2 = \sin x$ and $y + y^3 = \cos^{-1}\cos x$ is/are

(a) 0 (b) 1 (c) 3 (d) Infinite

45. Range of $f(x) = \sin^{-1}[x-1] + 2\cos^{-1}[x-2]$ ($[\cdot]$ denotes greatest integer function)

(a) $\left\{-\frac{\pi}{2}, 0\right\}$ (b) $\left\{\frac{\pi}{2}, 2\pi\right\}$ (c) $\left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$ (d) $\left\{\frac{3\pi}{2}, 2\pi\right\}$

Answers

1.	(a)	2.	(b)	3.	(b)	4.	(a)	5.	(a)	6.	(c)	7.	(a)	8.	(a)	9.	(d)	10.	(b)
11.	(c)	12.	(d)	13.	(a)	14.	(c)	15.	(b)	16.	(b)	17.	(b)	18.	(a)	19.	(d)	20.	(a)
21.	(b)	22.	(a)	23.	(b)	24.	(a)	25.	(b)	26.	(b)	27.	(a)	28.	(c)	29.	(d)	30.	(a)
31.	(b)	32.	(b)	33.	(c)	34.	(a)	35.	(b)	36.	(c)	37.	(d)	38.	(a)	39.	(a)	40.	(c)
41.	(b)	42.	(d)	43.	(a)	44.	(d)	45.	(d)										

Exercise-2 : One or More than One Answer is/are Correct

1. $f(x) = \sin^{-1}(\sin x), g(x) = \cos^{-1}(\cos x)$, then :

(a) $f(x) = g(x)$ if $x \in \left(0, \frac{\pi}{4}\right)$
(b) $f(x) < g(x)$ if $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$
(c) $f(x) < g(x)$ if $\left(\pi, \frac{5\pi}{4}\right)$
(d) $f(x) > g(x)$ if $x \in \left(\pi, \frac{5\pi}{4}\right)$

2. The solution(s) of the equation $\cos^{-1} x = \tan^{-1} x$ satisfy

(a) $x^2 = \frac{\sqrt{5}-1}{2}$
(b) $x^2 = \frac{\sqrt{5}+1}{2}$
(c) $\sin(\cos^{-1} x) = \frac{\sqrt{5}-1}{2}$
(d) $\tan(\cos^{-1} x) = \frac{\sqrt{5}-1}{2}$

3. If the numerical value of $\tan\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right)$ is $\left(\frac{a}{b}\right)$, where a, b are two positive integers and their H.C.F is 1

(a) $a + b = 23$
(b) $a - b = 11$
(c) $3b = a + 1$
(d) $2a = 3b$

4. A solution of the equation $\cot^{-1} 2 = \cot^{-1} x + \cot^{-1}(10 - x)$ where $1 < x < 9$ is :

(a) 7
(b) 3
(c) 2
(d) 5

5. Consider the equation $\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) + \cos^{-1} k = \frac{\pi}{2}$, then :

(a) the largest value of k for which equation has 2 distinct solution is 1
(b) the equation must have real root if $k \in \left(-\frac{1}{2}, 1\right)$
(c) the equation must have real root if $k \in \left(-1, \frac{1}{2}\right)$
(d) the equation has unique solution if $k = -\frac{1}{2}$

6. The value of x satisfying the equation

$$(\sin^{-1} x)^3 - (\cos^{-1} x)^3 + (\sin^{-1} x)(\cos^{-1} x)(\sin^{-1} x - \cos^{-1} x) = \frac{\pi^3}{16}$$

can not be equal to :

(a) $\cos\frac{\pi}{5}$
(b) $\cos\frac{\pi}{4}$
(c) $\cos\frac{\pi}{8}$
(d) $\cos\frac{\pi}{12}$

Answers

1.	(a, b, c)	2.	(a, c)	3.	(a, b, c)	4.	(a, b)	5.	(a, b, d)	6.	(a, b, d)

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Let $\cos^{-1}(4x^3 - 3x) = a + b\cos^{-1} x$

1. If $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, then $\sin^{-1}\left(\sin\frac{a}{b}\right)$ is :

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $-\frac{\pi}{6}$ (d) $\frac{\pi}{6}$

2. If $x \in \left(\frac{1}{2}, 1\right]$, then $\lim_{y \to a} b\cos y$ is :

(a) $-\frac{1}{3}$ (b) -3 (c) $\frac{1}{3}$ (d) 3

Answers

1.	(a)	**2.**	(d)

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	$\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=$	**(P)**	$\frac{\pi}{6}$
(B)	$\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=$	**(Q)**	$\frac{\pi}{2}$
(C)	If $A=\tan^{-1}\frac{x\sqrt{3}}{2\lambda-x}$, $B=\tan^{-1}\left(\frac{2x-\lambda}{\lambda\sqrt{3}}\right)$ then $A-B$ can be equal to	**(R)**	$\frac{\pi}{4}$
(D)	$\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}=$	**(S)**	π
		(T)	$\frac{\pi}{3}$

2.

	Column-I		Column-II
(P)	If $f(x)=\sin^{-1}x$ and $\lim\limits_{x\to\frac{1}{2}^+} f(3x-4x^3)=l-3\left(\lim\limits_{x\to\frac{1}{2}^+} f(x)\right)$ then $[l]=$ ([·] denotes greatest integer function)	**(P)**	3
(Q)	If $x>1$, then the value of $\sin\left(\frac{1}{2}\tan^{-1}\frac{2x}{1-x^2}-\tan^{-1}x\right)$ is	**(Q)**	-1
(R)	Number of values of x satisfying $\sin^{-1}x-\cos^{-1}x=\sin^{-1}(3x-2)$	**(R)**	2
(S)	The value of $\sin\left(\tan^{-1}3+\tan^{-1}\frac{1}{3}\right)$	**(S)**	1

3.

	Column-I		Column-II
(A)	If the first term of an arithmetic progression is 1, its second term is n, and the sum of the first n terms is $33n$	**(P)**	3
(B)	If the equation $\cos^{-1} x + \cot^{-1} x = k$ possess solution, then the largest integral value of k is	**(Q)**	4
(C)	The number of solution of equation $\cos\theta = \lvert 1 + \sin\theta \rvert$ in interval $[0, 3\pi]$, is	**(R)**	5
(D)	If the quadratic equation $x^2 - x - a = 0$ has integral roots where $a \in N$ and $4 \le a \le 40$, then the number of possible values of a is	**(S)**	9

4.

	Column-I		Column-II
(A)	The value of $\tan^{-1}([\pi]) + \tan^{-1}([-\pi] + 1) =$ ($[\cdot]$ denotes greatest integer function)	**(P)**	2
(B)	The number of solutions of the equation $\tan x + \sec x = 2\cos x$ in the interval $[0, 2\pi]$ is	**(Q)**	3
(C)	The number of roots of the equation $x + 2\tan x = \dfrac{\pi}{2}$ in the interval $[0, 2\pi]$ is	**(R)**	0
(D)	The number of solutions of the equation $x^3 + x^2 + 4x + 2\sin x = 0$ in the interval $[0, 2\pi]$ is	**(S)**	1

Answers

1. A → Q ; B → S ; C → P ; D → R

2. A → P ; B → Q ; C → R ; D → S

3. A → S ; B → R ; C → P ; D → Q

4. A → R ; B → P ; C → Q ; D → S

Exercise-5 : Subjective Type Problems

1. The complete set of values of x satisfying the inequality $\sin^{-1}(\sin 5) > x^2 - 4x$ is $(2-\sqrt{\lambda - 2\pi}, 2+\sqrt{\lambda - 2\pi})$, then $\lambda =$
2. In a ΔABC ; if $(II_1)^2 + (I_2I_3)^2 = \lambda R^2$, where I denotes incentre ; I_1, I_2 and I_3 denote centres of the circles escribed to the sides BC, CA and AB respectively and R be the radius of the circum circle of ΔABC. Find λ.
3. If $2\tan^{-1}\frac{1}{5} - \sin^{-1}\frac{3}{5} = -\cos^{-1}\frac{63}{\lambda}$, then $\lambda =$
4. If $2\tan^{-1}\frac{1}{5} - \sin^{-1}\frac{3}{5} = -\cos^{-1}\frac{9\lambda}{65}$, then $\lambda =$
5. If $\sum_{n=0}^{\infty} 2\cot^{-1}\left(\frac{n^2+n+4}{2}\right) = k\pi$, then find the value of k.
6. Find number of solutions of the equation $\sin^{-1}(|\log_6^2(\cos x) - 1|) + \cos^{-1}(|3\log_6^2(\cos x) - 7|) = \frac{\pi}{2}$, if $x \in [0, 4\pi]$.

Answers

1.	9	2.	16	3.	65	4.	7	5.	1	6.	4		

□□□

Chapter 26 – Vector & 3Dimensional Geometry

Vector & 3Dimensional Geometry

26 VECTOR & 3DIMENSIONAL GEOMETRY

Exercise-1 : Single Choice Problems

1. The minimum value of $x^2 + y^2 + z^2$ if $ax + by + cz = p$, is :

(a) $\left(\dfrac{p}{a+b+c}\right)^2$ (b) $\dfrac{p^2}{a^2+b^2+c^2}$ (c) $\dfrac{a^2+b^2+c^2}{p^2}$ (d) 0

2. If the angle between the vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\dfrac{\pi}{3}$ and the area of the triangle with adjacent sides equal to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is 3, then $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$ is equal to :

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) $\dfrac{\sqrt{3}}{2}$

3. A straight line cuts the sides AB, AC and AD of a parallelogram $ABCD$ at points B_1, C_1 and D_1 respectively. If $\overrightarrow{\mathbf{AB_1}} = \lambda_1 \overrightarrow{\mathbf{AB}}, \overrightarrow{\mathbf{AD_1}} = \lambda_2 \overrightarrow{\mathbf{AD}}$ and $\overrightarrow{\mathbf{AC_1}} = \dfrac{\lambda_3}{2}\overrightarrow{\mathbf{AC}}$, where λ_1, λ_2 and λ_3 are positive real numbers, then :

(a) λ_1, λ_3 and λ_2 are in AP (b) λ_1, λ_3 and λ_2 are in GP
(c) λ_1, λ_3 and λ_2 are in HP (d) $\lambda_1 + \lambda_2 + \lambda_3 = 0$

4. Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$. If $\vec{\mathbf{c}}$ is a vector such that $\vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = |\vec{\mathbf{c}}|, |\vec{\mathbf{c}} - \vec{\mathbf{a}}| = 2\sqrt{2}$ and the angle between $\vec{\mathbf{a}}\times\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is 30° then $\left|(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}\right|$ is equal to :

(a) $\dfrac{2}{3}$ (b) $\dfrac{3}{2}$ (c) 2 (d) 3

5. If acute angle between the line $\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \lambda(4\hat{\mathbf{i}} - 3\hat{\mathbf{k}})$ and xy-plane is θ_1 and acute angle between the planes $x + 2y = 0$ and $2x + y = 0$ is θ_2, then $(\cos^2\theta_1 + \sin^2\theta_2)$ equals to :

(a) 1 (b) $\dfrac{1}{4}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{3}{4}$

6. If a, b, c, x, y, z are real and $a^2+b^2+c^2=25$, $x^2+y^2+z^2=36$ and $ax+by+cz=30$, then $\dfrac{a+b+c}{x+y+z}$ is equal to :

(a) 1 (b) $\frac{6}{5}$ (c) $\frac{5}{6}$ (d) $\frac{3}{4}$

7. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are non-zero, non-collinear vectors such that $|\vec{\mathbf{a}}|=2$, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=1$ and angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\frac{\pi}{3}$. If $\vec{\mathbf{r}}$ is any vector such that $\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=2$, $\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=8$, $(\vec{\mathbf{r}}+2\vec{\mathbf{a}}-10\vec{\mathbf{b}})\cdot(\vec{\mathbf{a}}\times\vec{\mathbf{b}})=4\sqrt{3}$ and satisfy to $\vec{\mathbf{r}}+2\vec{\mathbf{a}}-10\vec{\mathbf{b}}=\lambda(\vec{\mathbf{a}}\times\vec{\mathbf{b}})$, then λ is equal to :

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) None of these

8. Let $\vec{\mathbf{a}}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$; $\vec{\mathbf{b}}=2(\hat{\mathbf{i}}+\hat{\mathbf{k}})$ and $\vec{\mathbf{c}}=4\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$. Sum of the values of '$\alpha$' for which the equation $x\vec{\mathbf{a}}+y\vec{\mathbf{b}}+z\vec{\mathbf{c}}=\alpha(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})$ has non-trivial solution is :

(a) -1 (b) 4 (c) 7 (d) 8

9. If $\vec{\mathbf{a}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{b}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{c}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$, then the value of $\begin{vmatrix}\vec{\mathbf{a}}\cdot\vec{\mathbf{a}} & \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} & \vec{\mathbf{a}}\cdot\vec{\mathbf{c}}\\ \vec{\mathbf{b}}\cdot\vec{\mathbf{a}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{b}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{c}}\\ \vec{\mathbf{c}}\cdot\vec{\mathbf{a}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{b}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{c}}\end{vmatrix}$ is equal to :

(a) 2 (b) 4 (c) 16 (d) 64

10. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two vectors such that $|\vec{\mathbf{a}}|=1$, $|\vec{\mathbf{b}}|=4$, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=2$. If $\vec{\mathbf{c}}=(2\vec{\mathbf{a}}\times\vec{\mathbf{b}})-3\vec{\mathbf{b}}$, then angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

11. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are unit vectors, then the value of $|\vec{\mathbf{a}}-2\vec{\mathbf{b}}|^2+|\vec{\mathbf{b}}-2\vec{\mathbf{c}}|^2+|\vec{\mathbf{c}}-2\vec{\mathbf{a}}|^2$ does not exceed to:

(a) 9 (b) 12 (c) 18 (d) 21

12. The adjacent side vectors $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{OB}}$ of a rectangle $OACB$ are $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ respectively, where O is the origin. If $16|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|=3(|\vec{\mathbf{a}}|+|\vec{\mathbf{b}}|)^2$ and θ be the acute angle between the diagonals OC and AB then the value of $\tan(\theta/2)$ is :

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}$

13. The vector $\overrightarrow{\mathbf{AB}}=3\hat{\mathbf{i}}+4\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{AC}}=5\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ are the sides of a triangle ABC. The length of the median through A is :

(a) $\sqrt{288}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{18}$

14. If $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$; $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$; $\vec{\mathbf{c}} = \lambda\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are linearly dependent vectors, then the number of possible values of λ is :

(a) 0 (b) 1 (c) 2 (d) More than 2

15. The scalar triple product $[\vec{\mathbf{a}}+\vec{\mathbf{b}}-\vec{\mathbf{c}}\ \ \vec{\mathbf{b}}+\vec{\mathbf{c}}-\vec{\mathbf{a}}\ \ \vec{\mathbf{c}}+\vec{\mathbf{a}}-\vec{\mathbf{b}}]$ is equal to :

(a) 0 (b) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ (c) $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ (d) $4[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

16. If $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are unit vectors then the vector defined as $\vec{\mathbf{V}} = (\hat{\mathbf{a}} \times \hat{\mathbf{b}}) \times (\hat{\mathbf{a}} + \hat{\mathbf{b}})$ is collinear to the vector :

(a) $\hat{\mathbf{a}} + \hat{\mathbf{b}}$ (b) $\hat{\mathbf{b}} - \hat{\mathbf{a}}$ (c) $2\hat{\mathbf{a}} - \hat{\mathbf{b}}$ (d) $\hat{\mathbf{a}} + 2\hat{\mathbf{b}}$

17. The sine of angle formed by the lateral face *ADC* and plane of the base *ABC* of the tetrahedron *ABCD*, where $A \equiv (3, -2, 1); B \equiv (3, 1, 5); C \equiv (4, 0, 3)$ and $D \equiv (1, 0, 0)$, is :

(a) $\dfrac{2}{\sqrt{29}}$ (b) $\dfrac{5}{\sqrt{29}}$ (c) $\dfrac{3\sqrt{3}}{\sqrt{29}}$ (d) $\dfrac{-2}{\sqrt{29}}$

18. Let $\vec{\mathbf{a}}_r = x_r\hat{\mathbf{i}} + y_r\hat{\mathbf{j}} + z_r\hat{\mathbf{k}}$, $r = 1, 2, 3$ be three mutually perpendicular unit vectors, then the value of $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$ is equal to :

(a) 0 (b) ± 1 (c) ± 2 (d) ± 4

19. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be three non-coplanar vectors and $\vec{\mathbf{r}}$ be any arbitrary vector, then the expression $(\vec{\mathbf{a}}\times \vec{\mathbf{b}}) \times (\vec{\mathbf{r}}\times \vec{\mathbf{c}}) + (\vec{\mathbf{b}}\times \vec{\mathbf{c}}) \times (\vec{\mathbf{r}}\times \vec{\mathbf{a}}) + (\vec{\mathbf{c}}\times \vec{\mathbf{a}}) \times (\vec{\mathbf{r}}\times \vec{\mathbf{b}})$ is always equal to :

(a) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]\,\vec{\mathbf{r}}$ (b) $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]\,\vec{\mathbf{r}}$ (c) $4[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]\,\vec{\mathbf{r}}$ (d) $\vec{\mathbf{0}}$

20. *E* and *F* are the interior points on the sides *BC* and *CD* of a parallelogram *ABCD*. Let $\overrightarrow{\mathbf{BE}} = 4\,\overrightarrow{\mathbf{EC}}$ and $\overrightarrow{\mathbf{CF}} = 4\,\overrightarrow{\mathbf{FD}}$. If the line *EF* meets the diagonal *AC* in *G*, then $\overrightarrow{\mathbf{AG}} = \lambda\,\overrightarrow{\mathbf{AC}}$, where λ is equal to :

(a) $\dfrac{1}{3}$ (b) $\dfrac{21}{25}$ (c) $\dfrac{7}{13}$ (d) $\dfrac{21}{5}$

21. If $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ are unit vectors and $\vec{\mathbf{c}}$ is such that $\vec{\mathbf{c}} = \vec{\mathbf{a}} \times \vec{\mathbf{c}} + \vec{\mathbf{b}}$, then the maximum value of $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ is :

(a) 1 (b) $\dfrac{1}{2}$ (c) 2 (d) $\dfrac{3}{2}$

22. Consider matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}; B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 2 & 3 \end{bmatrix}; C = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix}; D = \begin{bmatrix} 13 \\ 11 \\ 14 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that solutions of equation $AX = C$ and $BX = D$ represents two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ respectively in three dimensional space. If $P'Q'$ is the reflection of the line PQ in the plane $\prod : x + y + z = 9$, then the point which does not lie on $P'Q'$ is :

(a) (3, 4, 2) (b) (5, 3, 4) (c) (7, 2, 3) (d) (1, 5, 6)

23. The value of α for which point $M(\alpha\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$, lies in the plane containing three points $A(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$, $B(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$ and $C(3\hat{\mathbf{i}} - \hat{\mathbf{k}})$ is :

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

24. Q is the image of point $P(1, -2, 3)$ with respect to the plane $x - y + z = 7$. The distance of Q from the origin is :

(a) $\sqrt{\frac{70}{3}}$ (b) $\frac{1}{2}\sqrt{\frac{70}{3}}$ (c) $\sqrt{\frac{35}{3}}$ (d) $\sqrt{\frac{15}{2}}$

25. $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ and $\hat{\mathbf{a}} - \hat{\mathbf{b}}$ are unit vectors. The volume of the parallelopiped, formed with $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ and $\hat{\mathbf{a}} \times \hat{\mathbf{b}}$ as coterminous edges is :

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

26. A line passing through $P(3, 7, 1)$ and $R(2, 5, 7)$ meet the plane $3x + 2y + 11z - 9 = 0$ at Q. Then PQ is equal to :

(a) $\frac{5\sqrt{41}}{59}$ (b) $\frac{\sqrt{41}}{59}$ (c) $\frac{50\sqrt{41}}{59}$ (d) $\frac{25\sqrt{41}}{59}$

27. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are three non-zero non-coplanar vectors and $\vec{\mathbf{p}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} - 2\vec{\mathbf{c}}$; $\vec{\mathbf{q}} = 3\vec{\mathbf{a}} - 2\vec{\mathbf{b}} + \vec{\mathbf{c}}$ and $\vec{\mathbf{r}} = \vec{\mathbf{a}} - 4\vec{\mathbf{b}} + 2\vec{\mathbf{c}}$ are three vectors such that the volumes of the parallelopiped formed by $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{r}}$ as their coterminous edges are V_1 and V_2 respectively. Then $\frac{V_2}{V_1}$ is equal to :

(a) 10 (b) 15 (c) 20 (d) None of these

28. If the two lines represented by $x + ay = b$; $z + cy = d$ and $x = a'y + b'$; $z = c'y + d'$ be perpendicular to each other, then the value of $aa' + cc'$ is :

(a) 1 (b) 2 (c) 3 (d) 4

29. The distance between the line $\vec{\mathbf{r}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ and the plane $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ is :

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$ (c) $\frac{3}{10}$ (d) $\frac{10}{3}$

30. If $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = \vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$, where $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are any three vectors such that $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \neq 0$, $\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} \neq 0$, then $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ are :

(a) Inclined at an angle of $\frac{\pi}{3}$ (b) Inclined at an angle of $\frac{\pi}{6}$

(c) Perpendicular (d) Parallel

31. Let $\vec{r}$ be position vector of variable point in cartesian plane OXY such that $\vec{r}\cdot(\vec{r}+6\hat{j})=7$ cuts the co-ordinate axes at four distinct points, then the area of the quadrilateral formed by joining these points is :

(a) $4\sqrt{7}$ (b) $6\sqrt{7}$ (c) $7\sqrt{7}$ (d) $8\sqrt{7}$

32. If $|\vec{a}|=2, |\vec{b}|=5$ and $\vec{a}\cdot\vec{b}=0$, then $\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})))))$ is equal to :

(a) $64\vec{a}$ (b) $64\vec{b}$ (c) $-64\vec{a}$ (d) $-64\vec{b}$

33. If O (origin) is a point inside the triangle PQR such that $\vec{OP}+k_1\vec{OQ}+k_2\vec{OR}=0$, where k_1, k_2 are constants such that $\dfrac{\text{Area }(\Delta PQR)}{\text{Area }(\Delta OQR)}=4$, then the value of k_1+k_2 is :

(a) 2 (b) 3 (c) 4 (d) 5

34. Let PQ and QR be diagonals of adjacent faces of a rectangular box, with its centre at O. If $\angle QOR$, $\angle ROP$ and $\angle POQ$ are θ, ϕ and Ψ respectively then the value of '$\cos\theta+\cos\phi+\cos\Psi$' is :

(a) -2 (b) $-\sqrt{3}$ (c) -1 (d) 0

35. The value of $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{p} & \vec{b}\cdot\vec{p} & \vec{c}\cdot\vec{p} \\ \vec{a}\cdot\vec{q} & \vec{b}\cdot\vec{q} & \vec{c}\cdot\vec{q} \end{vmatrix}$ is equal to :

(a) $(\vec{p}\times\vec{q})[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$ (b) $2(\vec{p}\times\vec{q})[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$

(c) $4(\vec{p}\times\vec{q})[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$ (d) $(\vec{p}\times\vec{q})\sqrt{[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]}$

36. If $\vec{r}=a(\vec{m}\times\vec{n})+b(\vec{n}\times\vec{l})+c(\vec{l}\times\vec{m})$ and $[\vec{l}\ \vec{m}\ \vec{n}]=4$, find $\dfrac{a+b+c}{\vec{r}\cdot(\vec{l}+\vec{m}+\vec{n})}$:

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$ (c) 1 (d) 2

37. The volume of tetrahedron, for which three co-terminus edges are $\vec{a}$, $\vec{b}$ and $\vec{c}$, is k units. Then, the volume of a parallelepiped formed by $\vec{a}-\vec{b}$, $\vec{b}+2\vec{c}$ and $3\vec{a}-\vec{c}$ is :

(a) $6k$ (b) $7k$ (c) $30k$ (d) $42k$

38. The equation of a plane passing through the line of intersection of the planes :
$x+2y+z-10=0$ and $3x+y-z=5$ and passing through the origin is :

(a) $5x+3z=0$ (b) $5x-3z=0$

(c) $5x+4y+3z=0$ (d) $5x-4y+3z=0$

39. Find the locus of a point whose distance from x -axis is twice the distance from the point $(1, -1, 2)$:

(a) $y^2 + 2x - 2y - 4z + 6 = 0$
(b) $x^2 + 2x - 2y - 4z + 6 = 0$
(c) $x^2 - 2x + 2y - 4z + 6 = 0$
(d) $z^2 - 2x + 2y - 4z + 6 = 0$

Answers

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(b)	**5.**	(a)	**6.**	(c)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(d)
11.	(d)	**12.**	(d)	**13.**	(c)	**14.**	(c)	**15.**	(d)	**16.**	(b)	**17.**	(b)	**18.**	(b)	**19.**	(b)	**20.**	(b)
21.	(b)	**22.**	(a)	**23.**	(b)	**24.**	(a)	**25.**	(d)	**26.**	(d)	**27.**	(b)	**28.**	(a)	**29.**	(b)	**30.**	(d)
31.	(d)	**32.**	(d)	**33.**	(b)	**34.**	(c)	**35.**	(d)	**36.**	(a)	**37.**	(d)	**38.**	(b)	**39.**	(c)		

Exercise-2 : One or More than One Answer is/are Correct

1. If equation of three lines are :

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3};\frac{x}{2}=\frac{y}{1}=\frac{z}{3} \text{ and } \frac{x-1}{1}=\frac{2-y}{1}=\frac{z-3}{0}, \text{ then}$$

which of the following statement(s) is/are correct ?

(a) Triangle formed by the line is equilateral

(b) Triangle formed by the lines is isosceles

(c) Equation of the plane containing the lines is $x+y=z$

(d) Area of the triangle formed by the lines is $\frac{3\sqrt{3}}{2}$

2. If $\vec{\mathbf{a}}=\hat{\mathbf{i}}+6\hat{\mathbf{j}}+3\hat{\mathbf{k}}$; $\vec{\mathbf{b}}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\vec{\mathbf{c}}=(\alpha+1)\hat{\mathbf{i}}+(\beta-1)\hat{\mathbf{j}}+\hat{\mathbf{k}}$ are linearly dependent vectors and $|\vec{\mathbf{c}}|=\sqrt{6}$; then the possible value(s) of $(\alpha+\beta)$ can be :

(a) 1 (b) 2 (c) 3 (d) 4

3. Consider the lines :

$L_1:\frac{x-2}{1}=\frac{y-1}{7}=\frac{z+2}{-5}$

$L_2: x-4=y+3=-z$

Then which of the following is/are correct ?

(a) Point of intersection of L_1 and L_2 is $(1,-6,3)$

(b) Equation of plane containing L_1 and L_2 is $x+2y+3z+2=0$

(c) Acute angle between L_1 and L_2 is $\cot^{-1}\left(\frac{13}{15}\right)$

(d) Equation of plane containing L_1 and L_2 is $x+2y+2z+3=0$

4. Let $\hat{\mathbf{a}},\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ be three unit vectors such that $\hat{\mathbf{a}}=\hat{\mathbf{b}}+(\hat{\mathbf{b}}\times\hat{\mathbf{c}})$, then the possible value(s) of $|\hat{\mathbf{a}}+\hat{\mathbf{b}}+\hat{\mathbf{c}}|^2$ can be :

(a) 1 (b) 4 (c) 16 (d) 9

5. The value(s) of μ for which the straight lines $\vec{\mathbf{r}}=3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-4\hat{\mathbf{k}}+\lambda_1(\hat{\mathbf{i}}-\hat{\mathbf{j}}+\mu\hat{\mathbf{k}})$ and $\vec{\mathbf{r}}=5\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}+\lambda_2(\hat{\mathbf{i}}+\mu\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ are coplanar is/are :

(a) $\frac{5+\sqrt{33}}{4}$ (b) $\frac{-5+\sqrt{33}}{4}$ (c) $\frac{5-\sqrt{33}}{4}$ (d) $\frac{-5-\sqrt{33}}{4}$

6. If $\hat{\mathbf{i}}\times[(\vec{\mathbf{a}}-\hat{\mathbf{j}})\times\hat{\mathbf{i}}]+\hat{\mathbf{j}}\times[(\vec{\mathbf{a}}-\hat{\mathbf{k}})\times\hat{\mathbf{j}}]+\hat{\mathbf{k}}\times[(\vec{\mathbf{a}}-\hat{\mathbf{i}})\times\hat{\mathbf{k}}]=0$ and $\vec{\mathbf{a}}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$, then :

(a) $x+y=1$ (b) $y+z=\frac{1}{2}$ (c) $x+z=1$ (d) None of these

7. The value of expression $[\vec{a}\times\vec{b}\ \ \vec{c}\times\vec{d}\ \ \vec{e}\times\vec{f}]$ is equal to :

(a) $[\vec{a}\,\vec{b}\,\vec{d}][\vec{c}\,\vec{e}\,\vec{f}]-[\vec{a}\,\vec{b}\,\vec{c}][\vec{d}\,\vec{e}\,\vec{f}]$
(b) $[\vec{a}\,\vec{b}\,\vec{e}][\vec{f}\,\vec{c}\,\vec{d}]-[\vec{a}\,\vec{b}\,\vec{f}][\vec{e}\,\vec{c}\,\vec{d}]$
(c) $[\vec{c}\,\vec{d}\,\vec{a}][\vec{b}\,\vec{e}\,\vec{f}]-[\vec{c}\,\vec{d}\,\vec{b}][\vec{a}\,\vec{e}\,\vec{f}]$
(d) $[\vec{b}\,\vec{c}\,\vec{d}][\vec{a}\,\vec{e}\,\vec{f}]-[\vec{b}\,\vec{c}\,\vec{f}][\vec{a}\,\vec{e}\,\vec{d}]$

8. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the points A, B, C and D respectively in three dimensional space and satisfy the relation $3\vec{a}-2\vec{b}+\vec{c}-2\vec{d}=0$, then :

(a) A, B, C and D are coplanar
(b) The line joining the points B and D divides the line joining the point A and C in the ratio of $2:1$
(c) The line joining the points A and C divides the line joining the points B and D in the ratio of $1:1$
(d) The four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are linearly dependent.

9. If $OABC$ is a tetrahedron with equal edges and $\hat{p}, \hat{q}, \hat{r}$ are unit vectors along bisectors of $\overrightarrow{OA}, \overrightarrow{OB} : \overrightarrow{OB}, \overrightarrow{OC} : \overrightarrow{OC}, \overrightarrow{OA}$ respectively and $\hat{a}=\frac{\overrightarrow{OA}}{|\overrightarrow{OA}|}$, $\vec{b}=\frac{\overrightarrow{OB}}{|\overrightarrow{OB}|}$, $\vec{c}=\frac{\overrightarrow{OC}}{|\overrightarrow{OC}|}$, then :

(a) $\dfrac{[\hat{a}\,\hat{b}\,\hat{c}]}{[\hat{p}\,\hat{q}\,\hat{r}]}=\dfrac{3\sqrt{3}}{2}$
(b) $\dfrac{[\hat{a}+\hat{b}\ \ \hat{b}+\hat{c}\ \ \hat{c}+\hat{a}]}{[\hat{p}+\hat{q}\ \ \hat{q}+\hat{r}\ \ \hat{r}+\hat{p}]}=\dfrac{3\sqrt{3}}{4}$
(c) $\dfrac{[\hat{a}+\hat{b}\ \ \hat{b}+\hat{c}\ \ \hat{c}+\hat{a}]}{[\hat{p}\,\hat{q}\,\hat{r}]}=\dfrac{3\sqrt{3}}{2}$
(d) $\dfrac{[\hat{a}\,\hat{b}\,\hat{c}]}{[\hat{p}+\hat{q}\ \ \hat{q}+\hat{r}\ \ \hat{r}+\hat{p}]}=\dfrac{3\sqrt{3}}{4}$

10. Let $\hat{a}$ and $\hat{c}$ are unit vectors and $|\vec{b}|=4$. If the angle between $\hat{a}$ and $\hat{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$; and $\vec{b}-2\hat{c}=\lambda\hat{a}$, then the value of λ can be :

(a) 2
(b) -3
(c) 3
(d) -4

11. Consider the line $L_1: x=y=z$ and the line $L_2: 2x+y+z-1=0=3x+y+2z-2$, then :

(a) The shortest distance between the two lines is $\frac{1}{\sqrt{2}}$
(b) The shortest distance between the two lines is $\sqrt{2}$
(c) Plane containing the line L_2 and parallel to line L_1 is $z-x+1=0$
(d) Perpendicular distance of origin from plane containing line L_2 and parallel to line L_1 is $\frac{1}{\sqrt{2}}$

12. Let $\vec{r} = \sin x(\vec{a} \times \vec{b}) + \cos y(\vec{b} \times \vec{c}) + 2(\vec{c} \times \vec{a})$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors. It is given that $\vec{r}$ is perpendicular to $\vec{a} + \vec{b} + \vec{c}$. The possible value(s) of $x^2 + y^2$ is/are :

(a) π^2 (b) $\frac{5\pi^2}{4}$

(c) $\frac{35\pi^2}{4}$ (d) $\frac{37\pi^2}{4}$

13. If $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = h\vec{a} + k\vec{b} = r\vec{c} + s\vec{d}$, where $\vec{a}, \vec{b}$ are non-collinear and $\vec{c}, \vec{d}$ are also non-collinear then :

(a) $h = [\vec{b}\ \vec{c}\ \vec{d}]$ (b) $k = [\vec{a}\ \vec{c}\ \vec{d}]$

(c) $r = [\vec{a}\ \vec{b}\ \vec{d}]$ (d) $s = -[\vec{a}\ \vec{b}\ \vec{c}]$

14. Let a be a real number and $\vec{\alpha} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$, $\vec{\beta} = 2\hat{\mathbf{i}} + a\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$, $\vec{\gamma} = 12\hat{\mathbf{i}} + 20\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ be three vectors, then $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are linearly independent for :

(a) $a > 0$ (b) $a < 0$

(c) $a = 0$ (d) No value of a

15. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are $A(1, 0, 1)$; $B(2, 0, 0)$ and $C(0, 1, 0)$, then the position vectors of the vertex A_1 can be :

(a) (2, 2, 2) (b) (0, 2, 0)

(c) (0, –2, 2) (d) (0, –2, 0)

16. If $\vec{a} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, $\vec{b} = y\hat{\mathbf{i}} + z\hat{\mathbf{j}} + x\hat{\mathbf{k}}$, and $\vec{c} = z\hat{\mathbf{i}} + x\hat{\mathbf{j}} + y\hat{\mathbf{k}}$, then $\vec{a} \times (\vec{b} \times \vec{c})$ is :

(a) Parallel to $(y - z)\hat{\mathbf{i}} + (z - x)\hat{\mathbf{j}} + (x - y)\hat{\mathbf{k}}$

(b) Orthogonal to $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

(c) Orthogonal to $(y + z)\hat{\mathbf{i}} + (z + x)\hat{\mathbf{j}} + (x + y)\hat{\mathbf{k}}$,

(d) Orthogonal to $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

17. If a line has a vector equation, $\vec{r} = 2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \lambda(\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ then which of the following statements holds good ?

(a) the line is parallel to $2\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

(b) the line passes through the point $3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

(c) the line passes through the point $\hat{\mathbf{i}} + 9\hat{\mathbf{j}}$

(d) the line is parallel to xy plane

18. Let M, N, P and Q be the mid points of the edges AB, CD, AC and BD respectively of the tetrahedron $ABCD$. Further, MN is perpendicular to both AB and CD and PQ is perpendicular to both AC and BD. Then which of the following is/are correct :

(a) $AB = CD$ (b) $BC = DA$

(c) $AC = BD$ (d) $AN = BN$

19. The solution vectors $\vec{\mathbf{r}}$ of the equation $\vec{\mathbf{r}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{r}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} + \hat{\mathbf{i}}$ represent two straight lines which are :

(a) Intersecting (b) Non coplanar (c) Coplanar (d) Non intersecting

20. Which of the following statement(s) is/are incorrect ?

(a) The lines $\frac{x-4}{-3} = \frac{y+6}{-1} = \frac{z+6}{-1}$ and $\frac{x-1}{-1} = \frac{y-2}{-2} = \frac{z-3}{2}$ are orthogonal

(b) The planes $3x - 2y - 4z = 3$ and the plane $x - y - z = 3$ are orthogonal

(c) The function $f(x) = ln(e^{-2} + e^{x})$ is monotonic increasing $\forall\ x \in R$

(d) If g is the inverse of the function, $f(x) = \ln(e^{-2} + e^{x})$ then $g(x) = \ln(e^{x} - e^{-2})$

21. The lines with vector equations are; $\vec{\mathbf{r}}_1 = -3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \lambda(-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ and $\vec{\mathbf{r}}_2 = -2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + \mu(-4\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ are such that :

(a) they are coplanar

(b) they do not intersect

(c) they are skew

(d) the angle between them is $\tan^{-1}(3/7)$

Answers

1.	(b, c, d)	**2.**	(a, c)	**3.**	(a, b, c)	**4.**	(a, d)	**5.**	(a, c)	**6.**	(a, c)
7.	(a, b, c)	**8.**	(a, c, d)	**9.**	(a, d)	**10.**	(c, d)	**11.**	(a, d)	**12.**	(b, d)
13.	(b, c, d)	**14.**	(a, b, c)	**15.**	(a, d)	**16.**	(a, b, c, d)	**17.**	(b, c, d)	**18.**	(a, b, c, d)
19.	(b, d)	**20.**	(a, b)	**21.**	(b, c, d)						

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

The vertices of ΔABC are $A(2,0,0), B(0,1,0), C(0,0,2)$. Its orthocentre is H and circumcentre is S. P is a point equidistant from A, B, C and the origin O.

1. The z-coordinate of H is :

(a) 1 (b) 1/2 (c) 1/6 (d) 1/3

2. The y-coordinate of S is :

(a) 5/6 (b) 1/3 (c) 1/6 (d) 1/2

3. PA is equal to :

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

Paragraph for Question Nos. 4 to 6

Consider a plane $\pi: \vec{\mathbf{r}}\cdot\vec{\mathbf{n}} = d$ (where $\vec{\mathbf{n}}$ is not a unit vector). There are two points $A(\vec{\mathbf{a}})$ and $B(\vec{\mathbf{b}})$ lying on the same side of the plane.

4. If foot of perpendicular from A and B to the plane π are P and Q respectively, then length of PQ be:

(a) $\dfrac{|(\vec{\mathbf{b}}-\vec{\mathbf{a}})\cdot\vec{\mathbf{n}}|}{|\vec{\mathbf{n}}|}$ (b) $|(\vec{\mathbf{b}}-\vec{\mathbf{a}})\cdot\vec{\mathbf{n}}|$ (c) $\dfrac{|(\vec{\mathbf{b}}-\vec{\mathbf{a}})\times\vec{\mathbf{n}}|}{|\vec{\mathbf{n}}|}$ (d) $|(\vec{\mathbf{b}}-\vec{\mathbf{a}})\times\vec{\mathbf{n}}|$

5. Reflection of $A(\vec{\mathbf{a}})$ in the plane π has the position vector :

(a) $\vec{\mathbf{a}}+\dfrac{2}{(\vec{\mathbf{n}})^2}(d-\vec{\mathbf{a}}\cdot\vec{\mathbf{n}})\vec{\mathbf{n}}$ (b) $\vec{\mathbf{a}}-\dfrac{1}{(\vec{\mathbf{n}})^2}(d-\vec{\mathbf{a}}\cdot\vec{\mathbf{n}})\vec{\mathbf{n}}$

(c) $\vec{\mathbf{a}}+\dfrac{2}{(\vec{\mathbf{n}})^2}(d+\vec{\mathbf{a}}\cdot\vec{\mathbf{n}})\vec{\mathbf{n}}$ (d) $\vec{\mathbf{a}}+\dfrac{2}{(\vec{\mathbf{n}})^2}\vec{\mathbf{n}}$

6. If a plane π_1 is drawn from the point $A(\vec{\mathbf{a}})$ and another plane π_2 is drawn from point $B(\vec{\mathbf{b}})$ parallel to π, then the distance between the planes π_1 and π_2 is :

(a) $\dfrac{|(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot\vec{\mathbf{n}}|}{|\vec{\mathbf{n}}|}$ (b) $|(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot\vec{\mathbf{n}}|$ (c) $|(\vec{\mathbf{a}}-\vec{\mathbf{b}})\times\vec{\mathbf{n}}|$ (d) $\dfrac{|(\vec{\mathbf{a}}-\vec{\mathbf{b}})\times\vec{\mathbf{n}}|}{|\vec{\mathbf{n}}|}$

Paragraph for Question Nos. 7 to 9

Consider a plane $\prod : \vec{\mathbf{r}} \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 5$, a line $L_1 : \vec{\mathbf{r}} = (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + \lambda(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ and a point $A(3, -4, 1)$. L_2 is a line passing through A intersecting L_1 and parallel to plane Π.

7. Equation of L_2 is :

(a) $\vec{\mathbf{r}} = (1 + \lambda)\hat{\mathbf{i}} + (2 - 3\lambda)\hat{\mathbf{j}} + (1 - \lambda)\hat{\mathbf{k}}; \lambda \in R$

(b) $\vec{\mathbf{r}} = (3 + \lambda)\hat{\mathbf{i}} - (4 - 2\lambda)\hat{\mathbf{j}} + (1 + 3\lambda)\hat{\mathbf{k}}; \lambda \in R$

(c) $\vec{\mathbf{r}} = (3 + \lambda)\hat{\mathbf{i}} - (4 + 3\lambda)\hat{\mathbf{j}} + (1 - \lambda)\hat{\mathbf{k}}; \lambda \in R$

(d) None of the above

8. Plane containing L_1 and L_2 is :

(a) parallel to yz-plane
(b) parallel to x-axis
(c) parallel to y-axis
(d) passing through origin

9. Line L_1 intersects plane Π at Q and xy-plane at R the volume of tetrahedron $OAQR$ is : (where 'O' is origin)

(a) 0
(b) $\frac{14}{3}$
(c) $\frac{3}{7}$
(d) $\frac{7}{3}$

Paragraph for Question Nos. 10 to 11

Consider three planes :

$$2x + py + 6z = 8; x + 2y + qz = 5 \text{ and } x + y + 3z = 4$$

10. Three planes intersect at a point if :

(a) $p = 2, q \neq 3$
(b) $p \neq 2, q \neq 3$
(c) $p \neq 2, q = 3$
(d) $p = 2, q = 3$

11. Three planes do not have any common point of intersection if :

(a) $p = 2, q \neq 3$
(b) $p \neq 2, q \neq 3$
(c) $p \neq 2, q = 3$
(d) $p = 2, q = 3$

Paragraph for Question Nos. 12 to 14

The points A, B and C with position vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ respectively lie on a circle centered at origin O. Let G and E be the centroid of ΔABC and ΔACD respectively where D is mid point of AB.

12. If OE and CD are mutually perpendicular, then which of the following will be necessarily true ?

(a) $|\vec{\mathbf{b}} - \vec{\mathbf{a}}| = |\vec{\mathbf{c}} - \vec{\mathbf{a}}|$
(b) $|\vec{\mathbf{b}} - \vec{\mathbf{a}}| = |\vec{\mathbf{b}} - \vec{\mathbf{c}}|$
(c) $|\vec{\mathbf{c}} - \vec{\mathbf{a}}| = |\vec{\mathbf{c}} - \vec{\mathbf{b}}|$
(d) $|\vec{\mathbf{b}} - \vec{\mathbf{a}}| = |\vec{\mathbf{c}} - \vec{\mathbf{a}}| = |\vec{\mathbf{b}} - \vec{\mathbf{c}}|$

13. If GE and CD are mutually perpendicular, then orthocenter of ΔABC must lie on :

(a) median through A (b) median through C

(c) angle bisector through A (d) angle bisector through B

14. If $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AB} \times \overrightarrow{AC}] = \lambda[\overrightarrow{AE}\ \overrightarrow{AG}\ \overrightarrow{AE} \times \overrightarrow{AG}]$, then the value of λ is :

(a) -18 (b) 18 (c) -324 (d) 324

Paragraph for Question Nos. 15 to 16

Consider a tetrahedron D —ABC with position vectors if its angular points as

$$A\ (1, 1, 1);\ B(1, 2, 3);\ C(1, 1, 2)$$

and centre of tetrahedron $\left(\frac{3}{2}, \frac{3}{4}, 2\right)$.

15. Shortest distance between the skew lines AB and CD :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

16. If N be the foot of the perpendicular from point D on the plane face ABC then the position vector of N are :

(a) $(-1, 1, 2)$ (b) $(1, -1, 2)$ (c) $(1, 1, -2)$ (d) $(-1, -1, 2)$

Paragraph for Question Nos. 17 to 18

In a triangle AOB, R and Q are the points on the side OB and AB respectively such that $3OR = 2RB$ and $2AQ = 3QB$. Let OQ and AR intersect at the point P (where O is origin).

17. If the point P divides OQ in the ratio of $\mu : 1$, then μ is :

(a) $\frac{2}{19}$ (b) $\frac{2}{17}$ (c) $\frac{2}{15}$ (d) $\frac{10}{9}$

18. If the ratio of area of quadrilateral $PQBR$ and area of ΔOPA is $\frac{\alpha}{\beta}$ then $(\beta - \alpha)$ is (where α and β are coprime numbers) :

(a) 1 (b) 9 (c) 7 (d) 0

Answers

1.	(d)	**2.**	(c)	**3.**	(d)	**4.**	(c)	**5.**	(a)	**6.**	(a)	**7.**	(c)	**8.**	(b)	**9.**	(d)	**10.**	(b)
11.	(c)	**12.**	(a)	**13.**	(b)	**14.**	(d)	**15.**	(b)	**16.**	(b)	**17.**	(d)	**18.**	(d)				

Exercise-4 : Matching Type Problems

1.

	Column-I		Column-II
(A)	Lines $\frac{x-1}{-2}=\frac{y+2}{3}=\frac{z}{-1}$ and $\vec{\mathbf{r}}=(3\hat{i}-\hat{j}+\hat{k})+t(\hat{i}+\hat{j}+\hat{k})$ are	**(P)**	Intersecting
(B)	Lines $\frac{x+5}{1}=\frac{y-3}{7}=\frac{z+3}{3}$ and $x-y+2z-4=0=2x+y-3z+5$ are	**(Q)**	Perpendicular
(C)	Lines $(x=t-3,\ y=-2t+1,\ z=-3t-2)$ and $\vec{\mathbf{r}}=(t+1)\hat{i}+(2t+3)\hat{j}+(-t-9)\hat{k}$ are	**(R)**	Parallel
(D)	Lines $\vec{\mathbf{r}}=(\hat{i}+3\hat{j}-\hat{k})+t(2\hat{i}-\hat{j}-\hat{k})$ and $\vec{\mathbf{r}}=(-\hat{i}-2\hat{j}+5\hat{k})+s\left(\hat{i}-2\hat{j}+\frac{3}{4}\hat{k}\right)$ are	**(S)**	Skew
		(T)	Coincident

2.

	Column-I		Column-II
(A)	If $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are three mutually perpendicular vectors where $\lvert\vec{\mathbf{a}}\rvert=\lvert\vec{\mathbf{b}}\rvert=2$, $\lvert\vec{\mathbf{c}}\rvert=1$, then $[\vec{\mathbf{a}}\times\vec{\mathbf{b}}\ \ \vec{\mathbf{b}}\times\vec{\mathbf{c}}\ \ \vec{\mathbf{c}}\times\vec{\mathbf{a}}]$ is	**(P)**	-12
(B)	If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two unit vectors inclined at $\frac{\pi}{3}$, then $16[\vec{\mathbf{a}}\ \ \vec{\mathbf{b}}+(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\ \ \vec{\mathbf{b}}]$ is	**(Q)**	0
(C)	If $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are orthogonal unit vectors and $\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{a}}$ then $[\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}\ \ \vec{\mathbf{a}}+\vec{\mathbf{b}}\ \ \vec{\mathbf{b}}+\vec{\mathbf{c}}]$ is	**(R)**	16
(D)	If $[\vec{\mathbf{x}}\ \vec{\mathbf{y}}\ \vec{\mathbf{a}}]=[\vec{\mathbf{x}}\ \vec{\mathbf{y}}\ \vec{\mathbf{b}}]=[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]=0$, each vector being a non-zero vector, then $[\vec{\mathbf{x}}\ \vec{\mathbf{y}}\ \vec{\mathbf{c}}]$ is	**(S)**	1
		(T)	4

3.

	Column-I		Column-II
(A)	The number of real roots of equation $2^x + 3^x + 4^x - 9^x = 0$ is λ, then $\lambda^2 + 7$ is divisible by	**(P)**	2
(B)	Let ABC be a triangle whose centroid is G, orthocenter is H and circumcentre is the origin 'O'. If D is any point in the plane of the triangle such that not three of O, A, B, C and D are collinear satisfying the relation $\overrightarrow{AD} + \overrightarrow{BD} + \overrightarrow{CH} + 3\overrightarrow{HG} = \lambda\overrightarrow{HD}$, then $\lambda + 4$ is divisible by	**(Q)**	3
(C)	If A (adj A) $= \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $5\|A\| - 2$ is divisible by	**(R)**	4
(D)	$\vec{a}, \vec{b}, \vec{c}$ are three unit vector such that $\vec{a} + \vec{b} = \sqrt{2}\,\vec{c}$, then $\|6\vec{a} - 8\vec{b}\|$ is divisible by	**(S)**	6
		(T)	10

Answers

1. A → Q, S; B → R; C → P, Q; D → P

2. A → R; B → P; C → S; D → Q

3. A → P, R; B → P, Q, S ; C → P, Q, R, S; D → P, T

Exercise-5 : Subjective Type Problems

1. A straight line L intersects perpendicularly both the lines :
$$\frac{x+2}{2}=\frac{y+6}{3}=\frac{z-34}{-10} \text{ and } \frac{x+6}{4}=\frac{y-7}{-3}=\frac{z-7}{-2},$$
then the square of perpendicular distance of origin from L is

2. If $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ are non-coplanar unit vectors such that $[\hat{\mathbf{a}}\ \hat{\mathbf{b}}\ \hat{\mathbf{c}}] = [\hat{\mathbf{b}} \times \hat{\mathbf{c}}\ \ \hat{\mathbf{c}} \times \hat{\mathbf{a}}\ \ \hat{\mathbf{a}} \times \hat{\mathbf{b}}]$, then find the projection of $\hat{\mathbf{b}} + \hat{\mathbf{c}}$ on $\hat{\mathbf{a}} \times \hat{\mathbf{b}}$.

3. Let OA, OB, OC be coterminous edges of a cuboid. If l, m, n be the shortest distances between the sides OA, OB, OC and their respective skew body diagonals to them, respectively, then find
$$\frac{\left(\frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}\right)}{\left(\frac{1}{OA^2}+\frac{1}{OB^2}+\frac{1}{OC^2}\right)}.$$

4. Let $OABC$ be a tetrahedron whose edges are of unit length. If $\overrightarrow{\mathbf{OA}} = \vec{\mathbf{a}}$, $\overrightarrow{\mathbf{OB}} = \vec{\mathbf{b}}$ and d $\overrightarrow{\mathbf{OC}} = \alpha(\vec{\mathbf{a}}+\vec{\mathbf{b}}) + \beta(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$, then $(\alpha\beta)^2 = \frac{p}{q}$ where p and q are relatively prime to each other. Find the value of $\left[\frac{q}{2p}\right]$ where [·] denotes greatest integer function.

5. Let $\vec{\mathbf{v}}_0$ be a fixed vector and $\vec{\mathbf{v}}_0 = \begin{bmatrix}1\\0\end{bmatrix}$. Then for $n \ge 0$ a sequence is defined $\vec{\mathbf{v}}_{n+1} = \vec{\mathbf{v}}_n + \left(\frac{1}{2}\right)^{n+1}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}^{n+1}\vec{\mathbf{v}}_0$ then $\lim\limits_{n\to\infty}\vec{\mathbf{v}}_n = \begin{bmatrix}\alpha\\\beta\end{bmatrix}$. Find $\frac{\alpha}{\beta}$.

6. If A is the matrix $\begin{bmatrix}1 & -3\\-1 & 1\end{bmatrix}$, then $A - \frac{1}{3}A^2 + \frac{1}{9}A^3 \ldots\ldots + \left(-\frac{1}{3}\right)^n A^{n+1} + \ldots\ldots\infty = \frac{3}{13}\begin{bmatrix}1 & a\\b & 1\end{bmatrix}$. Find $\left|\frac{a}{b}\right|$.

7. A sequence of 2×2 matrices $\{M_n\}$ is defined as follows $M_n = \begin{bmatrix}\dfrac{1}{(2n+1)!} & \dfrac{1}{(2n+2)!}\\ \displaystyle\sum_{k=0}^{n}\frac{(2n+2)!}{(2k+2)!} & \displaystyle\sum_{k=0}^{n}\frac{(2n+1)!}{(2k+1)!}\end{bmatrix}$

then $\lim\limits_{n\to\infty}$ det. $(M_n) = \lambda - e^{-1}$. Find λ.

8. Let $|\vec{\mathbf{a}}| = 1$, $|\vec{\mathbf{b}}| = 1$ and $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3}$. If $\vec{\mathbf{c}}$ be a vector such that $\vec{\mathbf{c}} = \vec{\mathbf{a}} + 2\vec{\mathbf{b}} - 3(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$ and $p = |(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}|$, then find $[p^2]$. (where [ ] represents greatest integer function).

9. Let $\vec{\mathbf{r}} = (\vec{\mathbf{a}} \times \vec{\mathbf{b}})\sin x + (\vec{\mathbf{b}} \times \vec{\mathbf{c}})\cos y + 2(\vec{\mathbf{c}} \times \vec{\mathbf{a}})$, where $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are non-zero and non-coplanar vectors. If $\vec{\mathbf{r}}$ is orthogonal to $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}$, then find the minimum value of $\frac{4}{\pi^2}(x^2 + y^2)$.

10. The plane denoted by $\Pi_1 : 4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $\Pi_2 : 5x + 3y + 10z = 25$. If the plane in its new position be denoted by Π, and the distance of this plane from the origin is $\sqrt{53\,k}$ where $k \in N$, then find k.

11. $ABCD$ is a regular tetrahedron, A is the origin and B lies on x-axis. ABC lies in the xy-plane $|\vec{\mathbf{AB}}| = 2$. Under these conditions, the number of possible tetrahedrons is :

12. A, B, C, D are four points in the space and satisfy $|\vec{\mathbf{AB}}| = 3$, $|\vec{\mathbf{BC}}| = 7$, $|\vec{\mathbf{CD}}| = 11$ and $|\vec{\mathbf{DA}}| = 9$. Then find the value of $\vec{\mathbf{AC}} \cdot \vec{\mathbf{BD}}$.

13. Let $OABC$ be a regular tetrahedron of edge length unity. Its volume be V and $6V = \sqrt{p/q}$ where p and q are relatively prime. The find the value of $(p + q)$:

14. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are non zero, non collinear vectors and $\vec{\mathbf{a_1}} = \lambda\vec{\mathbf{a}} + 3\vec{\mathbf{b}}$; $\vec{\mathbf{b_1}} = 2\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}$; $\vec{\mathbf{c_1}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$. Find the sum of all possible real values of λ so that points A_1, B_1, C_1 whose position vectors are $\vec{\mathbf{a_1}}, \vec{\mathbf{b_1}}, \vec{\mathbf{c_1}}$ respectively are collinear is equal to .

15. Let P and Q are two points on curve $y = \log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) + \log_2\sqrt{4x^2 - 4x + 1}$ and P is also on $x^2 + y^2 = 10$. Q lies inside the given circle such that its abscissa is integer. Find the smallest possible value of $\vec{\mathbf{OP}} \cdot \vec{\mathbf{OQ}}$ where 'O' being origin.

16. In above problem find the largest possible value of $|\vec{\mathbf{PQ}}|$.

17. If $a, b, c, l, m, n \in R - \{0\}$ such that $al + bm + cn = 0$, $bl + cm + an = 0$, $cl + am + bn = 0$. If a, b, c are distinct and $f(x) = ax^3 + bx^2 + cx + 2$. Find $f(1)$:

18. Let $\vec{\mu}$ and $\vec{\nu}$ are unit vectors and $\vec{\omega}$ is vector such that $\vec{\mu} \times \vec{\nu} + \vec{\mu} = \vec{\omega}$ and $\vec{\omega} \times \vec{\mu} = \vec{\nu}$. The find the value of $[\vec{\mu}\ \vec{\nu}\ \vec{\omega}]$.

Answers

1.	5	**2.**	1	**3.**	2	**4.**	5	**5.**	2	**6.**	3	**7.**	1
8.	5	**9.**	5	**10.**	4	**11.**	8	**12.**	0	**13.**	0	**14.**	2
15.	4	**16.**	2	**17.**	2	**18.**	1						